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Numerical Study of Discretizations of Multistage Stochastic Programs

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Abstract: This paper presents a numerical study of a deterministic discretization procedure for multistage stochastic programs where the underlying stochastic process has a continuous probability distribution. The discretization procedure is based on quasi-Monte Carlo techniques originally developed for numerical multivariate integration. The solutions of the discretized problems are evaluated by statistical bounds obtained from random sample average approximations and out-of-sample simulations. In the numerical tests, the optimal values of the discretizations as well as their first-stage solutions approach those of the original infinite-dimensional problem as the discretizations are made finer.

Keywords: stochastic programming; discretization; integration quadratures; simulation;

AMS Subject Classification: 90C15; 49M25; 90C25;

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