

On the Structure of Continuous Uninorms

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Abstract: Uninorms were introduced by Yager and Rybalov [13] as a generalization of triangular norms and conorms. We ask about properties of increasing, associative, continuous binary operation U in the unit interval with the neutral element $e \in [0, 1]$. If operation U is continuous, then $e = 0$ or $e = 1$. So, we consider operations which are continuous in the open unit square. As a result every associative, increasing binary operation with the neutral element $e \in (0, 1)$, which is continuous in the open unit square may be given in $[0, 1]^2$ or $(0, 1]^2$ as an ordinal sum of a semigroup and a group. This group is isomorphic to the positive real numbers with multiplication. As a corollary we obtain the results of Hu, Li [7].

Keywords: uninorms; continuity; t -norms; t -conorms; ordinal sum of semigroups;

AMS Subject Classification: 06F05; 03E72; 03B52;

References

- [1] A. H. Clifford: Naturally totally ordered commutative semigroups. *Amer. J. Math.* 76 (1954), 631–646.
- [2] A. C. Climescu: Sur l'équation fonctionnelle de l'associativité. *Bull. Ecole Polytechn.* 1 (1946), 1–16.
- [3] E. Czogała and J. Drewniak: Associative monotonic operations in fuzzy set theory. *Fuzzy Sets and Systems* 12 (1984), 249–269.
- [4] J. Dombi: Basic concepts for a theory of evaluation: The aggregative operators. *European J. Oper. Res.* 10 (1982), 282–293.
- [5] J. Drewniak and P. Drygaś: Ordered semigroups in constructions of uninorms and nullnorms. In: *Issues in Soft Computing Theory and Applications* (P. Grzegorzewski, M. Krawczak, and S. Zadrozny, eds.), EXIT, Warszawa 2005, pp. 147–158.
- [6] J. Fodor, R. Yager, and A. Rybalov: Structure of uninorms. *Internat. J. Uncertain. Fuzziness Knowledge-Based Systems* 5 (1997), 411–427.

- [7] S.-K. Hu and Z.-F. Li: The structure of continuous uninorms. *Fuzzy Sets and Systems* 124 (2001), 43–52.
- [8] S. Jenei: A note on the ordinal sum theorem and its consequence for the construction of triangular norm. *Fuzzy Sets and Systems* 126 (2002), 199–205.
- [9] E. P. Klement, R. Mesiar, and E. Pap: *Triangular Norms*. Kluwer Academic Publishers, Dordrecht 2000.
- [10] Y.-M. Li and Z.-K. Shi: Remarks on uninorm aggregation operators. *Fuzzy Sets and Systems* 114 (2000), 377–380.
- [11] M. Mas, M. Monserrat, and J. Torrens: On left and right uninorms. *Internat. J. Uncertain. Fuzziness Knowledge-Based Systems* 9 (2001), 491–507.
- [12] W. Sander: Associative aggregation operators. In: *Aggregation Operators* (T. Calvo, G. Mayor, and R. Mesiar, eds), Physica-Verlag, Heidelberg 2002, pp. 124–158.
- [13] R. Yager and A. Rybalov: Uninorm aggregation operators. *Fuzzy Sets and Systems* 80 (1996), 111–120.