

On the Asymptotic Efficiency of the Multisample Location-Scale Rank Tests and Their Adjustment for Ties

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Abstract: Explicit formulas for the non-centrality parameters of the limiting chi-square distribution of proposed multisample rank based test statistics, aimed at testing the hypothesis of the simultaneous equality of location and scale parameters of underlying populations, are obtained by means of a general assertion concerning the location-scale test statistics. The finite sample behaviour of the proposed tests is discussed and illustrated by simulation estimates of the rejection probabilities. A modification for ties of a class of multisample location and scale test statistics, based on ranks and including the proposed test statistics, is presented. It is shown that under the validity of the null hypothesis these modified test statistics are asymptotically chi-square distributed provided that the score generating functions fulfill the imposed regularity conditions. An essential assumption is that the matrix, appearing in these conditions, is regular. Conditions sufficient for the validity of this assumption are also included.

Keywords: multisample rank test for location and scale; asymptotic non-centrality parameter; Pitman–Noether efficiency; adjustment for ties;

AMS Subject Classification: 62G10; 62G20;

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