

Spektralsatz

①

$$U (3 \times 3) \in \mathbb{R}, \quad U^T = U$$

$$U\varphi = \lambda\varphi \quad \lambda_k \in \mathbb{R}, \quad \varphi_k \in \mathbb{R}^3$$

$$\Phi = [\varphi_1 \ \varphi_2 \ \varphi_3] \quad \varphi_i^T \varphi_j = \delta_{ij} \Rightarrow \Phi^T \Phi = I$$

$$\Lambda = \text{diag} [\lambda_1 \ \lambda_2 \ \lambda_3]$$

$$U\Phi = [U\varphi_1 \ U\varphi_2 \ U\varphi_3] = [\lambda_1\varphi_1 \ \lambda_2\varphi_2 \ \lambda_3\varphi_3] = \Phi\Lambda$$

$$U\Phi = \Phi\Lambda \Rightarrow \boxed{U = \Phi\Lambda\Phi^T}$$

Odnorná matice

$$C \text{ sym. + def.} \quad \exists U \text{ sym + def.} \quad C = UU$$

$$C = U^2 \quad U = \sqrt{C}$$

Všichni odnorní je jednodušší.

$$\text{Př.} \\ C = \begin{bmatrix} 9 & 0 \\ 0 & 4 \end{bmatrix} \quad U = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \quad U \neq \begin{bmatrix} -3 & 0 \\ 0 & 2 \end{bmatrix}$$

Předp. C pro řešení. Předp. U pro jednodušší.

Existence:

$$C = \Phi \Lambda^2 \Phi^T \quad \Lambda^2 = \text{diag} [\mu_1, \mu_2, \mu_3] \quad \mu_k > 0$$

$$\lambda_k \stackrel{\text{def}}{=} |\sqrt{\mu_k}| \quad \Lambda = \text{diag} [\lambda_1, \lambda_2, \lambda_3] \quad \Lambda \Lambda = \Lambda^2$$

$$U \stackrel{\text{def}}{=} \Phi \Lambda \Phi^T$$

a) $U^2 = \Phi \Lambda \Phi^T \Phi \Lambda \Phi^T = \Phi \Lambda \Lambda \Phi^T = \Phi \Lambda^2 \Phi^T = C$

b) $U^T = (\Phi \Lambda \Phi^T)^T = \Phi \Lambda^T \Phi^T = \Phi \Lambda \Phi^T = U$ symmetrisch

c) + def: $\forall x \neq 0: x^T U x > 0$

$$x^T (\Phi \Lambda \Phi^T) x = \eta^T \Lambda \eta = \lambda_1 \eta_1^2 + \lambda_2 \eta_2^2 + \lambda_3 \eta_3^2 > 0$$

$$\Phi^T x = \eta \Rightarrow x = \Phi \eta, \quad \eta = 0 \Rightarrow x = 0$$

Eigenwerte: (Stephenson, 1980)

$$U^2 = \bar{U}^2 = C$$

$$(C - \mu I) \varphi = 0$$

$$(\bar{U}^2 - \mu I) \varphi = \underbrace{(\bar{U} + \lambda I)(\bar{U} - \lambda I)}_X \varphi = 0, \quad \lambda = |\sqrt{\mu}| > 0$$

$$\bar{U} x = -\lambda x, \quad \bar{U} + \text{def} \Rightarrow x = 0 \Rightarrow \bar{U} \varphi = \lambda \varphi$$

spektrale Zerlegung $\bar{U} = \Phi \Lambda \Phi^T = U$

Doplnek

$F(3 \times 3)$ regulární

$$J = \det |F| = F_{11} \det \begin{vmatrix} F_{22} & F_{23} \\ F_{32} & F_{33} \end{vmatrix} - F_{12} \det \begin{vmatrix} F_{21} & F_{23} \\ F_{31} & F_{33} \end{vmatrix} + F_{13} \det \begin{vmatrix} F_{21} & F_{22} \\ F_{31} & F_{32} \end{vmatrix} =$$

$$= F_{11} \bar{F}_{11} + F_{12} \bar{F}_{12} + F_{13} \bar{F}_{13} \quad \bar{F}_{ij} \text{ doplnek (kofaktor)}$$

Prum: \bar{F}_{ij} neobsluzuje prvky z i-teho radku

$$F_{ij}^{-1} = \frac{1}{J} \bar{F}_{ji} \quad \text{invertní matice}$$

$$\bar{F}_{ij} = J F_{ji}^{-1}$$

$$\frac{\partial J}{\partial F_{ij}} = \frac{\partial}{\partial F_{ij}} (F_{11} \bar{F}_{11} + F_{12} \bar{F}_{12} + F_{13} \bar{F}_{13}) = \bar{F}_{ij}$$

$$\frac{\partial J}{\partial F_{ij}} = J F_{ji}^{-1}$$

sym., antisym část t. z. v.

$$L = \underbrace{\frac{1}{2}(L + L^T)}_D + \underbrace{\frac{1}{2}(L - L^T)}_W \quad \begin{array}{l} D^T = D, \\ \text{sym.} \end{array} \quad \begin{array}{l} W^T = \frac{1}{2}(L^T - L) = -W \\ \text{antisym.} \end{array}$$

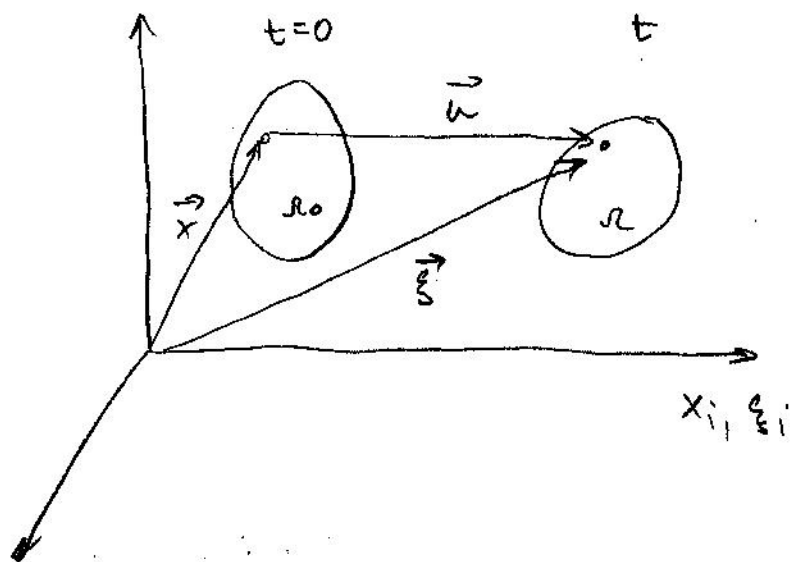
$$\left. \begin{array}{l} L = \bar{D} + \bar{W} \\ L^T = \bar{D} - \bar{W} \end{array} \right\} \quad \begin{array}{l} L + L^T = 2\bar{D} \\ L - L^T = 2\bar{W} \end{array}$$

jednoduchost $(X^T L X = X^T (D+W) X = X^T D X$

$$y = W X : \quad y^T X = X^T W^T X = -X^T W X = -X^T y = -y^T X = 0 \Rightarrow y \perp X$$

KINEMATIKA DEFORMACE

(4)



$$\vec{u}(\vec{x}, t)$$

$$\vec{\xi} = \vec{x} + \vec{u}$$

$$\vec{u}(\vec{r}, t)$$

x_i --- materialni s. (Lagrange)

ξ_i --- prostorni s. (Euler)

Pr $\vec{u}(x_i, t)$ materialni pole (Lagrangeho popis)

$\vec{u}(\xi_i, t)$ prostorni pole (Eulerho popis)

Lagrangeov' popis

grad posunuli'

deformacini' grad.

$$z_{ij} = \frac{\partial u_i}{\partial x_j}$$

$$F_{ij} = \frac{\partial \xi_i}{\partial x_j}$$

$$F_{ij} = \frac{\partial}{\partial x_j} (x_i + u_i) = \delta_{ij} + z_{ij}$$

$$F = I + Z$$

Deformacini' gradient

$$\xi_i(x_j, t) : \Omega_0 \rightarrow \Omega$$

Zobrazeni' je regularni' (\exists inverze)

$$J = \det \left| \frac{\partial \xi_i}{\partial x_j} \right| = \det |F| \neq 0 \quad \forall \Omega_0$$

vrstidnem le spritohi $J > 0$ nebo $J < 0$

$$V = \int_V dV = \int_{V_0} J dV_0$$

$$V_0 \rightarrow 0 : \int_{V_0} J dV_0 \approx J V_0 = V \quad \text{Jacobiana je yaver objemu}$$

$$V_0, V > 0 \Rightarrow \boxed{J > 0}$$

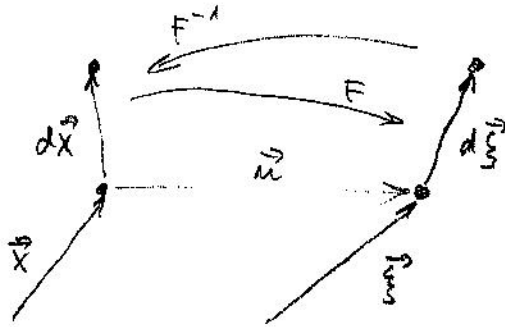
Prum $J < 0$ material' se obratit' nazad

$J = 0$ singularita (ulovecti' komprese)

Inverse: $F_{ij}^{-1} = \frac{\partial x_i}{\partial \xi_j}$

$F_{ik} F_{kj}^{-1} = \frac{\partial \xi_i}{\partial x_k} \frac{\partial x_k}{\partial \xi_j} = \frac{\partial \xi_i}{\partial \xi_j} = \delta_{ij}$ $FF^{-1} = I$

Geschieht's allgemein: $d\xi_i = \frac{\partial \xi_i}{\partial x_j} dx_j = F_{ij} dx_j$ $d\xi = F dx$



Green-Lagrange

$e \stackrel{\text{def}}{=} \frac{1}{2} (F^T F - I)$ symmetrisch

$(ds)^2 - (ds_0)^2 = d\xi^T d\xi - dx^T dx = dx^T F^T F dx - dx^T dx =$
 $= dx^T (F^T F - I) dx = 2 dx^T e dx$

$e = 0 \Rightarrow (ds)^2 - (ds_0)^2 = 0$

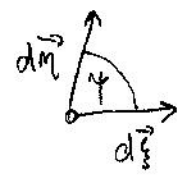
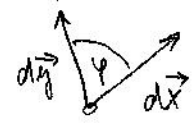
A: $x \rightarrow x'$ $x'_i = \lim_{\epsilon \rightarrow 0} x_{ij} + \epsilon$ $e' = \text{diag}[e_1, e_2, e_3]$

$dx^T e dx = dx^T A^T e' A dx = (dx')^T e' dx' = e_1 (dx'_1)^2 + e_2 (dx'_2)^2 + e_3 (dx'_3)^2$

$\forall dx \neq 0: dx^T e dx = 0 \Rightarrow e_1 = e_2 = e_3 = 0$

$(ds)^2 - (ds_0)^2 = 0 \Rightarrow e = 0$

e = 0



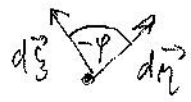
$d\vec{x} \cdot d\vec{y} = \|d\vec{x}\| \cdot \|d\vec{y}\| \cos \varphi$

$d\vec{\xi} \cdot d\vec{\eta} = \|d\vec{\xi}\| \cdot \|d\vec{\eta}\| \cos \psi = \|d\vec{x}\| \cdot \|d\vec{y}\| \cos \varphi$

$d\vec{\xi} \cdot d\vec{\eta} - d\vec{x} \cdot d\vec{y} = \|d\vec{x}\| \cdot \|d\vec{y}\| (\cos \psi - \cos \varphi)$

$d\vec{\xi}^T d\vec{\eta} - d\vec{x}^T d\vec{y} = d\vec{x}^T F^T F d\vec{y} - d\vec{x}^T d\vec{y} = d\vec{x}^T (F^T F - I) d\vec{y} = 2d\vec{x}^T e d\vec{y} = 0$

$\cos \psi = \cos \varphi$ $\Rightarrow |\psi| = |\varphi|$



J = +1

J = -1

von Matrix heißt *symmetrisch* & *hermitisch*.

$e = \frac{1}{2}(F^T F - I) = \frac{1}{2}[(z+I)^T(z+I) - I] = \frac{1}{2}(z+z^T+z^T z)$

$\varepsilon \stackrel{def}{=} \frac{1}{2}(z+z^T)$

$\varepsilon_{ij} = \frac{1}{2}(z_{ij} + z_{ji}) = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$

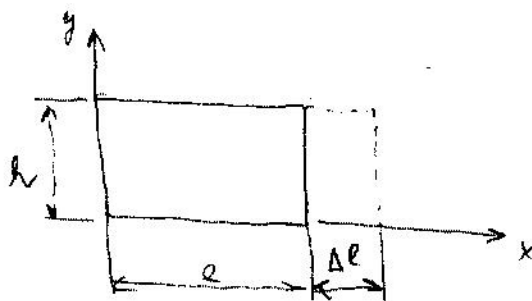
$e = \varepsilon + \frac{1}{2} z^T z$

$z \rightarrow 0: e \approx \varepsilon$

Pr

$$u(x,y) = \frac{\Delta l}{l} x$$

$$v(x,y) = 0$$

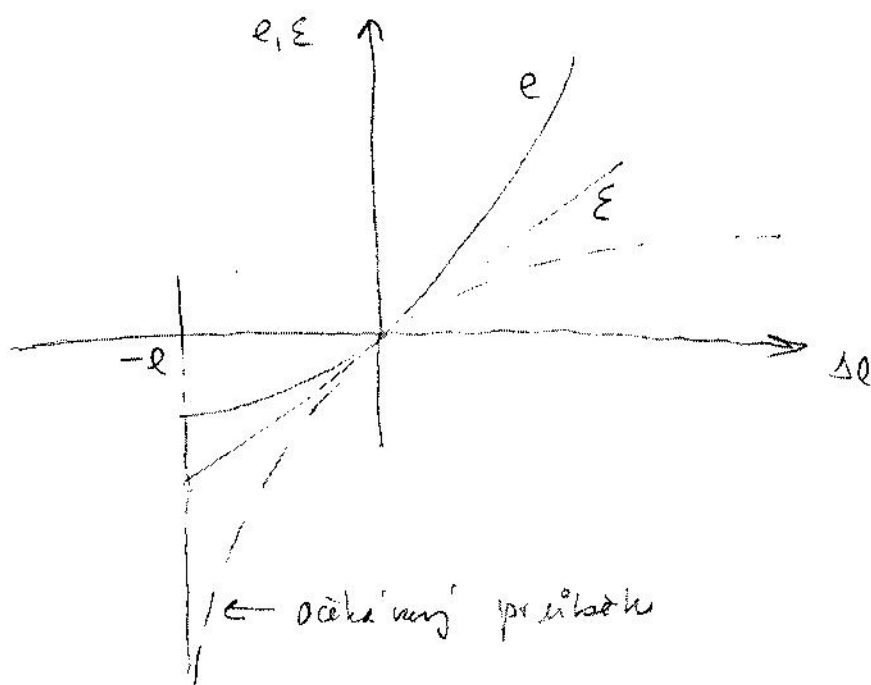


$$z = \begin{bmatrix} \frac{\Delta l}{l} & 0 \\ 0 & 0 \end{bmatrix}$$

$$F = \begin{bmatrix} 1 + \frac{\Delta l}{l} & 0 \\ 0 & 1 \end{bmatrix}$$

$$J = \det F = 1 + \frac{\Delta l}{l} = \frac{l + \Delta l}{l} = \frac{V}{V_0}$$

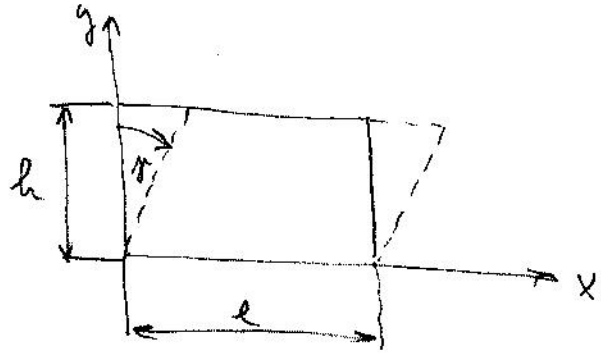
$$e = \begin{bmatrix} \frac{\Delta l}{l} + \frac{1}{2} \left(\frac{\Delta l}{l}\right)^2 & 0 \\ 0 & 0 \end{bmatrix} \quad e = \begin{bmatrix} \frac{\Delta l}{l} & 0 \\ 0 & 0 \end{bmatrix}$$



Pr

$$u(x,y) = M \log r$$

$$v(x,y) = 0$$



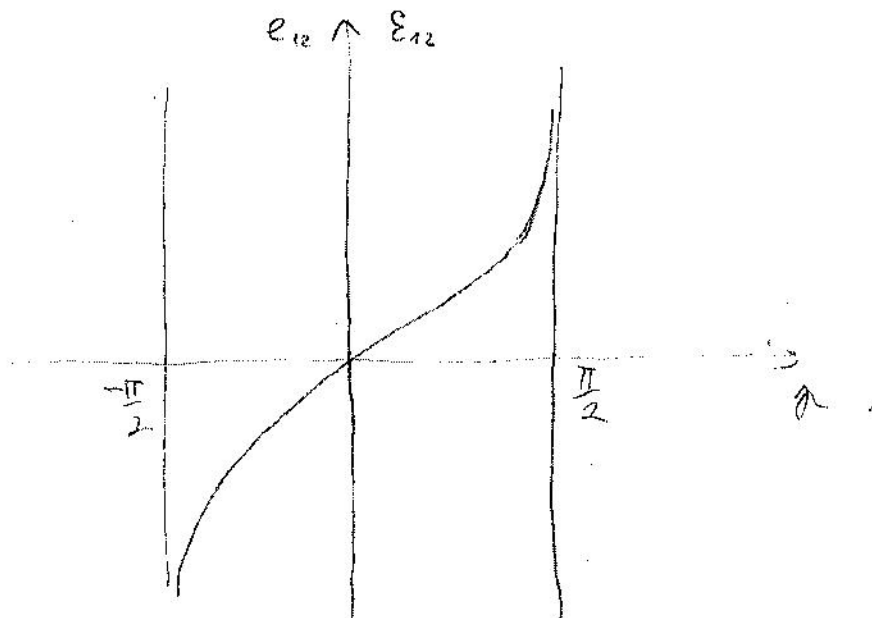
$$Z = \begin{bmatrix} 0 & \log r \\ 0 & 0 \end{bmatrix}$$

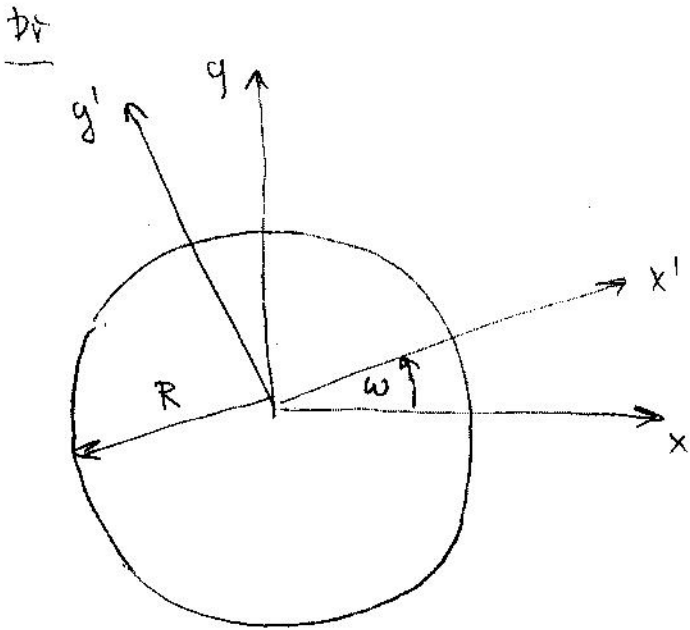
$$F = \begin{bmatrix} 1 & \log r \\ 0 & 1 \end{bmatrix}$$

$\exists = \det |F| = 1$ konformität bijektiv

$$e = \frac{1}{2} \begin{bmatrix} 0 & \log r \\ \log r & \frac{1}{r^2} \end{bmatrix} \quad \varepsilon = \frac{1}{2} \begin{bmatrix} 0 & \log r \\ \log r & 0 \end{bmatrix}$$

Prüf: $r \rightarrow 0, \log r \rightarrow \mu$





$$\xi' = X$$

$$A = \begin{bmatrix} \cos \omega & \sin \omega \\ -\sin \omega & \cos \omega \end{bmatrix}$$

$$\mu = \xi - X = A^T \xi' - X = (A^T - I) X$$

$$M(x, y) = X(\cos \omega - 1) - y \sin \omega$$

$$N(x, y) = X \sin \omega + y(\cos \omega - 1)$$

$$Z = \begin{bmatrix} \cos \omega - 1 & -\sin \omega \\ \sin \omega & \cos \omega - 1 \end{bmatrix}$$

$$F = \begin{bmatrix} \cos \omega & -\sin \omega \\ \sin \omega & \cos \omega \end{bmatrix}$$

$$J = \det |F| = \cos^2 \omega + \sin^2 \omega = 1 \quad (\text{isochronically det})$$

$$e = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$E = \begin{bmatrix} \cos \omega - 1 & 0 \\ 0 & \cos \omega - 1 \end{bmatrix}$$

E ulemi invarianti ni o' vaktari

Soubor

1) Jednosměrná deformace: $\epsilon = 2$, $e = 3$

Průřeh ϵ je lepší, avšak mít by jít $\rightarrow -\infty$
pro $\Delta l = -l$. Oba tenzory konválně používáme.

2) Prostý úhyb: $\epsilon = 2$, $e = 1$

by μ je lepší u nás soubor deformace než μ (z kos).
Srovnání tenzor navíc papíru protilomů k tomu d'šitý
(efekt 2. řádu)

3) Rotace: $\epsilon = 4$, $e = 1$

Tenzor malé deformace není invariantní vůči rotaci

Pozn: ϵ, μ, R může být rozloženo dx, dy, dR

Pozn: Libovolně přičtením lze vytvořit um. by ho
základní módy.

Pozn: Malé úhony ϵ měly by zho být tenzor první
řádu.

Polární matici F

$F = RU = VR$

$\det |F| = J > 0$

U sym. + def. p. m. strany koso. pr. strany

V sym. + def. l. m. strany —||—

$R^T R = I, \det |R| = 1$ matice rotace

Rotace je jednovrstevná

Algoritmus

1) $C \stackrel{\text{def}}{=} F^T F$ sym. + def. Cayley - Green

$x^T C x = x^T F^T F x = \|F x\|^2 > 0$ pro F regulární

2) $U \stackrel{\text{def}}{=} \sqrt{C} = \Phi \Lambda \Phi^T$ sym. + def.

3) $R \stackrel{\text{def}}{=} F U^{-1} = F \Phi \Lambda^{-1} \Phi^T$

$R^T R = U^{-T} F^T F U^{-1} = U^{-1} C U^{-1} = U^{-1} U^2 U^{-1} = I$

$\det |R| = \pm 1 \quad \det |U| > 0$

$\det |R| = \frac{\det |F U^{-1}|}{\det |U|} = \frac{J}{\det |U|} > 0 \Rightarrow \begin{cases} \det |R| = +1 \\ \det |U| = J \end{cases}$

Pozn: $V \stackrel{\text{def}}{=} R U^T \Rightarrow VR = R U^T = F$

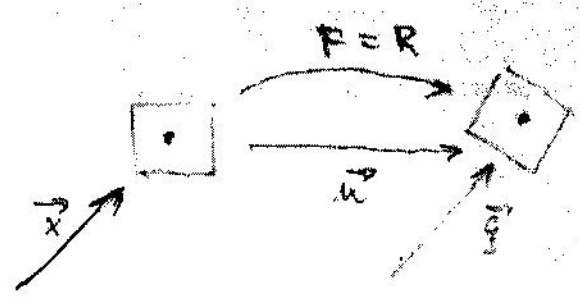
Identifikační: $F = RU = \bar{R} \bar{U} \quad C = F^T F = \bar{U}^T \bar{R}^T \bar{R} \bar{U} = \bar{U}^2$

$\bar{U} = \sqrt{C} = U \quad \bar{R} = F \bar{U}^{-1} = F U^{-1} = R$

Čistá rotace

$$F = R, \quad U = I$$

$$e = \frac{1}{2}(F^T F - I) = \frac{1}{2}(R^T R - I) = 0$$



Čistá deformace

$$F = U, \quad R = I$$

$$x'_i = \text{klavní osy } U, \quad U' = \text{diag}[\lambda_1, \lambda_2, \lambda_3]$$

$$df = F dx = U dx \quad df'_i = U'_i dx'_i$$

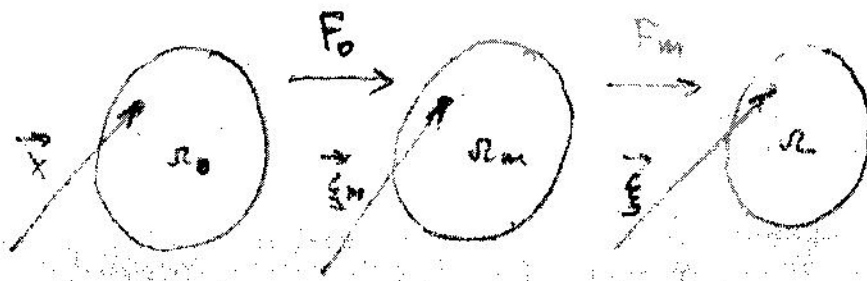
$$df'_1 = \lambda_1 dx'_1$$

$$df'_2 = \lambda_2 dx'_2$$

$$df'_3 = \lambda_3 dx'_3$$

$\lambda_1, \lambda_2, \lambda_3 > 0$ klavní protažení
 $\lambda_k < 1$ tlak $\lambda_k > 1$ tah

Multiplikation vektorů F



$$F_{ij}^0 = \frac{\partial \xi_i^m}{\partial x_j} \quad F_{ij}^m = \frac{\partial \xi_i^n}{\partial \xi_j^m} \quad F_{ij}^n = \frac{\partial \xi_i^n}{\partial x_j} = \frac{\partial \xi_i^n}{\partial \xi_k^m} \frac{\partial \xi_k^m}{\partial x_j} = F_{ik}^m F_{kj}^0$$

$$F = F_m F_0$$

$$d\xi = F_m d\xi_m = F_m F_0 dx = F dx$$

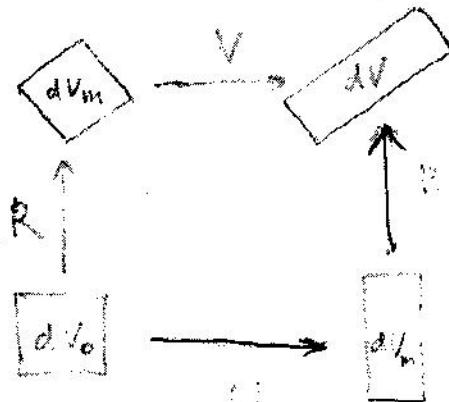
Poznámky Platí $F = F_e F_p$ multiplikativní

$$Z + I = \xi_e + \xi_p + z_e z_p + I$$

$$Z := z_e + z_p \Rightarrow \xi = \xi_e + \xi_p \quad \text{aditivní vektorů}$$

Palanci vektorů

Eulerovské tenzory



Lagrangeovské tenzory

Lagrangeův teorem definice

uvádíme hlavní osy U:

$$E_k \stackrel{\text{def}}{=} f(\lambda_k)$$

1) $f(\lambda) \neq 0$

2) $f(\lambda) \approx$ monotonně rostoucí

3) $f'(\lambda) \neq 1$ redukce na E

$$E = \Phi E' \Phi^T \quad E' = \text{diag}[E_1, E_2, E_3]$$

Hill

$$f(\lambda) = \frac{1}{m} (\lambda^m - 1) \quad m \in \mathbb{R}$$

m=0: $\lim_{m \rightarrow 0} \frac{\lambda^m - 1}{m} \stackrel{\text{L'H}}{=} \lim_{m \rightarrow 0} \frac{\lambda^m \ln \lambda}{1} = \ln \lambda$

$$E' = \frac{1}{m} (\mathcal{L}^m - I) \quad E = \frac{1}{m} (\Phi \mathcal{L}^m \Phi^T - I)$$

m = celé číslo $U^m = \Phi \mathcal{L} \Phi^T \dots \Phi \mathcal{L} \Phi^T = \Phi \mathcal{L}^m \Phi^T$

m = reálné $U^m \stackrel{\text{def}}{=} \Phi \mathcal{L}^m \Phi^T$

$$E = \frac{1}{m} (U^m - I)$$

m=2 Green-Lagrange

$$E^{(2)} = \frac{1}{2}(U^2 - I) = \frac{1}{2}(C - I) = \frac{1}{2}(F^T F - I) = e$$

m=1 Biot

$$E^{(1)} = U - I, \quad R = I \Rightarrow E^{(1)} = \varepsilon$$

m=0 Hencky

$$E^{(0)} = \phi(\ln \Lambda) \phi^T \stackrel{\text{def}}{=} \ln U$$

m = maximální proble R.W. Ogden

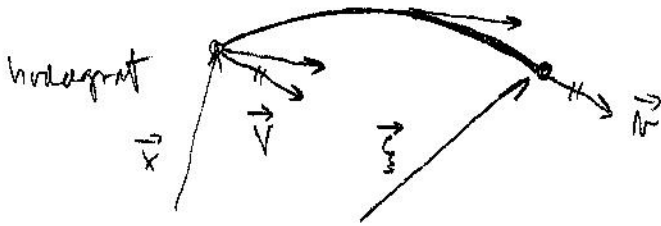
Pozn Nejlepší je E^0 . Ogdenův model dává $m=1-2$, což je zpravidla hodně dobré.

Pozn GL je efektivní, protože se nepotřebuje u šlá

Pozn: Uplatnění GL pro geom. nelineární úlohy

Rychlost

$$N_i(\xi_j, t) = N_i[\xi_j(x_k, t), t] = V_i(x_k, t)$$



vrtavý vektor

$$\vec{V} \neq \vec{v}$$

$$V_i = v_i$$

trajektorie $\vec{\xi}(x_k, t)$ při pevném \vec{x} .

$$\vec{V}(x_k, t) = \frac{\partial \vec{\xi}}{\partial t} = \dot{\vec{\xi}} = \dot{\vec{u}} \text{ materiálu derivace}$$

parallelu přenosu $\vec{x} \mapsto \vec{\xi} : \vec{N}(\xi_j, t)$

$$V_i(x_k, t) = V_i[x_k(\xi_j, t), t] = N_i(\xi_j, t)$$

Pr

$$\xi = x + at^2$$

$$\eta = y + bt + ct^2$$

$$x = \xi - at^2$$

$$\eta = \frac{\eta - ct^2}{1 + bt}$$

$$V = \begin{bmatrix} 2at \\ by + 2c \end{bmatrix}$$

$$N = \begin{bmatrix} 2at \\ \frac{b(\eta - ct^2)}{1 + bt} + 2c \end{bmatrix}$$

$$x = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad t = 1s \quad \xi = \begin{bmatrix} a \\ c \end{bmatrix}$$

$$V(0, 0, 1) = \begin{bmatrix} 2a \\ 2c \end{bmatrix} \quad N(a, c, 1) = \begin{bmatrix} 2a \\ 2c \end{bmatrix}$$

Probleme!

$\vec{A} = \dot{\vec{V}} = \sum \dot{\xi}_i \vec{e}_i = \dot{\vec{v}}$ materialna' derivate

parallelni' pruzor $\vec{x} \mapsto \vec{\xi} : \vec{a}(\xi_i, t)$

jin' izvrsit

$A_i = \frac{\partial v_i}{\partial t} = \frac{\partial v_i}{\partial \xi_k} \dot{\xi}_k + \frac{\partial v_i}{\partial t} = \frac{\partial v_i}{\partial \xi_k} v_k + \frac{\partial v_i}{\partial t} = a_i(\xi_i, t)$

$\dot{\vec{v}} = \frac{\partial \vec{v}}{\partial t} + \frac{\partial \vec{v}}{\partial \xi_k} v_k = \underbrace{\dot{\vec{v}}_t}_{\text{material' derivate}} + \underbrace{\vec{v} \cdot \text{grad } \vec{v}}_{\text{konvektiv' derivate}}$ materialna' derivate prostornih pull

Solution

$\vec{V}, \vec{v} \quad \dot{\vec{V}} = \frac{\partial \vec{V}}{\partial t} \quad \dot{\vec{v}} = \frac{\partial \vec{v}}{\partial t} + \frac{\partial \vec{v}}{\partial \xi_k} v_k$ mat. derivate

$\vec{A} = \dot{\vec{V}} \quad \vec{a} = \dot{\vec{v}}$

$A_i = a_i$

Ryhtymä gradientit

$$L_{ij} = \frac{d}{dt} \frac{\partial v_i}{\partial \xi_j} \quad dv = L d\xi \quad L = D + W$$

$$L_{ij} = \frac{\partial v_i}{\partial x_k} \frac{\partial x_k}{\partial \xi_j} = \frac{\partial \dot{\xi}_i}{\partial x_k} F_{kj}^{-1} = \dot{F}_{ik} F_{kj}^{-1} \quad L = \dot{F} F^{-1}$$

Symmetriset osat

$$D = \frac{1}{2} (L + L^T) \quad D_{ij} = \frac{1}{2} \left(\frac{\partial v_i}{\partial \xi_j} + \frac{\partial v_j}{\partial \xi_i} \right)$$

Positiivinen $D_{ij} \neq \dot{E}_{ij}$, $\int_0^t D dt$ riittää na integroimalla esittää

$$D_{ij} \neq \dot{E}_{ij}^{(m)}, \quad D_{ij} = 0 \iff \dot{E}_{ij}^{(m)} = 0$$

$$\begin{aligned} \frac{d}{dt} (dS)^2 &= \frac{d}{dt} (d\xi^T d\xi) = d\xi^T d\xi + d\xi^T d\xi = 2 d\xi^T d\xi = \\ &= 2 d\xi^T dv = 2 d\xi^T L d\xi = 2 d\xi^T D d\xi \end{aligned}$$

problemiä jotta piti analysoida Gullmannin kerosen

$$D = 0 \iff \text{ryhtymä def.} = 0$$

Arviointilaidä' ärist

$$W = \frac{1}{2} (L - L^T) \quad W_{ij} = \frac{1}{2} \left(\frac{\partial v_i}{\partial \xi_j} - \frac{\partial v_j}{\partial \xi_i} \right)$$

W = spin (vii'ny' teason)

muutinyne laidillei' kroyloy' polyt

$$d\vec{v} = \vec{w} \times d\vec{\xi} \quad dv_i = \mu_{ijk} \omega_j d\xi_k$$

$$W_{ik} \stackrel{\text{det}}{=} \mu_{ijk} \omega_j \quad dv_i = W_{ik} d\xi_k$$

$$dv = W d\xi \quad W = \begin{bmatrix} 0 & -\omega_2 & \omega_1 \\ \omega_2 & 0 & -\omega_3 \\ -\omega_1 & \omega_3 & 0 \end{bmatrix}$$

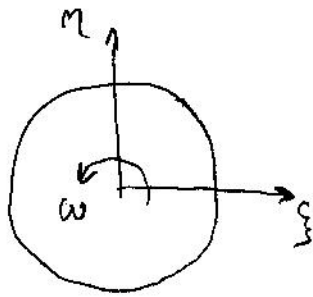
Prun $\forall W$ lre püirädiik axiaallei' nehoor \vec{w} .

Pv $F = I, \quad L = \dot{F}F^{-1} = \dot{F}, \quad L_{ij} = \frac{\partial \dot{\xi}_i}{\partial x_j} = \frac{\partial \dot{u}_i}{\partial x_j}$

$$D_{ij} = \frac{1}{2} \left(\frac{\partial \dot{u}_i}{\partial x_j} + \frac{\partial \dot{u}_j}{\partial x_i} \right) = \dot{\xi}_{ij} \quad W_{ij} = \frac{1}{2} \left(\frac{\partial \dot{u}_i}{\partial x_j} - \frac{\partial \dot{u}_j}{\partial x_i} \right) = \dot{\omega}_{ij}$$

lineaali' teorie ja lineaaliid' lusiokalanä'.

Pf



$$u_1(x, y, t) = x(\cos \omega t - 1) - y \sin \omega t$$

$$u_2(x, y, t) = x \sin \omega t + y(\cos \omega t - 1)$$

$$V_1 = -\omega x \sin \omega t - \omega y \cos \omega t = -\omega(u_2 + y) = -\omega \eta$$

$$V_2 = \omega x \cos \omega t - \omega y \sin \omega t = \omega(u_1 + x) = \omega \xi$$

$$N_1 = -\omega \eta$$

$$N_2 = \omega \xi$$

rotational field

$$a_1 = \frac{\partial N_1}{\partial t} + \frac{\partial N_1}{\partial \xi_k} v_k = -\omega N_2 = -\omega^2 \xi$$

$$a_2 = \frac{\partial N_2}{\partial t} + \frac{\partial N_2}{\partial \xi_k} v_k = \omega N_1 = -\omega^2 \eta$$

$$\vec{a} = -\vec{r} \omega^2$$

$$L = \begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix} = W, \quad D = 0, \quad \vec{\omega} = \begin{bmatrix} 0 \\ 0 \\ \omega_3 \end{bmatrix}$$