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Reasoning about Probabilities and Probabilistic Reasoning

After Dutch Books

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ABSTRACT. Three interpretations of the Dutch Book argument are considered, and are found to fail to establish that degrees of belief should obey the probability calculus. One way to overcome this failure is to consider different interpretations of the notion of belief. These alternative interpretations can make use of constructions other than Dutch Books. The representationalist account of belief is discussed in conjunction with a representation theorem due to de Groot. Prospects for this account of subjective probability are briefly considered.

1 Introduction

Dutch Book arguments are sometimes purported to establish that degrees of belief should obey the probability calculus. I will argue that they do not, and cannot, establish this. I do this by examining several competing interpretations of the arguments, and show that they share the same fundamental flaw, a connection to a dispositional account of belief. If we try to remove this account, there are better types of arguments for establishing links between degrees of belief and probabilities.

The first part of this paper covers the behaviourist interpretation of the Dutch Book argument, while the following two cover two different types of depragmatized Dutch Book arguments – Howson and Urbach's counterfactual interpretation and Howson's logical interpretation. While I conclude that Howson's logical interpretation is the best that can be made of the Dutch Book argument, in the next section I argue that there is a better argument, from de Groot, that degrees of belief should conform to the probability calculus. This argument can be made to fit nicely with Howson's logical interpretation. However, as I note in the next section, this argument entails significant philosophical commitments. In conclusion, I also note that there are other arguments which can avoid these particular philosophical commitments.

2 The Dutch Book argument

I take the Dutch Book argument to be composed of four elements. These are first, the bet (or a betting situation); second, an account of bets as representations of degrees of belief; third, the notion of a fair bet; and finally, the Ramsey-de Finetti theorem(s). (The following account follows the version of the argument in [HowUrb93].)

A bet on a proposition is a contract between a bettor and a bookie. The bettor agrees to give the bookie something of value b, if some proposition A comes to be accepted as false, and the bookie agrees to give the bettor something of value a if the proposition comes to be accepted as true (after, perhaps, consultation with an Oracle). The possible losses and gains from the bettor's perspective can be represented by a payoff table

A	Payoff
T	$+a$
F	$-b$

Normalizing the odds $p = (b/a)/(1 + b/a) = b/(a + b)$ gives the usual form

A	Payoff
T	$S(1-p)$
F	$-Sp$

p is known as the betting quotient, S as the stake. A bet against a proposition is one with the payoff signs reversed.

Betting quotients are taken as related to degrees of belief: the longer the odds a bettor is willing to accept against a proposition, the more certain the bettor is of the truth of that proposition. A *fair* betting quotient is one the bettor believes is fair to both sides in that he or she believes the potential loss or gain to be equal for both sides of the bet. It stands to reason that if a bettor thinks a betting quotient on A is fair, he or she should be willing to take either side of the bet on A (that is, either the bet on A or against A). The Ramsey-de Finetti theorem, or the Dutch Book argument, shows that fair betting quotients must be probabilities (that one side of the bet does not lead to a sure loss only if fair betting quotients obey the Kolmogorov axioms.) A converse argument shows that if fair betting quotients are probabilities, a bettor avoids sure losses.

These four ingredients when put together, it is argued, show that the probability calculus imposes consistency constraints on our degrees of belief. (De Finetti in for example [deF31] terms degrees of belief that obey the probability calculus *coherent*.) These constraints are claimed to give us an epistemology of great power (surveyed, for example, in [HowUrb93] and [HowUrb06]).[1]

[1] Another way to put the argument is: degrees of beliefs are betting quotients. To

3 Degrees of belief as behaviour

I shall now concentrate on the second ingredient, the link between bets and degrees of beliefs. It has been claimed that the link if very close indeed – that bets are actually degrees of beliefs, that is, that degrees of belief are nothing other than a psychological stand-in for odds actually given on propositions. This can be fairly called a behaviourist approach, and it is well-known to be unsatisfactory, since odds actually offered my bear no relation to actual strengths of beliefs. Still, it is worth looking at the details of why the behaviourist interpretation won't work, since the reasons it won't work apply to other interpretations of the Dutch Book argument as well.

Some people do not like to bet. Consider the Reverend, who considers gambling a sin: he isn't willing to give any odds.[2] Therefore, no bet will elicit his degree of belief, since he refuses to bet. However, he does have degrees of belief. Therefore, bets and degrees of belief cannot be equated. The Reverend's unwillingness to bet is just one extreme of a more general problem.

Even if someone is willing to bet, bets will not in general add up in such a way as to allow a determination of the value of joint bets. This means that even if bets did represent degrees of beliefs of propositions, they still might not represent degrees of beliefs in combinations of those propositions. The problem stems from the non-linear value of money: combined bets denominated in money may not simply be a combination of their component bets. For example, I may prefer to have a better deal when buying multiple bets, rather than buying them singly, as is required in the Dutch Book argument for the third axiom. I may have set aside a certain amount of money aside for my month's gambling, or I may just be strongly in favour of bulk discounts.

Another way to put it is that the fair betting quotient p is a ratio derived

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offer certain betting quotients is stupid. Therefore, certain degrees of beliefs are stupid. All and only betting quotients, and hence beliefs, that obey the probability calculus are non-stupid. Therefore, in order to be non-stupid, our degrees of beliefs should confirm to the probability calculus. 'Stupidity' should be understood here as a technical term meaning 'leading to a sure loss/gain'. One can deny that this technical use of 'stupidity' is a good explication of the ordinary language usage, cf. [Haj05].

[2] And not without scriptural support, of course. Matthew (5:33-37, KJV) has Jesus disallowing emphatic speech, and so expressions of degrees of certainty, at least: "Again, ye have heard that it hath been said by them of old time, Thou shalt not forswear thyself, but shalt perform unto the Lord thine oaths: But I say unto you, Swear not at all; neither by heaven; for it is God's throne: Nor by the earth; for it is his footstool: neither by Jerusalem; for it is the city of the great King. Neither shalt thou swear by thy head, because thou canst not make one hair white or black. But let your communication be, Yea, yea; Nay, nay: for whatsoever is more than these cometh of evil." [Kra96] alerted me to this passage, Marta Vlasaková helped me find it.

from a particular betting situation in which the bettor is willing to risk forfeiting some amount of money b to get an amount a if the proposition being betted on is true. However, b and a will obviously not be the same in all situations. So we cannot derive a general value for p. (This problem is well known: it is perhaps most forcefully expressed in [Sch86].)

There are three traditional responses. The first is that we should keep the amount of money involved in the bets small (I do not know the origins of this response: it can be found in [deF37]). But then we encounter a Goldilocks problem. If amount of money on offer is too small, I won't reveal my preferences since I can't be bothered to put out the effort to protect myself against losses (or to maximize my minimal gains). If the amount is too large we're right back where we started: I won't be willing to bet (or I may choose to hedge my bets). And this response does not address the problem of packages of bets: two separate bets might fall under threshold at which I will cease to add the value of my bets, but adding a third might push them above. The problem obviously gets worse the more bets are under consideration.

The second response is that bets should be denominated in a utility currency, in which each unit of currency has equal utility. Thus, any reasonable person would see that bets should add in the required way. Savage [Sav71], for example, following Smith [Smi61], refers to the construction of such a lottery. We find some lottery mechanism which our subject is willing to say is fair, in that each of the produced outcomes, say, tickets, coloured balls or spun pointers ending up at some point on a wheel, are for him or her equally likely. These can then serve as units of currency in a bet, each unit having the value of the prize divided by the total number of outcomes.

This response makes the strong assumption that for each bettor and betting situation, a suitable mechanism for producing a utility currency can be found. But there are people like the Reverend, for whom no such lottery exists. And we still cannot rule out the odd few who collect lottery tickets, are repulsed by spinning arrows, or have other attachments or aversions to chancy devices. These people may agree to the use of the currency, but it would not reveal their degrees of belief.

The final fix for the Dutch Book argument's problems is to require that players be compelled to bet (This seems to have been consider by de Finetti, albeit not wholeheartedly, in [deF37], p. 102). But this will not get *my* true odds: having a strong aversion to betting, I will respond by giving you odds at which I think I will lose, so I don't have to accept any filthy lucre. The Reverend will likewise be unmoved, for he welcomes martyrdom. It is also difficult to see how compulsion shows that degrees of belief should follow the probability calculus: it only seems to show that, unless you are willing

to be a martyr, you should try to please the compeller.[3]

The reader may wonder why I am wasting time beating a dead horse – the objections, and responses, I have discussed are well known. My aim in the foregoing has not been not to repeat what has been said at length, but to clearly spell out why a link between beliefs and betting behaviour is not strong enough to justify the argument for (at least) the Dutch Book argument for the third axiom. In the next section I will argue that for the same reasons that the link is not strong enough in the literal case, it will also not be strong enough in any interpretation which takes the Dutch Book argument to be an idealization.

4 The as-if interpretation

A standard defence of the Dutch Book argument is that it portrays an ideal agent, not caught up with worries about actual money, which we should find compelling. One way to idealize the betting situation is to not require that money change hands, but only to consider what would happen were money to change hands. This particular interpretation was put forward in [HowUrb93], where they gave a counterfactual interpretation of the Dutch Book argument:

> Attempts to measure the values of options in terms of utilities are traditionally the way people have sought to forge a link between belief and action, and much contemporary Bayesian literature takes this as its starting point. We do not want to deny that beliefs have behavioural consequences in appropriate conditions, they clearly do, but stating what those conditions are with any precision is a task fraught with difficulty, if not impossible... [T]he conclusion we want to derive, that beliefs infringing a certain condition are inconsistent, can be drawn merely by looking at the consequences of what *would* happen if anyone *were* to bet in the manner and in the conditions specified. ([HowUrb93], p. 77)

This interpretation seems superior to the literal one. It appears to avoid problems involving the currencies involved in betting, as well as those associated with unwilling bettors, since no money actually changes hands. This takes us some way to removing the extraneous elements of the Dutch Book argument.[4]

[3] For some, this might be enough. For example, Hobbes, whose account of psychology in Leviathan seems to be what proponents of traditional Dutch Book arguments have in mind, famously argued that a contract entered into under compulsion is valid (in Leviathan, Chapter XIV). Whatever the validity of Hobbes' view, it still does not show that degrees of belief are betting odds produced under compulsion, unless we are willing to accept a psychology in which degrees of belief just are betting odds, no matter how produced.

[4] Colin Howson no longer adheres to this interpretation, as we shall see. I discuss it, however, because it seems to me that this interpretation is the best possible of its kind, and determining where it goes wrong shows why the Dutch Book argument cannot be

Willingness to bet in this interpretation is, of course, a counterfactual, or subjunctive, matter. The standard semantics for dealing with counterfactuals are Lewis-Stalnaker semantics. Unfortunately, using this semantics will make the Dutch Book argument for the third axiom invalid, as can easily be seen:

If you were to bet on A you would regard p as a fair betting quotient.

If you were to bet on B you would regard q as a fair betting quotient.

Therefore

If you were to bet on A and on B, you would regard p and q as fair betting quotients.

This is an instance of the so-called counterfactual fallacy of strengthening the antecedent, that is the argument from 'If A were the case then C' to 'If A and B were the case then C'. To see that it is a fallacy, substitute "The match is struck" for A, "The match bursts into flame" for C, and "The match is soaked in water" for B. (This paragraph is based on an argument from [Ana93]. A discussion of strengthening the antecedent of counterfactual conditionals can be found in [Lew73], 17.)

The argument cannot be fixed by choosing some other semantics, as the following example shows. Consider Harold, known to all as Dirty Harry – deservedly so, given his hygienic proclivities. Dirty Harry lives in Prague, and it is summer. He is feeling suicidal (although he lives in a wonderful city, few people will approach him), and is disposed to kill himself. But, it's hot, and he is also disposed to have a beer. The beer drinking disposition may preclude his disposition for suicide. Perhaps he no longer feels suicidal because the beer is so good, or perhaps because he gets too drunk to remember his troubles. Or perhaps he accidentally stumbles in front of tram, and is run over before he can kill himself by his own hand. Conversely, suicide precludes beer drinking. So, Dirty Harry's beer drinking disposition can block his disposition to commit suicide, and vice versa.

Any logic which faithfully represents dispositions to behaviour will also represent dispositions in general not being serially or jointly realizable. This means that the Dutch Book argument, at least for the third axiom, is invalid, since it assumes that dispositions *are* so realizable. The as-if interpretation shows us exactly what is wrong with the Dutch Book argument, and why it cannot be saved. If we represent the Dutch Book as being about consequences, then even counterfactual bets and degrees of belief won't match. If we represent the Dutch Book argument as not being about consequences, it's not a Dutch Book argument, since there's no contract between a bettor

saved. As well, this interpretation also parallels Ramsey's view that "... the meaning of a sentence is to be defined by reference to the actions to which asserting it would lead, or, more vaguely still, by its possible causes and effects." [Ram27], p. 57.

and bookie.[5] If we are to save the Dutch Book argument, then, we must remove all considerations of dispositions to bet from the argument, and so of money, no matter how abstractly considered.

5 The 'logical' interpretation

Ramsey had made the claim that subjective probability involved consistency, like deductive logic. Logic has already been de-psychologized, so taking a cue from Ramsey, we might find some inspiration from logic for the interpretation of the Dutch Book argument. Colin Howson has recently taken up this task (for example [How03] and [HowUrb06]).

First, a familiar fact: a truth valuation for a set of sentences is consistent if it can be extended to cover all sentences in the language in accordance with the basic semantic definition, which lays down rules for truth assignments. To use Howson's example, if we were, using classical logic, to evaluate A $\rightarrow$ B and A as true, but B as false, there would be no assignment of truth values to all the other sentences of the language in accordance with the basic semantic definition. As he puts it, the problem is to solve a system of equations where we are given $v(\mathrm{A} \rightarrow \mathrm{B}) = 1$, $v(\mathrm{A}) = 1$ but $v(\mathrm{B}) = 0$. There is no such solution to such a system of equations of truth assignments, and so it is inconsistent.

According to Howson, the parallel notion of valuation for probability is that of assignments of fair betting quotients to propositions. An assignment of betting quotients to a set of propositions is consistent if it can be extended to an assignment over all propositions. Betting quotients serve as a model for degrees of belief in the same way that truth values serve to model the notion of truth: they share some common features of interest. But just as a truth valuation is mostly independent of a particular theory of truth, so a valuation of fair betting quotients is intended to be free of a substantial theory of uncertainty. Betting quotients are meant to serve as the semantic correlates of degrees of beliefs, not by any particular assignment to the betting quotients, but as groups of possible distributions of fair betting quotients. The notion of sure loss is no longer tied to eliciting particular degrees of belief, but serves as a heuristic to interpret fairness, as truth in a valuation is not tied to any particular distribution of truth values.

Given a particular assignment of fair betting quotients, we find that it

[5] If the Dutch Book is a dramatization, then either it has drama or not. With drama it is invalid. Without drama it is pointless. I am only claiming that an idealization of the Dutch Book argument that removes the notion of consequences is fatal to the argument. There are other idealizations of the argument that are, for some purposes, reasonable. For example, we could assume the bettor is logically omniscient, has betting quotients defined over all the states of the world, can tell the difference between the odds 89762:1 and 89761:1, and has access to an Oracle to settle wagers.

can only be extended to all propositions if it is fair. But we also know that betting quotients can only be fair if (and only if) they obey the rules of the probability calculus. Howson draws from this the lesson that we can treat fair betting quotients as semantic objects, the analogue of truth values. The associated syntax is simply the probability calculus. The Ramsey-de Finetti theorems serve as a kind of soundness and completeness theorems, showing the syntax and the semantics to be in complete agreement.

Adherence of degrees of belief to the probability calculus is therefore consistency, and this adherence is to be justified in the way that adherence to logical consistency is. According to Howson, after Frege adherence to consistency is not justified by appeal to actual or imagined consequences. Instead, it is justified in terms of adequately capturing some feature of reasoning of interest. We can amplify on Howson's account. There is no angry god of logic who hurls lightening bolts at the inconsistent. There will always be circumstances where inconsistency is harmless (or perhaps even helpful). Therefore, attempts to justify logic in terms of (good or bad, imagined or real) consequences will always fail. Instead, logic is a tool for exploring or modelling certain objects of interest, like, for example, certain notions of truth. Similarly, attempts to link probability and consequences are doomed. But probability is useful for modelling certain features of uncertainty.

This is a non-psychological, depragmatized, reading of the Bayesian interpretation: it is about the assignment of numbers, called fair betting quotients, to propositions. These fair betting quotients may serve as a heuristic, or as an explication of certain aspects of uncertainty. But, like the notion of truth in logic, the notion of uncertainty in Bayesianism remains, according to Howson, to a large degree independent of a substantive theory of uncertainty.

This interpretation of the Dutch Book argument of course severs any link between behaviour and probability on the one hand and belief on the other. It does so because belief does not, in fact, figure in the argument. The link between fair betting quotients and beliefs must therefore be provided by another argument. But we have already seen that the usual way of making such a link won't work.

We are interested in logics because they serve to model certain notions we have, for example, about truth. In fact, the link between the basic semantic definitions of first order logic and certain notions of truth is obvious, even if it is not the last word (which is why we are interested in different types of logics). In the case of fair betting quotients and degrees of belief we have a link provided by behaviour, but not by anything else. In other words, it is hard to see what appeal fair betting quotients have as semantic objects,

other than their being related to degrees of belief by betting, actual or not.[6]

6 Probability from likelihood

Dutch Book arguments (and, for similar reasons, utility theoretic arguments) will not provide the requisite link between probabilities and beliefs. There are, however, alternatives if we consider different conceptions of belief. For example, some take belief to be a representation – a propositional attitude, as opposed to a disposition. Individual beliefs and sets of beliefs are structured in certain ways. One part of this structure is that belief comes in degrees: we believe some things are more likely than others. Using a likelihood ordering and a means of calibration, Morris de Groot [DeG70] showed how to build a representation theorem for degrees of belief (which has mostly been ignored, excepting [Fre82] and [Fre88]).

The basis of the representation theorem is an ordering of beliefs, in terms of a likeliness relation, where the likeliness relation obeys the axioms of qualitative probability, and the propositions of the beliefs form a field. (It also requires that the certain event is more likely than the impossible event, and that every event is as least as likely as the impossible event.) The next step is to calibrate degrees of uncertainty by putting the members of the algebra into correspondence with a reference algebra with a known probability distribution. One way of doing this, as French showed, is to use a visual representation. Although nothing turns on which representation we use, French uses a wheel of fortune – a disk with a (perfectly balanced and oiled) spinning arrow. The arrow's landing at some particular point is an event. We can then construct a field of events which contains points, intervals and combinations of intervals. Using the obvious flat distribution over the events, we can use the normalized length of the arcs to calibrate the uncertainty in the original algebra by matching up each event with a point or an interval on the wheel, equating the certain event with the entire circumference of the wheel, and the impossible event with length 0. Probability then turns out to be the percentage an event takes up on the circumference of the wheel of fortune.

Just as the Dutch Book argument is tied to an account of disposition, the de Groot construction is tied to an account of beliefs as attitudes towards proposition-like entities. The constraints of the construction are taken as

[6] I do not believe that there is a strong distinction between 'pragmatic' and 'epistemic' interpretations of the Dutch Book argument, since any epistemic theory without consequences is hardly of interest. But theories of truth without discernable pragmatic consequences are of interest. This is why truth in a model is more closely related to the notion of truth than a depragmatized notion of betting is linking to the notion of belief. Of course, truth may be pragmatic notion. But it would be a severe theory that grounded truth on immediate consequences, which is what the Dutch Book does with belief.

plausibly being an explication of the structure of the attitudes and their contents. This fits well with a logical interpretation like Howson's. The underlying semantics are proposition-like entities with some structure. The linking of the events in the underlying algebra with the reference distribution serves to show that the probability calculus completely captures the underlying likeliness relation. There is no longer any need for actualizations of dispositions, and the semantics of the likeliness relation is naturally linked to the syntax of the probability calculus.

I am not arguing that we now have set once and forever the structure of partial beliefs. Two obviously questionable assumptions are the total ordering of beliefs and the sharp correlation of the reference experiment and the algebra of beliefs. Further investigation is required to show in which ways the construction can be plausibly weakened and generalized, for example.

7 Problems with probabilities from likelihood, further prospects

There are several reasons why one might express scepticism about the prospects of this representationalist programme of justification. The representationalist account of belief entails significant philosophical commitments. It takes beliefs to be open to introspection. It further assumes that beliefs are proposition-like entities, or at least that they can be represented as such. It requires the assumption of a flat distribution. Since there is no element of risk or compulsion, one could lie about one's degrees of beliefs.

I begin with the last: how, absent a sanction, we can give a reason for someone to conform their degrees of beliefs to the probability calculus? The traditional Dutch Book argument at least aims to show that if your degrees of belief are somehow not aligned with the probability calculus, something bad will happen to you. There are no sanctions in the de Groot construction.[7] So according to the representationalist account, someone could lie about their degrees of belief, or even have incoherent beliefs (which they do not reveal). However, if I am right, the Dutch Book argument also does not establish that someone could not lie about their beliefs, much less suffer from those lies. It is certainly true that we cannot determine if someone accurately represents their degrees of beliefs, but this is beside the point, for representationalist does not equate belief with action.

The question of introspection is much trickier, turning as it does on a

[7] I should also point out that, even though I do not advocate such an approach, sanctions can be added to the construction by introducing penalties via the reference experiment. For example, using the wheel of fortune, areas can be normalized to betting quotients, and the usual machinery then applied.

major debate in the philosophy of mind. The representationalist view does indeed entail significant philosophical commitments, as does a dispositional view of belief. The point of this paper is that views about what beliefs are determine what sorts of arguments should be made for equating degrees of belief with probabilities. It is true that it would be a strong assumption that we can always conjure up an appropriate representation to scale the probabilities. Even though flat distributions, which can be used as reference distributions, seem very natural in many cases (witness the popularity of logical interpretations of probability based on the maximum entropy principle), they need not apply in every case. Indeed, it probably is the case that there will be no appropriate reference distribution for some propositions: but then it is a strong assumption that there should be a corresponding probability for all propositions.

I have argued that the search for universal sanctions to enforce the probability calculus is pointless. Instead we might undertake the task of explicating the notion of partial belief relative to a given set of purposes. This is, in fact, what most accounts of interpretations of probability focus upon, at least in the philosophy of science. So instead of wondering which way we may be struck down for heresy, we question whether the Bayesian solution to the Duhem-Quine is the correct one. In other words, the foregoing objections turn on what one expects from an interpretation of probability. If we take it that an interpretation should tell us when, on pain of some penalty, we should obey the probability calculus, then a de Groot style representation is probably not ideal. If, however, we take interpretations of probability to be explications of certain notions about belief, then a de Groot style representation might be what we want given a representationalist account.

Still, those unhappy with the Dutch Book argument, and those uncomfortable with propositional attitudes might wish to simply drop beliefs out of the picture completely, and concentrate on something else. There are several alternatives, the most prominent being James Joyce's [Joy98] and Cox's [Cox46]. Neither of their constructions refers directly to belief, and so are free of the difficulties inherent its explication. If we approach these arguments as not being about belief, then presumably they will be taken as explications of pre-theoretical notions. The notion of there being such explications is very much out of fashion. But if we are to make any headway in debates over the foundations of subjective probabilities, then the debate must take place at the level of the explication of belief, or one of its substitutes.[8]

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