

H_2 -optimal Rejection with Preview: Geometric Constraints and Dynamic Feedforward Solutions via Spectral Factorization

Elena Zattoni

Abstract: In this work, a feedforward dynamic controller is devised in order to achieve H_2 -optimal rejection of signals known with finite preview, in discrete-time systems. The feedforward approach requires plant stability and, more generally, robustness with respect to parameter uncertainties. On standard assumptions, those properties can be guaranteed by output dynamic feedback, while dynamic feedforward is specifically aimed at taking advantage of the available preview of the signals to be rejected, in compliance with a two-degree-of-freedom control structure. The geometric constraints which prevent achievement of perfect rejection are first discussed. Then, the procedure for the design of the feedforward dynamic compensator is presented. Since the approach proposed in this work is based on spectral factorization via Riccati equation of a real rational matrix function directly related to the original to-be-controlled system, the delays introduced to model the preview of the signals to be rejected do not affect the computational burden intrinsic in the solution of the appropriate algebraic Riccati equation. A numerical example helps to illustrate the geometric constraints and the procedure for the design of the feedforward dynamic unit.

Keywords: optimal design; geometric approach; linear systems; discrete-time systems;

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References

- [1] G. Basile and G. Marro: Controlled and Conditioned Invariants in Linear System Theory. Prentice Hall, Englewood Cliffs, NJ 1992.
- [2] S. Bittanti, A. J. Laub, and J. C. Willems (eds.): The Riccati Equation. Springer-Verlag, Berlin–Heidelberg 1991.
- [3] J. Chen, Z. Ren, S. Hara, and L. Qiu: Optimal tracking performance: Preview control and exponential signals. IEEE Trans. Automat. Control 46 (2001), 10, 1647–1653.
- [4] D. J. Clements: Rational spectral factorization using state-space methods. Systems Control Lett. 20 (1993), 335–343.

- [5] P. Colaneri, J. C. Geromel, and A. Locatelli: Control Theory and Design: An RH_2 and RH_∞ Viewpoint. Academic Press, London 1997.
- [6] M. J. Grimble: Polynomial matrix solution to the standard H_2 -optimal control problem. *Internat. J. Systems Sci.* 22 (1991), 5, 793–806.
- [7] D. N. Hoover, R. Longchamp, and J. Rosenthal: Two-degree-of-freedom ℓ_2 -optimal tracking with preview. *Automatica* 40 (2004), 1, 155–162.
- [8] K. J. Hunt, M. Šebek, and V. Kučera: Polynomial solution of the standard multivariable H_2 -optimal control problem. *IEEE Trans. Automat. Control* 39 (1994), 7, 1502–1507.
- [9] H. Imai, M. Shinozuka, T. Yamaki, D. Li, and M. Kuwana: Disturbance decoupling by feedforward and preview control. *ASME J. Dynamic Systems, Measurements and Control* 105 (1983), 3, 11–17.
- [10] A. Kojima and S. Ishijima: LQ preview synthesis: Optimal control and worst case analysis. *IEEE Trans. Automat. Control* 44 (1999), 2, 352–357.
- [11] P. Lancaster and L. Rodman: Algebraic Riccati Equations. Oxford University Press, New York 1995.
- [12] G. Marro, D. Prattichizzo, and E. Zattoni: A unified setting for decoupling with preview and fixed-lag smoothing in the geometric context. *IEEE Trans. Automat. Control* 51 (2006), 5, 809–813.
- [13] G. Marro and E. Zattoni: H_2 -optimal rejection with preview in the continuous-time domain. *Automatica* 41 (2005), 5, 815–821.
- [14] G. Marro and E. Zattoni: Signal decoupling with preview in the geometric context: exact solution for nonminimum-phase systems. *J. Optim. Theory Appl.* 129 (2006), 1, 165–183.
- [15] A. A. Moelja and G. Meinsma: H_2 control of preview systems. *Automatica* 42 (2006), 6, 945–952.
- [16] M. Vidyasagar: Control System Synthesis: A Factorization Approach. The MIT Press, Cambridge, MA 1985.
- [17] M. Šebek, H. Kwakernaak, D. Henrion, and S. Pejchová: Recent progress in polynomial methods and polynomial toolbox for Matlab version 2.0. In: Proc. 37th IEEE Conference on Decision and Control, Tampa 1998.
- [18] J. C. Willems: Feedforward control, PID control laws, and almost invariant subspaces. *Systems Control Lett.* 1 (1982), 4, 277–282.
- [19] W. M. Wonham: Linear Multivariable Control: A Geometric Approach. Third edition. Springer-Verlag, New York 1985.
- [20] M. Yamada, Y. Funahashi, and Z. Riadh: Generalized optimal zero phase tracking controller design. *Trans. ASME – J. Dynamic Systems, Measurement and Control* 121 (1999), 2, 165–170.

- [21] E. Zattoni: Decoupling of measurable signals via self-bounded controlled invariant subspaces: Minimal unassignable dynamics of feedforward units for prestabilized systems. *IEEE Trans. Automat. Control* 52 (2007), 1, 140–143.