MLE OF CAPABILITY INDEX C_p AND ITS APPLICATION IN TESTING HYPOTHESES

Jiří Michálek

Institute of Information Theory and Automation Czech Academy of Sciences
(e-mail: michalek@utia.cas.cz)

ABSTRACT: The contribution deals with the construction of MLE of capability index C_p using different standard deviation estimates of within subgroup variability. Then the corresponding likelihood ratioes are used in the construction of statistical tests on process capability.

KEYWORDS: Capability index, MLE, statistical hypothesis, confidence interval, power of a statistical test

1 Formulation of the problem

We can meet very often the following problem in practice. A quality variable is measured on a product, the manufacturing process is controlled by a control chart and the information obtained from data in the chart is also used for evaluation of process capability. It means we have together with the analysis of a chart an estimate of capability index C_p. Of course under the assumption that the manufacturing proces is in control. Usually such a situation occurs when we use a software for controlling the process, the result of analysis is in the form of a control chart and a point estimate of C_p is given. The original data from subgroups are in many cases definitely lost. But we need to estimate the capability of the process in a long term period, let us say within a couple of days and we have point estimates of index C_p from each control chart at disposal only. The problem is how to construct a new estimate of C_p that must be based on marginal estimates of this index obtained from each control chart separately. In the language of mathematical statistics this problem can be formulated as follows: measured values of a quality variable are collected in subgroups of n observations, each control chart contains k subgroups and we have together N charts. Very often in practice we can assume mutual stochastic independence among observations. The cases with stochastic dependence would have to be solved quite individually. Let us denote by $C_p^{(1)}, \ldots\ldots C_p^{(N)}$ the point estimates from N control charts. Now it is very important to realize the fact that the estimate of index C_p is based on the estimate of inherent variability level that is estimated within subgroups. The quality of the point estimate is then strongly dependent on the form of used standard deviation estimation.Unfortunately, this situation is often ignored in practice. There are three possibilities usually used how rto estimate the level of inherent variability: average sample range, average sample standard deviation and pooled standard deviation.

2 MLE of index C_p

First we must derive probability density functions for each type of the estimate of C_p . The case with pooled standard deviation is solved by use of χ^2-distribution, the other cases are somewhat complicated as we must use the normal distribution approximation for describing the asymptotic behaviour of the sample range and sample standard deviation. Then the common probability density fuctions for all three cases are derived together with the form of the corresponding MLE of C_p. The density function for pooled standard deviation has a very important property because it belongs to the exponential density family. This property is not present by the other considered densities. There is no problem to prove consistency of all three MLE´s.

3 Tests on capability and their power

The answer on the question whether a manufacturing process is capable or not is very important for practice. It is directly connected with hypothesis H: $C_p=C^0$ against alternative hypothesis $C_p<C^0$, where C^0 is a demanded value and also the opposite question can be interested in some cases. When pooled standard deviation is used then the corresponding likelihood ratio has the monotony ratio property and therefore there exists uniformly most powerful test even. Unfortunately, this situation does not occur in two other cases and one or two sided test can be based on the construction of confidence intervals for theoretical values of index C_p. Finally, the suggested tests are compared with respect to their powers regarding the number of observations. The formulas for test powers are derived and it is shown that the application of pooled standard deviation gives a somewhat better results than using average sample range or average sample standard deviation.

References

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