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DESIGN OF A MODEL FOLLOWING CONTROL SYSTEM FOR NONLINEAR DESCRIPTOR SYSTEM IN DISCRETE TIME

Shujing Wu, Shigenori Okubo and Dazhong Wang

A model following control system (MFCS) can output general signals following the desired ones. In this paper, a method of nonlinear MFCS will be extended to be a nonlinear descriptor system in discrete time. The nonlinear system studied in this paper has the property of norm constraint $||f(v(k))|| \le \alpha + \beta||v(k)||^{\gamma}$, where $\alpha \ge 0$, $\beta \ge 0$, $0 \le \gamma < 1$. In this case, a new criterion is proposed to ensure the internal states be stable.

Keywords: discrete-time system, descriptor, model following control system, nonlinear control system, disturbance

AMS Subject Classification: 93E12, 62A10, 62F15

1. INTRODUCTION

This paper studies the design of a model following control system (MFCS) for nonlinear descriptor system in discrete time. In previous studies, a method of nonlinear model following control system with disturbances was proposed by Okubo [8], and also a nonlinear model following control system with unstable zero points of the linear part [10], a nonlinear model following control system with containing inputs in nonlinear parts [9], and a nonlinear model following control system using stable zero assignment [11]. In this paper, the method of MFCS will be extended to discrete-time descriptor systems, and the effectiveness of the method will be verified by numerical simulation.

2. EXPRESSIONS OF THE PROBLEMS

The controlled object is described below, which is a nonlinear descriptor system in discrete time:

$$\begin{aligned} Ex(k+1) &= Ax(k) + Bu(k) + B_f f(v(k)) + d(k) && (1)\\ v(k) &= C_f x(k) && (2)\\ y(k) &= Cx(k) + d_0(k). && (3) \end{aligned}$$

The reference model is given below, which is assumed controllable and observable:

$$\begin{aligned} x_m(k+1) &= A_m x_m(k) + B_m r_m(k) && (4)\\ y_m(k) &= C_m x_m(k) && (5)\end{aligned}$$

where $x(k) \in \mathbb{R}^n$, $d(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^l$, $y(k) \in \mathbb{R}^l$, $y_m(k) \in \mathbb{R}^l$, $d_0(k) \in \mathbb{R}^l$, $f(v(k)) \in \mathbb{R}^{l_f}$, $v(k) \in \mathbb{R}^{l_f}$, $r_m(k) \in \mathbb{R}^{l_m}$, $x_m(k) \in \mathbb{R}^{n_m}$, $y(k)$ is the available states output vector, $v(k)$ is the measurement output vector, $u(k)$ is the input vector, $x(k)$ is the internal state vector whose elements are available, $d(k), d_0(k)$ are bounded disturbances, $y_m(k)$ is the model output.

The basic assumptions are as follows:

(1) Assume that (C, A, B) is controllable and observable, i. e.

$$\operatorname{rank}[zE - A, B] = n, \quad \operatorname{rank}\begin{bmatrix} zE - A \\ C \end{bmatrix} = n.$$

(2) In order to guarantee the existence and uniqueness of the solution and have exponential function mode but an impulse one for (1), the following conditions are assumed:

$$|zE - A| \not\equiv 0, \qquad \operatorname{rank} E = \deg|zE - A| = r \le n.$$

(3) Zeros of $C[zE - A]^{-1}B$ are stable.

In this system, the nonlinear function $f(v(k))$ is available and satisfies the following constraint:

$$||f(v(k))|| \le \alpha + \beta||v(k)||^{\gamma}$$

where $\alpha \ge 0$, $\beta \ge 0$, $0 \le \gamma < 1$, $||\cdot||$ is the Euclidean norm, disturbances $d(k), d_0(k)$ are bounded and satisfy

$$\begin{aligned} D_d(z)d(k) &= 0 && (6)\\ D_d(z)d_0(k) &= 0. && (7)\end{aligned}$$

Here, $D_d(z)$ is a scalar characteristic polynomial of disturbances. Output error is given as

$$e(k) = y(k) - y_m(k). \tag{8}$$

The aim of the control system design is to obtain a control law which makes the output error zero and keeps the internal states be bounded.

3. DESIGN OF A NONLINEAR MODEL FOLLOWING CONTROL SYSTEM

Letting z be the shift operator, (1) can be rewritten as follows:

$$\begin{aligned} C[zE - A]^{-1}B &= N(z)/D(z)\\ C[zE - A]^{-1}B_f &= N_f(z)/D(z)\end{aligned}$$

where $D(z) = |zE - A|$, $\partial_{r_i}(N(z)) = \sigma_i$ and $\partial_{r_i}(N_f(z)) = \sigma_{f_i}$.

Then, the representations of input-output equation is given as

$$D(z)y(k) = N(z)u(k) + N_f(z)f(v(k)) + w(k). \tag{9}$$

Here $w(k) = C\text{adj}[zE - A]d(k) + D(z)d_0(k)$, (C_m, A_m, B_m) is controllable and observable. Hence,

$$C_m[zI - A_m]^{-1}B_m = N_m(z)/D_m(z).$$

Then, we have

$$D_m(z)y_m(k) = N_m(z)r_m(k) \tag{10}$$

where $D_m(z) = |zI - A_m|$ and $\partial_{r_i}(N_m(z)) = \sigma_{m_i}$.

Since the disturbances satisfy (6) and (7), and $D_d(z)$ is a monic polynomial, one has

$$D_d(z)w(k) = 0. \tag{11}$$

The first step of design is that a monic and stable polynomial $T(z)$, which has the degree of $\rho(\rho \geq n_d + 2n - n_m - 1 - \sigma_i)$, is chosen. Then, $R(z)$ and $S(z)$ can be obtained from

$$T(z)D_m(z) = D_d(z)D(z)R(z) + S(z) \tag{12}$$

where the degree of each polynomial is: $\partial T(z) = \rho$, $\partial D_d(z) = n_d$, $\partial D_m(z) = n_m$, $\partial D(z) = n$, $\partial R(z) = \rho + n_m - n_d - n$ and $\partial S(z) \leq n_d + n - 1$.

From (8) $\sim$ (12), the following form is obtained:

$$\begin{aligned} T(z)D_m(z)e(k) &= D_d(z)R(z)N(z)u(k) \\ &\quad + D_d(z)R(z)N_f(z)f(v(k)) + S(z)y(k) - T(z)N_m(z)r_m(k). \end{aligned}$$

The output error $e(k)$ is represented as

$$\begin{aligned} e(k) &= \frac{1}{T(z)D_m(z)}\{[D_d(z)R(z)N(z) - Q(z)N_r]u(k) + Q(z)N_r u(k) \\ &\quad + D_d(z)R(z)N_f(z)f(v(k)) + S(z)y(k) - T(z)N_m(z)r_m(k)\}. \end{aligned} \tag{13}$$

Suppose $\Gamma_r(N(z)) = N_r$, where $\Gamma_r(\cdot)$ is the coefficient matrix of the element with maximum of row degree, as well as $|N_r| \neq 0$. The next control law $u(k)$ can be obtained by making the right-hand side of (13) be equal to zero. Thus,

$$\begin{aligned} u(k) &= -N_r^{-1}Q^{-1}(z)\{D_d(z)R(z)N(z) - Q(z)N_r\}u(k) \\ &\quad - N_r^{-1}Q^{-1}(z)D_d(z)R(z)N_f(z)f(v(k)) \\ &\quad - N_r^{-1}Q^{-1}(z)S(z)y(k) + u_m(k) \end{aligned} \tag{14}$$

$$u_m(k) = N_r^{-1}Q^{-1}(z)T(z)N_m(z)r_m(k). \tag{15}$$

Here, $Q(z) = \text{diag}[z^{\delta_i}]$, $\delta_i = \rho + n_m - n + \sigma_i$ $(i = 1, 2, \ldots, n)$, and $u(k)$ of (14) is obtained from $e(k) = 0$. The model following control system can be realized if the system internal states are bounded.

4. PROOF OF THE BOUNDED PROPERTY OF INTERNAL STATES

System inputs are both reference input signal $r_m(k)$ and disturbances $d(k), d_0(k)$, which are all assumed to be bounded. The boundedness can be easily proved if there is no nonlinear part $f(v(k))$. But if $f(v(k))$ exists, the bound has a relation with it. The state space expression of $u(k)$ is

$$\begin{aligned} u(k) &= -H_1\xi_1(k) - E_2y(k) - H_2\xi_2(k) \\ &\qquad - E_3f(v(k)) - H_3\xi_3(k) + u_m(k) \qquad (16) \\ u_m(k) &= E_4r_m(k) + H_4\xi_4(k). \qquad (17) \end{aligned}$$

The followings must be satisfied:

$$\begin{aligned} \xi_1(k+1) &= F_1\xi_1(k) + G_1u(k) \qquad (18) \\ \xi_2(k+1) &= F_2\xi_2(k) + G_2y(k) \qquad (19) \\ \xi_3(k+1) &= F_3\xi_3(k) + G_3f(v(k)) \qquad (20) \\ \xi_4(k+1) &= F_4\xi_4(k) + G_4r_m(k). \qquad (21) \end{aligned}$$

Here,

$$|zI - F_i| = |Q(z)|, \quad (i = 1, 2, 3, 4).$$

Note that there are connections between the polynomial matrices and the system matrices, as follows:

$$\begin{aligned} N_r^{-1}Q^{-1}(z)\{D_d(z)R(z)N(z) - Q(z)N_r\} &= H_1(zI - F_1)^{-1}G_1 \qquad (22) \\ N_r^{-1}Q^{-1}(z)S(z) &= H_2(zI - F_2)^{-1}G_2 + E_2 \qquad (23) \\ N_r^{-1}Q^{-1}(z)D_d(z)R(z)N_f(z) &= H_3(zI - F_3)^{-1}G_3 + E_3 \qquad (24) \\ N_r^{-1}Q^{-1}(z)T(z)N_m(z) &= H_4(zI - F_4)^{-1}G_4 + E_4. \qquad (25) \end{aligned}$$

Firstly, remove $u(k)$ from (1) $\sim$ (3) and (18) $\sim$ (21). Then, the representation of the overall system can be obtained as follows:

$$\begin{bmatrix} E & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \begin{bmatrix} x(k+1) \\ \xi_1(k+1) \\ \xi_2(k+1) \\ \xi_3(k+1) \end{bmatrix}$$

$$= \begin{bmatrix} A - BE_2C & -BH_1 & -BH_2 & -BH_3 \\ -G_1E_2C & F_1 - G_1H_1 & -G_1H_2 & -G_1H_3 \\ G_2C & 0 & F_2 & 0 \\ 0 & 0 & 0 & F_3 \end{bmatrix} \begin{bmatrix} x(k) \\ \xi_1(k) \\ \xi_2(k) \\ \xi_3(k) \end{bmatrix}$$

$$+ \begin{bmatrix} BH_4 \\ G_1H_4 \\ 0 \\ 0 \end{bmatrix} \xi_4(k) + \begin{bmatrix} B_f - BE_3 \\ -G_1E_3 \\ 0 \\ G_3 \end{bmatrix} f(v(k))$$

$$+\begin{bmatrix} BE_4 \\ G_1E_4 \\ 0 \\ 0 \end{bmatrix} r_m(k) + \begin{bmatrix} d(k) - BE_2d_0(k) \\ -G_1E_2d_0(k) \\ G_2d_0(k) \\ 0 \end{bmatrix} \tag{26}$$

$$\xi_4(k+1) = F_4\xi_4(k) + G_4r_m(k) \tag{27}$$

$$v(k) = \begin{bmatrix} C_f & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x(k) \\ \xi_1(k) \\ \xi_2(k) \\ \xi_3(k) \end{bmatrix} \tag{28}$$

$$y(k) = \begin{bmatrix} C & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x(k) \\ \xi_1(k) \\ \xi_2(k) \\ \xi_3(k) \end{bmatrix} + d_0(k). \tag{29}$$

In equation (27), the $\xi_4(k)$ are bounded, because $|zI - F_4| = |Q(z)|$ is a stable polynomial and $r_m(k)$ is reference input. Let $z(k), A_s, \tilde{E}, d_s(k), B_s, C_v, C_s$ be as follows, respectively:

$$z(k) = \begin{bmatrix} x^T(k) & \xi_1^T(k) & \xi_2^T(k) & \xi_3^T(k) \end{bmatrix}^T$$

$$A_s = \begin{bmatrix} A - BE_2C & -BH_1 & -BH_2 & -BH_3 \\ -G_1E_2C & F_1 - G_1H_1 & -G_1H_2 & -G_1H_3 \\ G_2C & 0 & F_2 & 0 \\ 0 & 0 & 0 & F_3 \end{bmatrix}$$

$$\tilde{E} = \begin{bmatrix} E & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}$$

$$d_s(k) = \begin{bmatrix} Bu_m(k) + d(k) - BE_2d_0(k) \\ G_1u_m(k) - G_1E_2d_0(k) \\ G_2d_0(k) \\ 0 \end{bmatrix}$$

$$B_s = \begin{bmatrix} B_f - BE_3 \\ -G_1E_3 \\ 0 \\ G_3 \end{bmatrix}$$

$$C_v = \begin{bmatrix} C_f & 0 & 0 & 0 \end{bmatrix}$$

$$C_s = \begin{bmatrix} C & 0 & 0 & 0 \end{bmatrix}.$$

With the consideration that $\xi_4(k)$ is bounded, the necessary parts to an easy proof of the bounded property are arranged as

$$\begin{aligned}
\tilde{E}z(k+1) &= A_s z(k) + B_s f(v(k)) + d_s(k) && (30)\\
v(k) &= C_v z(k) && (31)\\
y(k) &= C_s z(k) + d_0(k) && (32)
\end{aligned}$$

where the contents of $A_s, \tilde{E}, d_s(k), B_s, C_v, C_s$ are constant matrices, and $f(v(k)), d_s(k)$ are bounded. Thus, the internal states are bounded if $z(k)$ can be proved to be bounded. So, it needs to prove that $|z\tilde{E} - A_s|$ is a stable polynomial. The characteristic polynomial of A_s is calculated next.

From (26), $|z\tilde{E} - A_s|$ can be shown as

$$|z\tilde{E} - A_s| = \begin{vmatrix} zE - A + BE_2C & BH_1 & BH_2 & BH_3 \\ G_1E_2C & zI - F_1 + G_1H_1 & G_1H_2 & G_1H_3 \\ -G_2C & 0 & zI - F_2 & 0 \\ 0 & 0 & 0 & zI - F_3 \end{vmatrix}. \qquad (33)$$

Prepare the following formulas:

$$\begin{aligned}
\begin{vmatrix} X & Y \\ W & Z \end{vmatrix} &= |Z||X - YZ^{-1}W|, \; (|Z| \neq 0)\\
I - X(I + YX)^{-1}Y &= (I + XY)^{-1}\\
|I + XY| &= |I + YX|.
\end{aligned}$$

Using the above formulas, $|z\tilde{E} - A_s|$ is described as

$$\begin{aligned}
& |z\tilde{E} - A_s| \\
= \; & |zI - F_3||zI - F_2||zI - F_1||I + H_1[zI - F_1]^{-1}G_1| \\
& \cdot |zE - A + B\{I - H_1[zI - F_1 + G_1H_1]^{-1}G_1\} \\
& \cdot \{E_2 + H_2[zI - F_2]^{-1}G_2\}C| \\
= \; & |Q(z)|^3|I + H_1[zI - F_1]^{-1}G_1||zE - A \\
& + B\{I + H_1[zI - F_1]^{-1}G_1\}^{-1}\{E_2 + H_2[zI - F_2]^{-1}G_2\}C| \\
= \; & |Q(z)|^3|J_1||zE - A||I + BJ_1^{-1}J_2[zE - A]^{-1}| \\
= \; & |Q(z)|^3|zE - A||J_1 + J_2[zE - A]^{-1}B|. && (34)
\end{aligned}$$

Here

$$\begin{aligned}
J_1 &= I + H_1[zI - F_1]^{-1}G_1 && (35)\\
J_2 &= \{E_2 + H_2[zI - F_2]^{-1}G_2\}C. && (36)
\end{aligned}$$

From (22), (23), (35) and (36), we have

$$\begin{aligned}
J_1 &= N_r^{-1}Q^{-1}(z)D_d(z)R(z)N(z) && (37)\\
J_2 &= N_r^{-1}Q^{-1}(z)S(z)C. && (38)
\end{aligned}$$

Using $C[zE-A]^{-1}B = N(z)/D(z)$ and $D(z) = |zE-A|$, furthermore, $|z\tilde{E}-A_s|$ is shown as

$$|z\tilde{E}-A_s| \;=\; T^l(z)D_m^l(z)|Q(z)|^2\frac{|N(z)||N_r|^{-1}}{D^{l-1}(z)}$$

and $V(z)$ is the zeros polynomial of $C[zE-A]^{-1}B = N(z)/D(z) = U^{-1}(z)V(z)$ (left coprime decomposition), $|U(z)| = D(z)$, that is, $|N(z)| = D^{l-1}(z)|V(z)|$. So $|z\tilde{E}-A_s|$ can be rewritten as

$$|z\tilde{E}-A_s| \;=\; |N_r|^{-1}T^l(z)D_m^l(z)|Q(z)|^2|V(z)|. \tag{39}$$

As $|N_r|^{-1},\ T(z),\ D_m(z),\ |Q(z)|,\ |V(z)|$ are all stable polynomials, A_s is a stable system matrix.

Consider the following:

$$z(k) \;=\; Q\bar{z}(k) = Q\begin{bmatrix} \bar{z}_1(k) \\ \bar{z}_2(k) \end{bmatrix}. \tag{40}$$

Using (40), one obtains

$$P\tilde{E}Q\bar{z}(k+1) \;=\; PA_sQ\bar{z}(k) + PB_sf(v(k)) + Pd_s(k).$$

Namely,

$$\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \bar{z}_1(k+1) \\ \bar{z}_2(k+1) \end{bmatrix} = \begin{bmatrix} A_{s1} & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} \bar{z}_1(k) \\ \bar{z}_2(k) \end{bmatrix} + \begin{bmatrix} B_{s1} \\ B_{s2} \end{bmatrix} f(v(k)) + \begin{bmatrix} d_{s1}(k) \\ d_{s2}(k) \end{bmatrix}. \tag{41}$$

One can rewritten (41) as

$$\bar{z}_1(k+1) \;=\; A_{s1}\bar{z}_1(k) + B_{s1}f(v(k)) + d_{s1}(k) \tag{42}$$

$$0 \;=\; \bar{z}_2(k) + B_{s2}f(v(k)) + d_{s2}(k) \tag{43}$$

where $\bar{z}(k), Pd_s(k), PA_sQ, PB_s$ can be represented by

$$\bar{z}(k) = \begin{bmatrix} \bar{z}_1(k) \\ \bar{z}_2(k) \end{bmatrix}, \qquad Pd_s(k) = \begin{bmatrix} d_{s1}(k) \\ d_{s2}(k) \end{bmatrix},$$

$$PA_sQ = \begin{bmatrix} A_{s1} & 0 \\ 0 & I \end{bmatrix}, \quad PB_s = \begin{bmatrix} B_{s1} \\ B_{s2} \end{bmatrix}. \tag{44}$$

Let $C_vQ = [C_{v1}, C_{v2}], (|C_{v1}| \neq 0)$. Then

$$v(k) \;=\; C_{v1}\bar{z}_1(k) + C_{v2}\bar{z}_2(k). \tag{45}$$

From (43) and (45), we have

$$v(k) + C_{v2}B_{s2}f(v(k)) \;=\; C_{v1}\bar{z}_1(k) - C_{v2}d_{s2}(k). \tag{46}$$

From(46), we have

$$\frac{\partial}{\partial v^T(k)}(v(k)+C_{v2}B_{s2}f(v(k))) \;=\; I+C_{v2}B_{s2}\frac{\partial f(v(k))}{\partial v^T(k)}.$$

Existing condition of $v(k)$ is

$$|I+C_{v2}B_{s2}\frac{\partial f(v(k))}{\partial v^T(k)}| \;\neq\; 0. \tag{47}$$

From (44), we have

$$\begin{aligned} |P||z\tilde{E}-A_s||Q| &= \alpha_{PQ}|z\tilde{E}-A_s| \\ &= \alpha_{PQ}\begin{vmatrix} zI-A_{s1} & 0 \\ 0 & -I \end{vmatrix} \\ &= \alpha_I|zI-A_{s1}|. \end{aligned} \tag{48}$$

Here, α_{PQ} and α_I are fixed. So, from (39), A_{s1} is a stable system matrix. Consider a quadratic Lyapunov function candidate

$$V(k) \;=\; \bar{z}_1^T(k)P_s\bar{z}_1(k). \tag{49}$$

The difference of $V(k)$ along the trajectories of system (42) is given by

$$\begin{aligned} \Delta V(k) &= \bar{z}_1^T(k+1)P_s\bar{z}_1(k+1)-V(k) \\ &= [A_{s1}\bar{z}_1(k)+B_{s1}f(v(k))+d_{s1}(k)]^TP_s \\ &\quad\cdot[A_{s1}\bar{z}_1(k)+B_{s1}f(v(k))+d_{s1}(k)]-V(k) \end{aligned} \tag{50}$$

$$A_{s1}^TP_sA_{s1}-P_s=-Q_s \tag{51}$$

where Q_s and P_s are symmetric positive definite matrices defined by (51). If A_{s1} is a stable matrix, we can get a unique P_s from (51) when Q_s is given. As $d_{s1}(k)$ is bounded and $0\leq\gamma<1$, $\Delta V(k)$ satisfies

$$\begin{aligned} \Delta V(k) &\leq -\bar{z}_1^T(k)Q_s\bar{z}_1(k)+X_1||\bar{z}_1(k)||||f(v(k))|| \\ &\quad + X_2||\bar{z}_1(k)||+\mu_2||f(v(k))||^2+X_3||f(v(k))||+X_4. \end{aligned} \tag{52}$$

From(31), (43) and (45), we have

$$||\bar{z}_1(k)|| \;\leq\; M||z(k)||. \tag{53}$$

Here, M is positive constant. From (52), (53), we have

$$\begin{aligned} \Delta V(k) &\leq -\mu_1||z(k)||^2+X_5||z(k)||^{1+\gamma}+X_6 \\ &\leq -\mu_c||z(k)||^2+X \\ &\leq -\mu_{c1}||\bar{z}_1(k)||^2+X \\ &\leq -\mu_mV(k)+X \end{aligned} \tag{54}$$

where $0 < \mu_1 = \lambda_{\min}(Q_s)$, $\mu_2 \geq 0$ and $0 < \mu_m < \mu_c < \min(\mu_1, 1)$. Also, $\mu_1, \mu_2, X_i\,(i = 1 \sim 6)$ and X are positive constants. As a result of (54), $V(k)$ is bounded:

$$V(k) \leq V(0) + X/\mu_m. \tag{55}$$

Hence, $\bar{z}_1(k)$ is bounded. From (43), $\bar{z}_2(k)$ is also bounded. Therefore, $z(k)$ is bounded. The above result is summarized as Theorem.

Theorem. In the nonlinear system

$$\begin{aligned} Ex(k+1) &= Ax(k) + Bu(k) + B_f f(v(k)) + d(k) \\ v(k) &= C_f x(k) \\ y(k) &= Cx(k) + d_0(k) \end{aligned}$$

where $x(k) \in \mathbb{R}^n, y(k) \in \mathbb{R}^l, v(k) \in \mathbb{R}^{l_f}, d(k) \in \mathbb{R}^n$, $d_0(k) \in \mathbb{R}^l$, $f(v(k)) \in \mathbb{R}^{l_f}$, $d(k)$ and $d_0(k)$ are assumed to be bounded. All the internal states are bounded and the output error $e(k) = y(k) - y_m(k)$ asymptotically converges to zero in the design of the model following control system for a nonlinear descriptor system in discrete time, if the following conditions are held:

1. Both the controlled object and the reference model are controllable and observable.
2. $|N_r| \neq 0$.
3. Zeros of $C[zE - A]^{-1}B$ are stable.
4. $||f(v(k))|| \leq \alpha + \beta||v(k)||^\gamma$, $(\alpha \geq 0, \beta \geq 0, 0 \leq \gamma < 1)$.
5. Existing condition of $v(k)$ is $\left| I + C_{v2}B_{s2}\dfrac{\partial f(v(k))}{\partial v^T(k)} \right| \neq 0$.
6. $|zE - A| \not\equiv 0$ and $\operatorname{rank} E = \deg|zE - A| = r \leq n$.

5. NUMERICAL SIMULATION

An example is given as follows:

$$\begin{aligned} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} x(k+1) &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.2 & -0.5 & 0.6 \end{bmatrix} x(k) \\ &\quad + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} u(k) + \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} f(v(k)) + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} d(k) \\ v(k) &= \begin{bmatrix} 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix} x(k) \end{aligned}$$

$$
\begin{aligned}
y(k) &= \begin{bmatrix} 0 & 0.1 & 0 \\ 0.1 & 0 & 0.1 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} d_0(k) \\
f(v(k)) &= \frac{3v(k)^3 + 4v(k) + 1}{1 + v(k)^4}.
\end{aligned}
$$

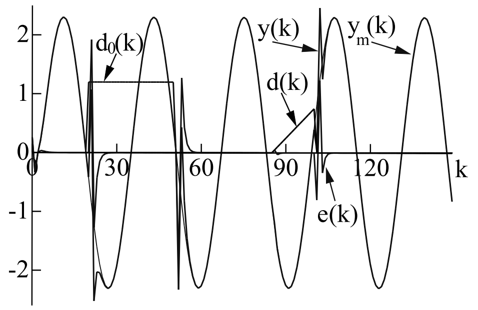

Fig. Responses of the system for nonlinear descriptor system in discrete time.

Reference model is given by

$$
\begin{aligned}
x_m(k+1) &= \begin{bmatrix} 0 & 1 \\ -0.12 & 0.7 \end{bmatrix} x_m(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r_m(k) \\
y_m(k) &= \begin{bmatrix} 1 & 0 \end{bmatrix} x_m(k) \\
r_m(k) &= \sin(k\pi/16).
\end{aligned}
$$

In this example, disturbances $d(k)$ and $d_0(k)$ are step and ramp disturbances, respectively. Then, $d(k)$ and $d_0(k)$ are given as

$$
\begin{aligned}
d_0(k) &= 1.2, (20 \le k \le 50) \\
d(k) &= 0.05(k - 85), (85 \le k \le 100).
\end{aligned}
$$

We show a result of simulation in Figure. It can be concluded that the output signal follows the reference even if disturbances exist in the system.

6. CONCLUSION

In the responses (Figure) of the nonlinear discrete-time descriptor model following control system, the output signal follows the reference even though disturbances exist in the system. The effectiveness of this method has thus been verified. This is a topic in the future that the condition of nonlinear parameter which is bigger than $\gamma \ge 1$ will be proved and analyzed.

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