An Overview of Multichannel Image Restoration Techniques

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Abstract

In this paper we provide a general overview of multichannel (MC) restoration techniques. We do not give a detailed description of individual methods but rather provide a classification of MC techniques. We present a general MC degradation model in a vector-matrix notation and we state some useful properties of matrices in use. Classification of restoration techniques is done according to several different projections: probabilistic, algorithmic, amount of prior knowledge, and a relation of MC techniques to single-channel (SC) techniques. The MC-SC relation is discussed in detail with a stress on blind reconstructions, where the advantages of using MC techniques over SC ones are particularly noticeable.

1 Introduction

By the term multichannel (MC) image we mean multiple image planes (channels) that are typically obtained by an imaging system that measures the same scene using multiple sensors and/or an imaging system that measures the same scene at different time instants. If the channels are different frequency bands we talk about multispectral images. Color images represent a special case of great interest, see [25], [12], [24]. Other examples are wave radiometric images and image sequences. By the latter case we mean a time sequential measurement of the same scene and the channels represent the different time frames. Channels do not have to be necessarily different frequency bands or time samples, but they could also be different resolution representations of the original image. This is for example the case of reconstruction of a high-resolution image from multiple low-resolution images [6] or a wavelet subband decomposition of a single-channel (SC) image [2].

In many imaging applications including astronomical speckle images, remote sensing, and medical imaging, an observed image $g(x,y)$ can be accurately represented by the sum of additive noise $n(x,y)$ and the 2-D convolution of the true image $f(x,y)$ with a linear shift-invariant blur, also known as the *point-spread function* (PSF), $h(x,y)$. That is,

$$g(x,y) = (h * f)(x,y) + n(x,y) \ , \qquad (1)$$

in which $*$ denotes the 2-D linear convolution operator. The blurring function $h(x,y)$ results from a relative motion of measured objects to imaging systems, defocused lenses, and a turbulence of a media in which imaging systems measure (atmospheric turbulence), among others. The noise term is not negligible. Common types of noise are electronic, photoelectronic, film grain and quantization noise. The problem of restoring the the original image $f(x,y)$ requires the *deconvolution* of the PSF $h(x,y)$ from the degraded image $g(x,y)$.

We can deal with the MC restoration problem on a SC level. Thus restoring individual channels separately but because MC images exhibit strong between-channel correlations this approach is not what we want. All the channels of the MC image should be processed as one entity. There are two advantages of restoring images in a multichannel framework. First, the cross-correlation terms are exploited to achieve better restoration results. Second, the MC framework allows for the possibility of having cross-channel degradations. Such cross-channel degradations may occur in the form of channel crosstalks, leakages in detectors, and spectral blurs. The phenomenon of spectral blurs is also called chromatic aberration or dispersion and occurs in certain devices, e.g. optical fibers. There are some special restoration techniques in the MC framework for which no true SC counterparts exist, see [20], [10]. Such intrinsically MC restoration techniques introduce an additional mild assumption of *coprimeness*, i.e. degradation (blurring) operators in the z-domain do not have any common factor except of a scalar constant. The intrinsically MC restoration was successfully applied to SC problems by dividing the single frame into subframes and thus entering the MC framework.

An important assumption for a good restoration of M-C images, that is not present in SC processes and that must be satisfied, is the channel registration. Before the restoration process can take place the image channels must be properly registered. In the case of multispectral images we often assume that the assumption is automatically satisfied, but in the case of image sequences registration could be a serious and very difficult task (for an overview of registration methods see ....).

The literature of the SC image restoration is very extensive and there exist several overview articles, e.g. [3], [14]. Most of the MC restoration algorithms are just a straightforward generalization of the SC restoration algorithms but the notation is somewhat cumbersome. Therefor, we highly recommend the reader to start with the SC restoration techniques and then the MC restoration techniques will be comprehensible.

The paper is organized as follows. In Section 2 the vector-matrix notation of the MC degradation model is introduced. Structures of blurring matrices are discussed together with their special properties that enable us to handle the matrices in a computationally feasible way. In Section 3 several approaches how to categorize the MC restoration algorithms are proposed and some techniques are sketched in more detail.

2 The MC image restoration theory

We refer to images in their digital (discrete) form. Common mathematical representation of sampled images is in a vector form. For notational conveniences and without loss of generality we assume square images. Then an image of support $M \times M$ samples (pixels) is represented by a vector of size N ($N = M^2$), where the image samples were ordered lexicographically by stacking either the rows or the columns of the original image [3]. Let a MC image has P channels f_i each of support $M \times M$, when $\mathbf{f_i}$ is the i-th channel in the above vector form, then such a MC image could be represented by the vector $\mathbf{f}$ of size PN in two different ways as follows:

$$\mathbf{f} = [\mathbf{f_1}^T \mathbf{f_2}^T \ldots \mathbf{f_p}^T]^T \tag{2}$$

or

$$\begin{aligned} \mathbf{f} \;=\; & [f_1(0) f_2(0) \ldots f_P(0) f_1(1) f_2(1) \ldots \\ & f_P(1) \ldots f_1(N-1) \ldots f_P(N-1)]^T, \end{aligned} \tag{3}$$

where T denotes the transpose of a matrix or a vector. In representation (2) the channel vectors are stacked and in representation (3) the channel vectors are interlaced. Using the image vector notation one can define linear image operators in a matrix form. When the vector-matrix representation is used, the linear degradation model (1) can be rewritten as

$$\mathbf{g} = \mathbf{H}\mathbf{f} + \mathbf{n}\ , \tag{4}$$

where $\mathbf{g}$, $\mathbf{f}$, and $\mathbf{n}$ are the degraded (observed) MC image, the original MC image, and the noise, respectively. These vectors are formed according to (2) or (3) and have size PN. The square matrix $\mathbf{H}$ of dimension $PN \times PN$ represents intrachannel and interchannel blur operators, i.e. linear 2-D convolution of channels with shift-invariant blurs. For the stacked-vector representation (2) the blur matrix is of the form

$$\mathbf{H}_S = \begin{bmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} & \ldots & \mathbf{H}_{1P} \\ \mathbf{H}_{21} & \mathbf{H}_{22} & \ldots & \mathbf{H}_{2P} \\ \vdots & \vdots & & \vdots \\ \mathbf{H}_{P1} & \mathbf{H}_{P2} & \ldots & \mathbf{H}_{PP} \end{bmatrix}, \tag{5}$$

where the submatrices $\mathbf{H_{ii}}$ and $\mathbf{H_{ij}}$, for $i \neq j$ are of dimension $N \times N$ and represent the within-channel and the between-channel degradation, respectively. If we assume shift-invariant blurs then $\mathbf{H_{ij}}$ become block-Toeplitz matrices. By padding $\mathbf{g}$ and $\mathbf{f}$ appropriately with zeros, so that results of linear and circular convolution are the same, $\mathbf{H_{ij}}$ become block circulant matrices, However, $\mathbf{H}_S$ is not block circulant since shift-invariance is not assumed across channels, i.e. $\mathbf{H_{ij}} \neq \mathbf{H_{i+k,j+k}}$. The matrix $\mathbf{H}_S$ is called block block-semi circulant (BBSC) of order (P,N).

When we use the interlaced-vector representation (3) in (4) the blur matrix is of the form

$$\mathbf{H}_I = \begin{bmatrix} \mathbf{H}(0) & \mathbf{H}(1) & \ldots & \mathbf{H}(M-1) \\ \mathbf{H}(M-1) & \mathbf{H}(0) & \ldots & \mathbf{H}(M-2) \\ \vdots & \vdots & & \vdots \\ \mathbf{H}(1) & \mathbf{H}(2) & \ldots & \mathbf{H}(0) \end{bmatrix} \tag{6}$$

and it is circulant at the block level. In general each $\mathbf{H(i)}$ is an arbitrary $PM \times PM$ matrix, but if shift-invariant circular convolution is assumed $\mathbf{H(i)}$ becomes

$$\mathbf{H}(i) = \begin{bmatrix} \mathbf{H}(i,0) & \mathbf{H}(i,1) & \ldots & \mathbf{H}(i,M-1) \\ \mathbf{H}(i,M-1) & \mathbf{H}(i,0) & \ldots & \mathbf{H}(i,M-2) \\ \vdots & \vdots & & \vdots \\ \mathbf{H}(i,1) & \mathbf{H}(i,2) & \ldots & \mathbf{H}(i,0) \end{bmatrix}, \tag{7}$$

which is also circulant at the block level $\mathbf{H(i,j)}$. Each $P \times P$ submatrix $\mathbf{H(i,j)}$ has the form

$$\mathbf{H}(i,j) = \begin{bmatrix} h_{11}(i,j) & h_{12}(i,j) & \ldots & h_{1P}(i,j) \\ h_{21}(i,j) & h_{22}(i,j) & \ldots & h_{2P}(i,j) \\ \vdots & \vdots & & \vdots \\ h_{P1}(i,j) & h_{P2}(i,j) & \ldots & h_{PP}(i,j) \end{bmatrix}, \tag{8}$$

where $h_{kl}(i,j)$ is the value of the PSF at the position (i,j) for degradation between channels k and l. $\mathbf{H}_I$ is called a block semi-block circulant (BSBC) matrix of order (P,N). Note that, for example, a three-channel image of moderate size 100×100 requires a 30000×30000 matrix $\mathbf{H}$. Clearly, to handle such matrices is virtually impossible in any readily available software. Katsaggelos *et al.* in [12] described and analyzed several basic properties of BSBC matrices in an one-dimensional MC model that provide computationally feasible manipulation with BSBC's and BBSC's. The results also apply directly to our two-dimensional M-C model, compare with [2]. It has been shown that BSBC and BBSC matrices are closed under addition, multiplication and inversion. Two fundamental properties of BSBC and BBSC matrices that allow the extension of frequency domain SC image processing techniques to MC images are the following.

PROPERTY 1 An BSBC(P,N) matrix $\mathbf{H}$ can be reduced to a block diagonal matrix $\mathcal{H}$ by the similarity transformation, $\mathbf{H} = \mathbf{\Omega}_{PN}\mathcal{H}\mathbf{\Omega}_{PN}^{-1}$, where $\mathcal{H}$ is given by

$$\mathcal{H} = \begin{bmatrix} \mathcal{H}_{00} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathcal{H}_{11} & \dots & \mathbf{0} \\ \vdots & \vdots & & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathcal{H}_{(N-1)(N-1)} \end{bmatrix} \tag{9}$$

with $\mathcal{H}_{ii}$ a $P \times P$ matrix, where $i = 0, \dots, N-1$. The $PN \times PN$ matrix $\mathbf{\Omega}_{PN}$ is the P-channel 2-D discrete Fourier transform (DFT) for the interlaced-vector representation (3).

The matrix $\mathbf{\Omega}_{PN}$ is given by $\mathbf{\Omega}_{PN} = W \otimes I_P$, where $\otimes$ denotes the Kronecker product, W is the 2-D DFT for images with $M \times M = N$ support ordered lexicographically, and I_P is a $P \times P$ identity matrix. For BBSC matrices a similar property holds.

PROPERTY 2 An BBSC(P,N) matrix $\mathbf{D}$ can be reduced to a sparse matrix $\mathcal{D}$ by the similarity transformation, $\mathbf{D} = \mathbf{\Gamma}_{PN}\mathcal{D}\mathbf{\Gamma}_{PN}^{-1}$, where $\mathcal{D}$ is given by

$$\mathcal{D} = \begin{bmatrix} \mathcal{D}_{11} & \mathcal{D}_{12} & \dots & \mathcal{D}_{1P} \\ \mathcal{D}_{21} & \mathcal{D}_{22} & \dots & \mathcal{D}_{2P} \\ \vdots & \vdots & & \vdots \\ \mathcal{D}_{P1} & \mathcal{D}_{P2} & \dots & \mathcal{D}_{PP} \end{bmatrix} \tag{10}$$

with $\mathcal{D}_{ij}$ a $N \times N$ diagonal matrix, $i, j = 1, \dots, P$, and $PN \times PN$ matrix $\mathbf{\Gamma}_{PN}$ is the P-channel 2-D DFT for the stacked-vector representation (2).

The matrix $\mathbf{\Gamma}_{PN}$ is given by $\mathbf{\Gamma}_{PN} = I_P \otimes W$. The inverse DFT's $\mathbf{\Omega}^{-1}$ and $\mathbf{\Gamma}^{-1}$ are related to their Hermitian conjugate by $\mathbf{\Omega}^{-1} = (1/N)\mathbf{\Omega}^H$ and $\mathbf{\Gamma}^{-1} = (1/N)\mathbf{\Gamma}^H$, respectively. The relation between elements of $\mathcal{H}$ and $\mathcal{D}$ is defined as $\mathcal{H}_{ii}(p,q) = \mathcal{D}_{pq}(i,i)$, $i = 0, \dots, N-1$; $p, g = 1, \dots, P$.

Properties of BBSC and BSBC matrices together with the FFT provide us with a powerful tool in the image restoration. From the above discussion it follows that any linear equation with BBSC or BSBC matrices can be handled in a computationally efficient way in the frequency domain.

3 Categorization of MC algorithms

We may approach the problem of categorization from several different points of view: *probabilistic*, *algorithmic*, *prior knowledge*. For the time being let us consider any general image restoration method (not necessarily only MC). From a probabilistic point of view we distinguish two categories: *stochastic* and *deterministic*. Stochastic methods model images as random fields and estimate an original image (or images in the MC model) as a most probable realization of a certain random process. The well known Wiener filter — linear minimum mean square error (LMMSE), the maximum likelihood (ML) approach [21], and the maximum a posterior (MAP) approach [5] belong to the stochastic category. They utilize several statistical quantities, e.g. noise variance, auto-correlation and cross-correlation of images. Estimating the statistics is often the key point in a good image restoration and stochastic methods are very sensitive to perturbations in estimates of the statistics. Another limitation is the fact that stochastic methods are computationally feasible only when homogeneous random processes are considered and the noise and image random processes are assumed to be uncorrelated. Deterministic methods find an estimate of the original image by minimizing a norm of a certain residuum. Although often no probabilistic estimates are required here, deterministic methods are subject to other constraints. The generalized inverse filter is the most simple direct deterministic method which does not require any constraints. However it usually gives quite useless results because the restoration problem is generally ill-posed and the inverse filtering cannot handle such cases. Constrained least squares (CLS) restoration [25] is a deterministic method that incorporates a regularizing term. This term is essentially a smoothness constraint which suggests that most images are relatively flat. Other deterministic methods are the iterative blind deconvolution (IBD) method [1], the simulated annealing (SA) algorithm and the NAS-RIF algorithm. They apply several constraints like nonnegativity of the original image, smoothness of the estimate, finite support of convolution masks, etc.

In the framework of algorithms we distinguish three categories of image restoration techniques: *one-step*, *iterative*, and *recursive*. One-step methods estimate the original image in a process in which prior estimates are not considered. Typical representatives are the inverse filter and the Wiener filter in its classical formulation. Iterative methods, on the other hand, use previous estimates at each iteration to form a new estimate.The iterative restoration methods conduct a search for extrema on an error metric, where the error metric is defined as a quantitative measure of how much the current estimate violates the known constraints. The SA and NAS-RIF methods implement the conjugate-gradient (CG) or the steepest descent (SD) numerical minimization algorithms to search for extrema. The popular IBD method of Ayers and Dainty [1] and many of its different implementations [13, 17, 18] cannot be equated with the steepest-descent search and lack the stable convergence property. The expectation maximization (EM) iterative method in [15] is a good choice for solving the ML estimate and the generalized cross-validation (GCV) iterative method in [22] suits well the CLS restoration. The iterative methods are a very powerful tool especially for a blind restoration but often suffer from a common drawback of local extrema existence. Recursive filters have degraded images on a serial input and thus estimate a new pixel of the original image from pixels already processed. They permit spatial adaptive restoration to be easily incorporated into the filter model and they usually require less memory for storage than one-step or iterative methods. The recursive equivalent of the Wiener filter is the discrete Kalman filter.

Any partial prior knowledge of the convolution operator ($\mathbf{H}$), the original image ($\mathbf{f}$) or the noise ($\mathbf{n}$) plays a fundamental role in the image restoration process. Considering this, we may categorize restoration algorithms according to the amount of prior knowledge as follows: *blind*, *non-blind* and something in between. The blind restoration estimates $\mathbf{f}$ without any precise knowledge of PSF. In its most general form, when only the degraded image is available and no other constraints are given, the blind restoration problem is virtually unsolvable. The practical difficulties with estimating both PSFs and $\mathbf{f}$ include a high computational complexity for PSFs with large support, instabilities of estimation algorithms, and ambiguities of solutions. To overcome these problems following assumptions are commonly made: $\mathbf{f}$ and PSFs are positive, the PSF is symmetric and zero-phase, and the PSF has a known parametric form consisting of only a few parameters. More assumptions we have then smaller is the search space. However, it was proved in [10] that in the MC framework PSFs can be determined exactly in the absence of noise and stably estimated in its presence, if two or more degraded images are used and if the z-transforms of their PSFs are coprime. An overview of blind deconvolution techniques is given in [14].

None of the above categorization strategies emphasize appropriately the differences in MC methods. Before we try to do so, let us have a look at two MC model specifications.

3.1 MC model specifications

When we talk about the MC restoration (deconvolution) we must note that two distinguishable restoration goals exist. Firstly, we may want to estimate P channels of a P-channel degraded image. Secondly, we may want to estimate one original image from P differently degraded images. In both cases we talk about the MC restoration. Note that in the latter case we use the term image instead of channel because it reflects more our intuitive notion of what are the differences between the two MC models.

The 1-to-P degradation model

In the 1-to-P configuration we obtain P degraded images from an imaging system and the restoration goal is to estimate an original image. The degradation model in the vector-matrix notation is given by (4) in general, but for this specific configuration when the stacked-vector image representation (2) is used, the model looks as follows

$$\begin{bmatrix} \mathbf{g}_1 \\ \mathbf{g}_2 \\ \vdots \\ \mathbf{g}_P \end{bmatrix} = \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \\ \vdots \\ \mathbf{H}_P \end{bmatrix} \mathbf{y} + \mathbf{n} \;, \tag{11}$$

where $\mathbf{g}_i$ and $\mathbf{H}_i$ are the i-th degraded image and the blur operator for $i = 1, \ldots, P$, respectively. The vectors $\mathbf{y}$ and $\mathbf{n}$ denote the original image and the noise. In a similar fashion we derive the equation for the interlaced-vector representation (3) of $\mathbf{g}$ and $\mathbf{H}$. The 1-to-P degradation model is typical for image sequences where the degraded images $\mathbf{g}_i$ represent, for example, images acquired at different time instances or at different focus settings. Many MC restoration techniques like in [13, 18, 6, 21, 16, 7, 4] are based on the 1-to-P model.

The P-to-P degradation model

The P-to-P degradation model has P original channels on the input and P degraded channels on the output. The restoration goal is to estimate all the P original channels. The P-to-P degradation model is again given by (4) and construction of $\mathbf{H}$ and $\mathbf{f}$ was described in Section 2 in detail. Clearly, this model is more general than the 1-to-P model and enables between-channel degradations. To simulate the 1-to-P model we set $\mathbf{f}_1 = \mathbf{f}_2 = \ldots = \mathbf{f}_P$ in (2) and (3), off-diagonal elements $\mathbf{H}_{kl} = \mathbf{0}$ for $k \neq l$ in (5), $h_{kl}(i,j) = 0$ for $k \neq l$, and $\forall x, y = 0, \ldots, (M-1)$ in (6). The P-to-P model is associated with broad band imaging systems, which produce multispectral images, and restoration techniques given in [2, 24, 25, 12] are based on it.

3.2 MC algorithm specification

We have seen that the MC degradation models directly extend the SC model and the SC techniques can also be used to deal with MC problems. How exactly the SC techniques are utilized is the question we are going to discuss. The following categorization of MC algorithms is thus based on the way how the SC techniques are incorporated into MC algorithms. We will also see that some new MC approaches exist that does not have a SC version.

SC restoration techniques with additional MC a prior constraints

A naive MC restoration approach is to transform a M-C image into multiple SC images and then apply readily available SC algorithms. This approach was proposed by Hunt and Kubler in [11] for multispectral images (P-to-P model). First a transform that spectrally decorrelates the channels is found and then in the transformation domain the decorrelated images are restored in the SC framework independently. Difficulties arise when a transform that decorrelated the channels cannot be found. Furthermore, this transform does not work when between-channel blurring is present.

Several attempts where made in the blind deconvolution of multiple speckle images, when the degradation model is 1-to-P and the restoration goal is to find the original speckle image. Miura *et al.* in [18] proposed to use the SC IBD algorithm for P degraded images in parallel. The IBD algorithm runs iteratively. In the i-th iteration step it finds P new estimates from corresponding P degraded images using an averaged estimate of the $i-1$ step. The $i-1$ averaged estimate is determined by averaging the best estimates of the $i-1$ step. More robust blind deconvolution of speckle images was suggested in [16], where a SC iterative algorithm based on CG minimization is implemented. An ensemble of speckle patterns enables one to obtain information concerning either the modulus or the phase of the Fourier transform of an original object by either interferometry or bispectral techniques. Thus at each iteration step of the CG algorithm the Fourier modulus or phase of the estimate is set to the estimated modulus or phase of the original image.

Extended SC restoration techniques in the MC framework

Most of the successful SC methods were transformed to the MC framework in recent past years. Taking the vector-matrix notation given in Section 2 we may derive the MC restoration equations from their SC counterparts in a relatively straightforward but tedious manner. A presence of cross-channel blurs and cross-channel correlations are not the only advantages of working in the MC framework. It was already mentioned that the degradation matrix in (4) is ill-conditioned (ill-posed). In other words, some of the singular values of $\mathbf{H}$ are close to zero and thus when the generalized inverse of $\mathbf{H}$ is multiplied by $\mathbf{g}$ to obtain an estimate of $\mathbf{f}$ these singular values amplify the noise present in $\mathbf{g}$. In the frequency domain this corresponds to the fact that the DFT of $\mathbf{H}$ has some elements close to zero and dividing the DFT of $\mathbf{g}$ by these elements results in noise amplification. However, if for example the 1-to-P model in (11) is assumed, $\mathbf{H} = \left[\mathbf{H}_1^T \mathbf{H}_2^T \ldots \mathbf{H}_P^T\right]^T$ becomes in general better conditioned and so is the restoration problem. In the frequency domain this is associated with the fact that the DFTs of different $\mathbf{H}_i$ have in general zeros at different locations.

The MC Wiener filter is well presented in [12] and the MC blind ML restoration using the EM algorithm is proposed in [24]. Both methods as they are presented use the P-to-P model with the interlaced image representation (3). Results in the frequency domain are almost identical to the ones presented previously for the SC cases, see [3, 15] for comparison. In the computational perspective the main difference from the SC case is that when the interlaced image representation is used, the frequency element of a P-channel image is a $P \times 1$ vector and the frequency element of a P-channel operator (matrix) is a $P \times P$ matrix, instead of a scalar. Extension of the CLS restoration to the MC case in which the CV algorithm is used, is given in [25]. The P-to-P model with the stacked image representation (2) is assumed here and the derived technique is similar to the SC case in [22]. The stacked representation results in BBSC matrices and from the Property 2 follows that the final vector-matrix equations are sparse and can be solved quickly.

Several extensions of the SC techniques also exist for the 1-to-P model, e.g. the LMMSE [4], the CLS restoration [7], the ML estimation [21, 23], the MC restoration based on a modified IBD algorithm [13], and the MAP estimation [6] used to restore a superresolution image from several undersampled images.

Any SC problem may be transformed to the 1-to-P model by dividing the degrade image to subimages and assuming that all the subimages are blurred by a single mask. Banham *et al.* in [2] proposed a more complex transformation based on a wavelet subband decomposition. A SC degradation model (4) is first transformed to the P-to-P model by multiplying both sides of the equation with an appropriate wavelet-packet decomposition matrix. Transformed vectors $\mathbf{g}$ and $\mathbf{f}$ are then compositions of the observed image subbands and the original image subbands, respectively. Once we are in the P-to-P model we may apply any of the above methods. The M-C restoration with the previous subband decomposition performs better than a simple SC restoration because the common unrealistic assumption that the whole image is a homogeneous random process is reduced to the assumption that the random process is homogeneous only within subbands.

Intrinsically MC restoration techniques

So far we have discussed the MC techniques that were based on the SC techniques. A very common configuration is what we have called the 1-to-P model in which none of the P channel degradations are known a priory and thus we face the MC blind deconvolution problem. The iterative techniques like EM, GCV, IBD were successfully applied to the blind deconvolution problem in the SC framework when a certain prior knowledge about $\mathbf{H}$, $\mathbf{y}$ and/or $\mathbf{n}$ in (11) is provided. Intuitively we see that the MC case carries more information about the degraded image $\mathbf{y}$ than the SC one if the blur operators $\mathbf{H}_i$ are different. Restoration techniques that estimate the original image by considering the differences in degraded images $\mathbf{g}_i$ we call intrinsically MC. Such techniques operate only in the MC framework.

An interesting approach that is intrinsically MC and which has its origin in 1-D signal processing is based on the high order cepstral information. Petropulu *et al.* in [19] proposed an algorithm that assumed the one-dimensional 1-to-2 degradation model, i.e.,

$$g_i(x) = (h_i * f)(x) + n(x), \qquad i = 1, 2;\ x = 0, \ldots, M \tag{12}$$

where g_i, h_i, f, and n denote the i-th degraded signal, the i-th channel impulse response (in the 2-D case it is called PSF), the original signal of support M+1, and the noise, respectively. The symbol $*$ denotes the convolution operator. This equation is just a rewritten eq. (4). In the absence of noise the cepstra coefficients of g_i are equal to the sum of the cepstra coefficients of f and h_i, and so one can reconstruct the channel impulse responses h_i from the difference of the cepstra coefficients of the degraded signals g_1 and g_2. Once h_i is known any of the standard non-blind deconvolution techniques may be used. Several assumptions must be made for the algorithm to work properly: $h_1(x)$ and $h_2(x)$ are FIR sequences; $f(x)$, $h_1(x)$ and $h_2(x)$ have no zeros on the unit circle; $h_1(x)$ and $h_2(x)$ have no common zeros; there are no zero-pole cancellations between $f(x)$ and the channels $h_1(x)$, $h_2(x)$. Unfortunately, the cepstra coefficients are defined only for 1-D signals and to use the proposed technique for images the PSF $h_i(x, y)$ in (12) rewritten for the 2-D case must be separable, i.e., $h_i(x, y) = h_i^h(x)h_i^v(y)$.

Different intrinsically MC techniques that are being explored just recently are based on the greatest common divisor. In noiseless conditions and taking the z-transforms on both sides of (12) we get

$$G_i(z) = H_i(z)F(z), \qquad i = 1, 2 \tag{13}$$

Let $F(z)$ and $G_i(z)$ be polynomials of degree $R \geq 1$ and $M \geq 1$ with coefficient values $\{f_j\}, j = 0, \ldots, R$ and $\{g_{i_j}\}, j = 0, \ldots, M$, respectively. Then $H_i(z)$ is a polynomial of degree $M - R$. From (13), if the two PSFs $H_1(z)$ and $H_2(z)$ are relatively coprime, $F(z)$ is the greatest common divisor (GCD) of $G_1(z)$ and $G_2(z)$, i.e, if $\mathrm{GCD}\{H_1(z), H_2(z)\} = 1$ then $\mathrm{GCD}\{G_1(z), G_2(z)\} = F(z)$. For the estimates e_i of h_i (in the z-domain $e_i \leftrightarrow E_i(z) = e_{i_0} + e_{i_1}z + \ldots + e_{i_{(M-R)}}z^{M-R}$) follows that

$$E_1(z)G_2(z) - E_2(z)G_1(z) = 0, \qquad E_i(z) = \alpha H_i(z) \tag{14}$$

for some scalar α and by equating the coefficients of like powers of z on both sides of (14), it has the matrix equivalence of

$$\mathcal{S}\mathbf{x} = \mathbf{0}, \tag{15}$$

where the vector $\mathbf{x}$ $(N + M - 2R + 2) \times 1$ contains coefficients $\{e_{1_j}\}$ and $\{e_{2_j}\}$, and the matrix $\mathcal{S}$ contains $\{g_{1_j}\}$ and $\{g_{2_j}\}$ and has size $(N+M-R+1)\times(N+M-2R+2)$. The least square solution of (15) for $\mathbf{x}$ in the noiseless case is the singular vector that corresponds to the zero singular value of $\mathcal{S}$. More generally when the noise is present matrix $\mathcal{S}$ is not singular and the solution corresponds to the singular vector associated with the minimum singular value. If R is not known, which is often the case, we can estimate R by building $\mathcal{S}$ for $R = 1$ and then the number of small singular values is approximately equal to the correct value of R. Extension of the GCD algorithm to the 2-D case is straightforward but because the size of $\mathcal{S}$ increases with the power of two, it is not computationally feasible to use the singular value decomposition (SVD). One way how the 1-D GCD algorithm can be used in the 2-D blind deconvolution problem in a computationally efficient manner is described in [20], where a combination of the Fourier transform and the z-transform is used.

The subspace technique, which was proposed by Harikumar and Bresler [10, 9], is a intrinsically MC approach for the 1-to-P model . Their work extends the 1-D subspace techniques EVAM [8] to the 2-D scenario. Although it is not specifically mentioned the subspace technique is based on GCD. Let us have the original image $f(x, y)$ of size $R \times R$ and $P \geq 2$ degraded images $g_i(x, y)$, $i = 1, \ldots, P$ each of size $M \times M$ that are relatively coprime. Then (14) in the 2-D case holds true for any two g_i, g_j $(i \neq j)$ and by taking the inverse z-transform of (14) we get

$$(e_i * g_j)(x, y) - (e_j * g_i)(x, y) = 0, \qquad 1 \leq i < j \leq P \tag{16}$$

Now, if we form a vector $\mathbf{e}$ $(P(M-R)^2 \times 1)$ from the estimates e_i $((M-R)\times(M-R))$ according to the stacked-vector notation (2), equation (16) can be rewritten as

$$\mathcal{G}\mathbf{e} = \mathbf{0} \tag{17}$$

where the matrix $\mathcal{G}$ consists of block-Toeplitz submatrices of size $M^2 \times (M-R)^2$, which define convolutions with g_i. It has been proved [10] that $\mathcal{G}$ is singular with the nullity equal to one if the size of the original image $f(x, y)$ is correctly estimated as $R \times R$ and the noise is absent. As in the case (15), SVD can be used to find the solution of (17) for $\mathbf{e}$. The computation cannot be performed directly because of the large size of the matrix $\mathcal{G}$. However, the solution of (17) is also equal to the eigenvector of $\mathcal{G}^T\mathcal{G}$ that corresponds to the smallest eigenvalue. The matrix $\mathcal{G}^T\mathcal{G}$ is much smaller than the original $\mathcal{G}$ and its construction is far less complex. Once the estimates e_i of the PSFs h_i are determined the original image can be reconstructed by any of the non-blind deconvolution approaches. At low SNR the subset technique does not give very accurate estimates and an iterative algorithm based on the ML estimates is proposed in [10] as follows

$$\{h_i^{ML}\} = \arg\min_{\mathbf{h}} \mathbf{h}^T(\mathcal{G}^T[\mathcal{B}\mathcal{B}^T]^+\mathcal{G})\mathbf{h} \qquad (18)$$

where $\mathbf{h}$ is formed from h_i according to (2) and $+$ denotes the pseudo-inverse. $\mathcal{B}$ has a similar structure to $\mathcal{G}$ in (17) but for convolutions with the transposed h_i . At the beginning $\mathcal{B}\mathcal{B}^T$ is set to the identity matrix. At each iteration $\mathbf{h}$ is set to an eigenvector of a minimum eigenvalue of $\mathcal{G}^T[\mathcal{B}\mathcal{B}^T]^+\mathcal{G}$ and then the new $\mathcal{B}$ from $\mathbf{h}$ is constructed. The first iterate of the ML algorithm coincides with the subset technique and successive iterates further refine the solution.

4 Summary

We have presented two different ways how to code a M-C image in a vector form: stacked or interlaced vector representation. The corresponding blur matrices were derived. We have used the image degradation model in the vector-matrix notation which has a similar form both for the SC and the MC cases. This enables us to transform existing SC methods to the MC framework in a relatively straightforward manner. Apart from introducing between-channel blurs and cross-correlation terms in the restoration problem, the MC framework also has the advantage of degradation matrices that are in general better conditioned. We have categorized the restoration algorithms according to four different criteria to give a better understanding of different techniques used in the reconstruction problem. The blind deconvolution problem is of a special interest in the present time not only because it occurs frequently in real-life phenomena but also it was not solved in its “purest” version for the SC case in a satisfying way. Emerging intrinsically MC approaches, like the subspace technique, are the promising answer to the blind deconvolution question.

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