# Search of the correspondence between the control points for registration of the projectively deformed images

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## ABSTRACT

The algorithm for search of the correspondence between the points from two planar finite point sets is presented. The projective transformation between the point sets is supposed, but points without correspondence in the other set can be presented in both sets. The algorithm is based on the comparison of two projective and permutation invariants of five-tuples of the points. The most hopeful five-tuples are then used for the computation of the projective transformation and that with the maximum of corresponding points is used.

Key words: registration, projective transform, control points, projective invariants, permutation invariants

## 1. INTRODUCTION

One of the important tasks in image processing and remote sensing is a registration of images captured under different viewing angles. If the scene is planar, then the distortion between two frames can be described by *projective transform*

$$
\begin{aligned}
x' &= \frac{a_0 + a_1 x + a_2 y}{1 + c_1 x + c_2 y} \\
y' &= \frac{b_0 + b_1 x + b_2 y}{1 + c_1 x + c_2 y},
\end{aligned} \qquad (1)
$$

where $x, y$ are the coordinates in the first frame called input image and $x', y'$ are the coordinates in the second frame called reference image.

We can imagine the following situation. We have selected candidates for control points in both images by some method, but we don't know the correspondence between them. In addition to that, some candidates haven't partner in the other image.

Some papers have dealt with the similar situation, e.g. [1], but they only considered the affine transform between the images and determined the correspondence between the points according to the similarity of neighborhood of the points.

This paper deals with the possibilities of looking for the correspondence between the points only according to their location.

## 2. THE FULL SEARCH ALGORITHM

The simplest algorithm may be the full search of all possibilities. We can examine each four points from the input image against each four points from the reference image. If we have $n$ points in the input image and $k$ points in the reference one, we must examine $\binom{n}{4}\binom{k}{4}4!$ possibilities.

The examination means the computation of the projective transform, the transformation of the points in the input image and judgement of the quality of the transform. We performed this judgement by the following way. The two nearest points are found and removed and again two nearest points from remaining ones are found. The searching is finished when the distance between the nearest points exceeds the suitable threshold. The number of corresponding points is used as the criterion of the quality of the transform. If the number is the same, the distance of the last two points is used. The best transform according to this criterion is used as the solution.

The threshold must correspond to the precision of the selection of the points. The threshold 5 pixels proved its suitability in usual situations.

If $n$ and $k$ are approximately the same and high, this algorithm has the computing complexity $O\left(n^{11}\right)$ and in our experience it is too time consuming.

## 3. THE PROJECTIVE AND PERMUTATION INVARIANTS

Now the projective and permutation invariants are known [2,3]. Let's denote $P(1,2,3)$ the area of the triangle with the vertices $[x_1, y_1]$, $[x_2, y_2]$ and $[x_3, y_3]$, i.e.

$$P(1,2,3) = (x_1 y_2 - x_2 y_1 + x_2 y_3 - x_3 y_2 + x_3 y_1 - x_1 y_3) / 2\,. \qquad (2)$$

If the vertices are numbered clockwise, we will consider the area negative.

The five-point cross-ratio

$$r = \frac{P(1,2,3)\,P(1,4,5)}{P(1,2,4)\,P(1,3,5)}\,, \qquad (3)$$

is the projective invariant. The point 1 is contained in all four triangles and is called the common point of the cross-ratio. If the common point stays the same and the other points are permuted, then the values of the cross-ratios are

$$r_1 = r, \quad r_2 = \frac{1}{r}, \quad r_3 = 1 - r, \quad r_4 = \frac{1}{1-r}, \quad r_5 = \frac{r}{r-1}, \quad r_6 = \frac{r-1}{r} \tag{4}$$

If we construct a function $F(r)$, which has the same value for all these values of $r$, it is invariant to the permutation of four points. If we change the common point, we receive another value of the cross-ratio, let us say $s$, and a function

$$I(r,s) = F(r) + F(s) + F(\frac{r}{s}) + F(\frac{r-1}{s-1}) + F(\frac{r(s-1)}{s(r-1)}) \tag{5}$$

is the five point projective and permutation invariant.

We used these functions $F$

$$\begin{gathered} F_1 = \frac{8}{5} \frac{r^2 (1-r)^2}{(r^2 - r + 1)^3} \\ F_2 = 3 \frac{r^2 (1-r)^2}{(2r^2 - 2r + 1)(r^2 - 2r + 2)(r^2 + 1)}. \end{gathered} \tag{6}$$

The corresponding invariants $I_1$ and $I_2$ are normalized into the interval from 0 to 1, but they utilize this space ineffectively (see Fig. 1).

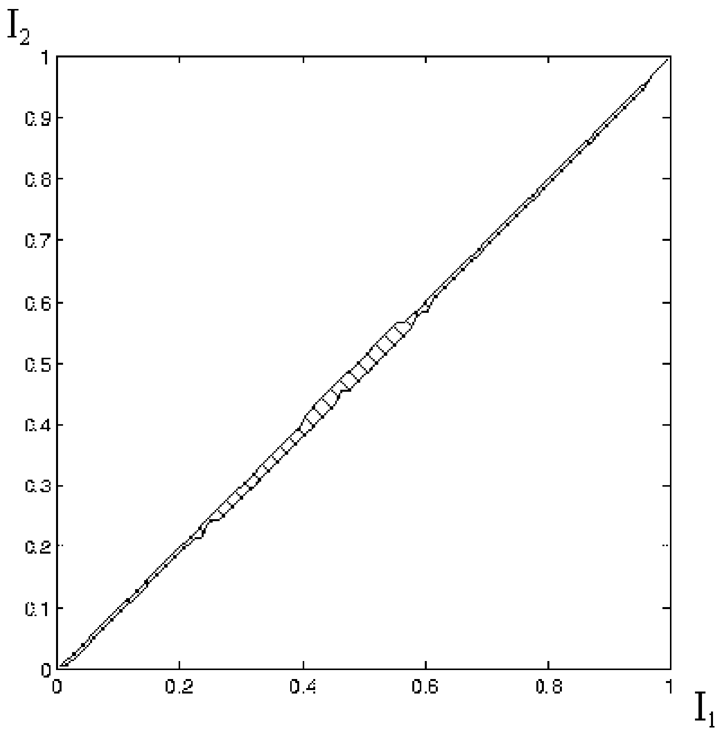


Figure 1, Possible values of the invariants $I_1, I_2$

A better result can be obtained as the sum and the difference (see Fig. 2)

$$I_1' = (I_1 + I_2)/2$$
$$I_2' = (I_1 - I_2 + 0.006) \cdot 53\,, \tag{7}$$

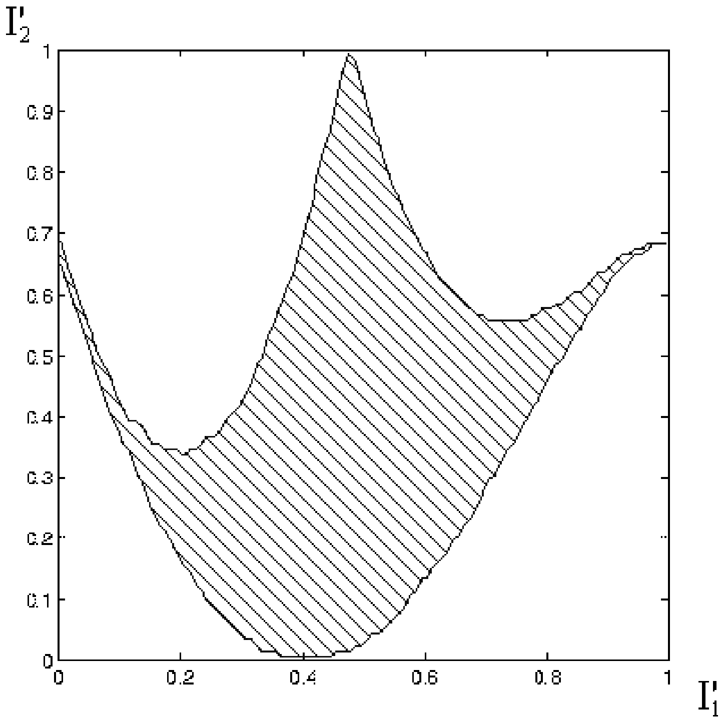


Figure 2, Possible values of the invariants $I_1', I_2'$

but subtraction and division reach good utilization of the given range by the polynomials (see Fig. 3)

$$I_1'' = I_1'$$
$$I_2'' = \frac{1 - I_2' + \mathrm{p}(I_1')}{d(I_1')}\,, \tag{8}$$

where

$$\mathrm{p}(I_1') = 10.110488 \cdot I_1'^6 - 27.936483 \cdot I_1'^5 + 31.596612 \cdot I_1'^4 - \\ -16.504259 \cdot I_1'^3 - 0.32251158 \cdot I_1'^2 + 3.0473587 \cdot I_1' - 0.66901966\,,$$

if $I_1' < 0.475$ then

$$d(I_1') = 17.575974 \cdot I_1'^4 - 16.423212 \cdot I_1'^3 + 9.1115270 \cdot I_1'^2 - 0.43942294 \cdot I_1' + 0.016542258 ,$$

if $I_1' \geq 0.475$ then

$$d(I_1') = 3.9630392 \cdot I_1'^4 - 13.941518 \cdot I_1'^3 + 21.672754 \cdot I_1'^2 - 17.304971 \cdot I_1' + 5.6198814 .$$

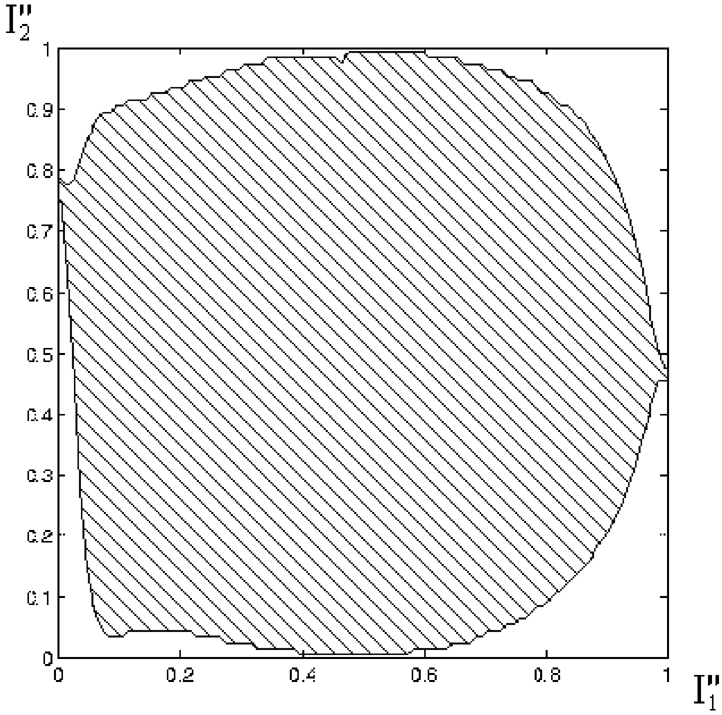


Figure 3, Possible values of the invariants $I_1''$, $I_2''$

## 4. PAIRING ALGORITHM BY MEANS OF PROJECTIVE AND PERMUTATION INVARIANTS

We can compute the distance in the feature space between invariants of each five points from the input image against invariants of each five points in the reference image. Nevertheless, it was found that wrong five points often match one another randomly, this false match can be better than the correct one and we must search not only the best match, but also each good match.

We carried out experiments with a number of searching algorithms. We consider following as the best one. We find the first $m$ best matches and the full search algorithm from Section 2 is applied on each pair of five-tuples corresponding each match.

The number $m$ was chosen as $\binom{\max(n,k)}{5}$, but this number is not critical. Note: the total number of pairs of five-tuples is $\binom{n}{5}\binom{k}{5}$.

In [2] convex hull constraints are proposed. It is based on idea that usual projective transforms, as occur also in remote sensing, preserve position of points on or inside the convex hull. Then the pairs of five-tuples with the different number of points on the convex hull need not be considered. In the same work the idea of partial search is proposed. The authors randomly chose about one fifteenth of all pairs and tried to search them only. They found the decrease of reliability relatively small. These constraints can be used in our algorithm too, but the following numerical experiment used the general algorithm without these constraints.

## 5. NUMERICAL EXPERIMENT

A cut of a Landsat Thematic Mapper image of north-east Bohemia (surroundings of the town Trutnov) from 29 august 1990 ($256 \times 256$) was used as the reference image (see Fig. 4) and an aerial image from 1984 ($180 \times 256$) with relatively strong projective distortion was used as the input one (see Fig. 5).

16 points was selected in the input image (see Fig. 5), 18 points was selected in the reference one (see Fig. 4) and 10 points in both images had partners in the other image (number 1 to 10 in the input correspond number 9 to 18 in the reference). The first $\binom{18}{5} = 8568$ best matches were examined and the 476$^{th}$ one was correct. All ten pairs of control points were found, the distance of the tenth pair was 2.23 pixels. The result is shown on Fig. 6. The final parameters of the transform was computed from all ten control points by means of the least square method. The deviations on the control points was from 0.17 to 1.71 pixels, the average was 0.94 pixel.

The time of the search of the best matches was about one hour and a half on the workstation HP 9000/700 and the time of the examination of all 8568 matches was about two hours and a half, but the correct 476$^{th}$ match was found in 8 minutes.

| no. | x | y |
|---|---|---|
| 1 | 22 | 26 |
| 2 | 161 | 27 |
| 3 | 117 | 189 |
| 4 | 64 | 214 |
| 5 | 8 | 31 |
| 6 | 50 | 15 |
| 7 | 90 | 52 |
| 8 | 61 | 73 |
| 9 | 35 | 114 |
| 10 | 96 | 142 |
| 11 | 65 | 143 |
| 12 | 14 | 116 |
| 13 | 35 | 153 |
| 14 | 145 | 149 |
| 15 | 139 | 51 |
| 16 | 161 | 75 |

Table 1, The coordinates of the selected points in the input image

| no. | x | y |
|---|---|---|
| 1 | 35 | 42 |
| 2 | 233 | 106 |
| 3 | 104 | 166 |
| 4 | 253 | 147 |
| 5 | 16 | 243 |
| 6 | 202 | 235 |
| 7 | 73 | 39 |
| 8 | 130 | 40 |
| 9 | 55 | 111 |
| 10 | 176 | 67 |
| 11 | 172 | 197 |
| 12 | 152 | 215 |
| 13 | 47 | 122 |
| 14 | 72 | 86 |
| 15 | 126 | 118 |
| 16 | 114 | 146 |
| 17 | 113 | 181 |
| 18 | 155 | 182 |

Table 2, The coordinates of the selected points in the reference image

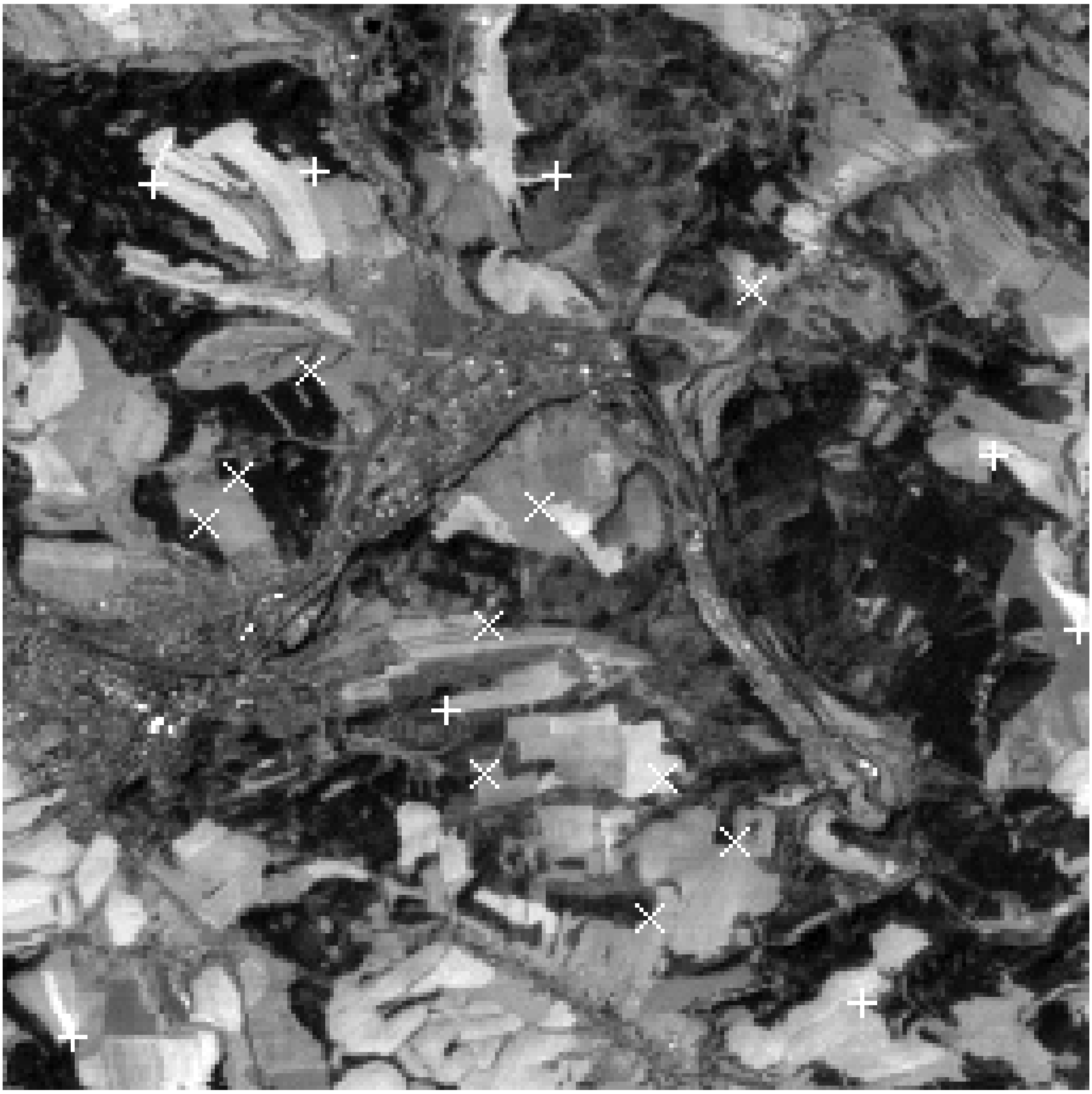

Figure 4, The satellite image used as reference one

(× the control points with partners in the input image, + the points without partners)

Figure 5, The aerial image used as the input one

(× the control points with partners in the reference image, + the points without partners)

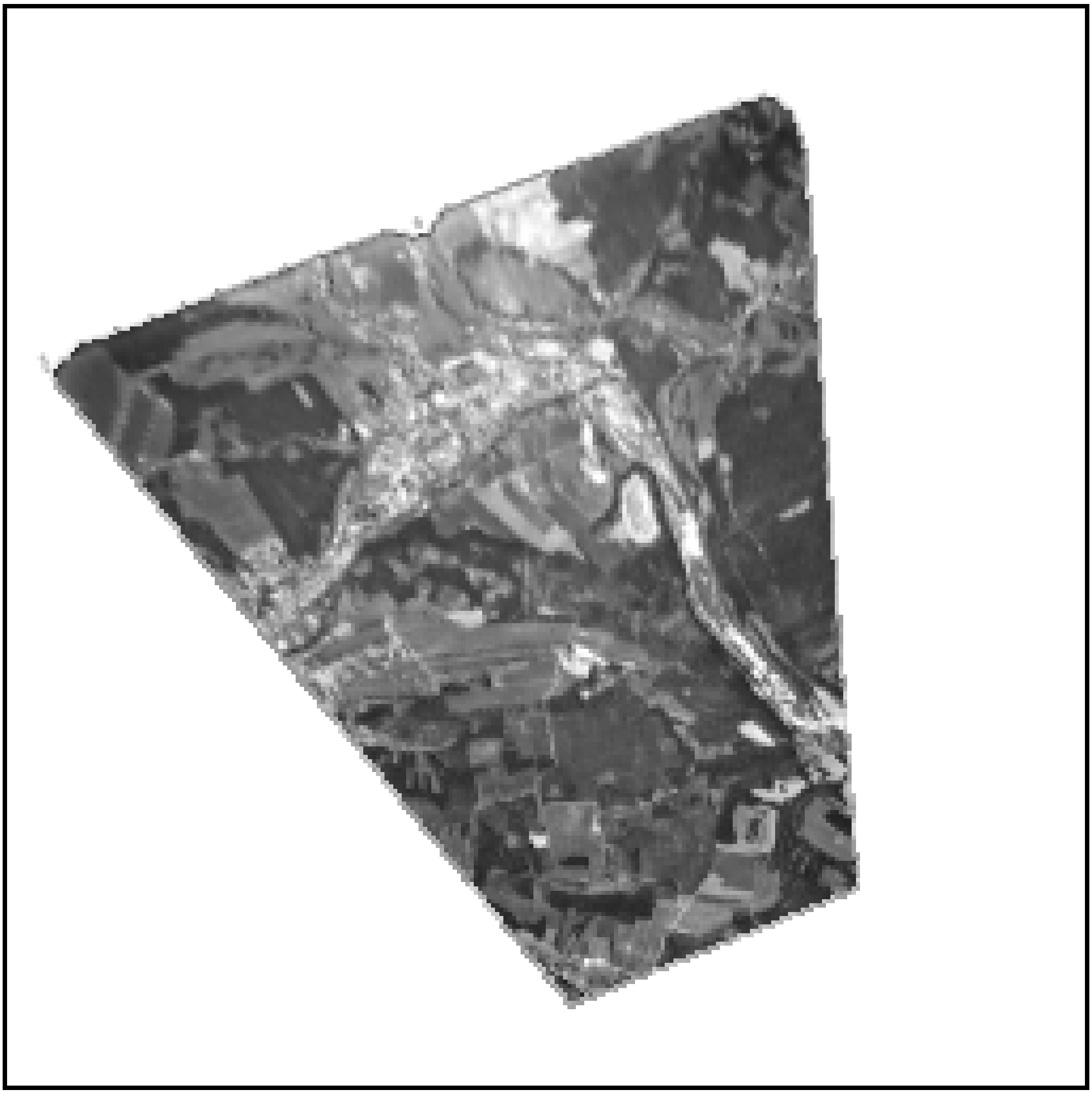

Figure 6, The registered image

## 6. CONCLUSION

If the affine and simpler transforms can be used for approximation of the distortion between the images, other methods are suitable. In case of strong projective distortion between the images, the described algorithm is one of the possible solutions of the task.

The minimum number of corresponding pairs is 6 and correspondence between point sets with fewer pair of the points cannot be found principally. In case of 6 corresponding pairs once the wrong correspondence was found during tens of experiments, in case of more than 6 correspond pairs no error was found. It means that in case of more than 6 corresponding pairs the hope of successful result is very good.

## 7. ACKNOWLEDGEMENT

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