ZÁKONY MECHANIKY KONTINUA

Zákon zachování hmotnosti

$$\boxed{m(V) = m(V_0)}$$

Lagrange

$$\left.\begin{aligned} m(V) &= \int_V \rho \, dV = \int_{V_0} J\rho \, dV_0 \\ m(V_0) &= \int_{V_0} \rho_0 \, dV \end{aligned}\right\} \qquad \underline{\forall V_0:} \quad \boxed{J\rho = \rho_0}$$

Př $\quad M = \int_{V_0} \rho_0 H^T H \, dV_0 = \int_{V_0} J\rho H^T H \, dV_0 = \int_V \rho H^T H \, dV$

Euler

$$\dot{\rho}_0 = \dot{J}\rho + J\dot{\rho} = 0$$

$$\dot{J} = \frac{\partial J}{\partial F_{ij}} \dot{F}_{ij} = J F^{-1}_{ji} \dot{F}_{ij}$$

$$L = \dot{F}F^{-1} \Rightarrow L_{ik} = \dot{F}_{ij} F^{-1}_{jk} \Rightarrow L_{ii} = \dot{F}_{ij} F^{-1}_{ji}$$

$$\dot{J} = J L_{ii} \Rightarrow J L_{ii} \rho + J\dot{\rho} = 0 \, , \qquad J \neq 0$$

$$L_{ii} = \frac{\partial v_i}{\partial \xi_i} = \operatorname{div} \vec{v} \qquad \text{Porov} \quad \operatorname{Div} \vec{V} = \frac{\partial V_i}{\partial X_i}$$

$$\boxed{\dot{\rho} + \rho \operatorname{div} \vec{v} = 0} \qquad \text{rovnice kontinuity}$$

Zákon zachování hybnosti

$$\vec{p} \overset{\text{def}}{=} \int_V \rho \vec{v} \, dV \qquad \text{hybnost}$$

$$\boxed{\frac{d\vec{p}}{dt} = \int_V \vec{f} \, dV + \int_S \vec{t} \, dS}$$

$$\frac{dp_i}{dt} = \frac{d}{dt} \int_V \rho v_i \, dV = \frac{d}{dt} \int_{V_0} J \rho V_i \, dV = \int_{V_0} \rho_0 \dot{V}_i \, dV_0 \qquad \text{Lagrange}$$

$$= \int_{V_0} J \rho \dot{V}_i \, dV_0 = \int_V \rho \dot{v}_i \, dV \qquad \text{Euler}$$

Euler

$$\int_S t_i \, dS = \int_S \sigma_{ij} n_j \, dS = \int_V \frac{\partial \sigma_{ij}}{\partial \xi_j} \, dV$$

$$\boxed{\frac{\partial \sigma_{ij}}{\partial \xi_j} + f_i = \rho a_i} \qquad \text{pohybové rovnice} \quad (a_i = \dot{v}_i)$$

Lagrange

$$\int_S t_i \, dS = \int_{S_0} T_i \, dS_0 = \int_{S_0} \Sigma_{ij} N_j \, dS_0 = \int_{V_0} \frac{\partial \Sigma_{ij}}{\partial x_j} \, dV_0$$

$$\int_V f_i \, dV = \int_{V_0} J f_i \, dV_0 = \int_{V_0} F_i \, dV_0 , \qquad \vec{F} \overset{\text{def}}{=} J \vec{f}$$

$$\boxed{\frac{\partial \Sigma_{ij}}{\partial x_j} + F_i = \rho A_i} \qquad (A_i = \dot{V}_i)$$

Zákon zachování momentu hybnosti

$$\frac{d}{dt}\int_V \vec{\xi} \times \rho \vec{v} \, dV = \int_V \vec{\xi} \times \vec{f} \, dV + \int_S \vec{\xi} \times \vec{t} \, dS$$

$$(\vec{\xi} \times \vec{v})_i = \varepsilon_{ijk} \xi_j v_k \qquad \frac{\partial \sigma_{ij}}{\partial \xi_j} + f_i - \rho \ddot{v}_i = 0$$

<u>Euler</u> $\quad \int_V \varepsilon_{ijk} \sigma_{jk} \, dV = 0$

$$\varepsilon_{123} \sigma_{23} + \varepsilon_{132} \sigma_{32} = 0 \quad \Rightarrow \quad \sigma_{23} - \sigma_{32} = 0$$

$$\boxed{\sigma_{ij} = \sigma_{ji}} \qquad \boxed{\sigma^T = \sigma} \qquad \text{symetrie}$$

<u>Lagrange</u>

$$\Sigma = J \sigma F^{-T} \Rightarrow \sigma = \frac{1}{J} \Sigma F^T$$

$$\sigma^T = \sigma \Rightarrow \boxed{F \Sigma^T = \Sigma F^T} \quad \text{momentová podmínka na 1PK}$$

<u>Pozn:</u> Vektor nominálního napětí $\vec{T}$ tedy "nesplňuje" momentovou rovnováhu vzhledem k $\vec{X}$.

Zákon zachování energie

$$\dot{W}(V) = \int_V \sigma_{ij} D_{ij}\, dV = \int_{V_0} s_{ij} \dot{e}_{ij}\, dV_0 = \int_{V_0} S_{ij}^{(m)} \dot{E}_{ij}^{(m)}\, dV_0$$

$$\dot{Q}(V) = \int_V r\, dV - \int_S \vec{h} \cdot \vec{n}\, dS = \int_V (r - \operatorname{div} \vec{h})\, dV$$

Lagrange:

$$\int_V r\, dV = \int_{V_0} J r\, dV_0 = \int_{V_0} R\, dV_0$$

$\boxed{R \overset{def}{=} J r}$ tepelný zdroj v Ω_0

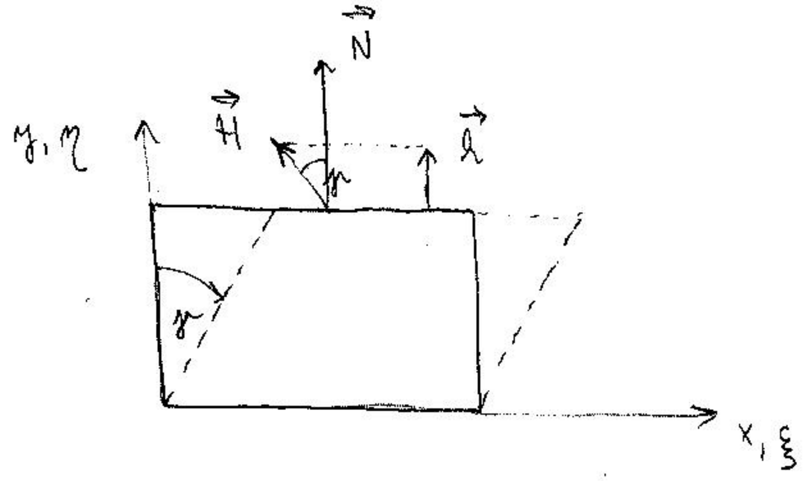
$$\int_S \vec{h} \cdot \vec{n}\, dS = \int_S h_i n_i\, dS = \int_{S_0} h_i J F_{ji}^{-1} N_j\, dS_0 =$$

$$= \int_{S_0} H_j N_j\, dS_0 = \int_{S_0} \vec{H} \cdot \vec{N}\, dS_0$$

$H_j \overset{def}{=} J F_{ji}^{-1} h_i$ $\quad \boxed{H = J F^{-1} h}$ Piola

$$\dot{Q}(V) = \int_V (r - \operatorname{div} \vec{h})\, dV = \int_{V_0} (R - \operatorname{Div} \vec{H})\, dV_0$$

Př



$$\vec{h} = \begin{bmatrix} 0 \\ a \end{bmatrix}$$

$$F = \begin{bmatrix} 1 & \operatorname{tg}\gamma \\ 0 & 1 \end{bmatrix} \qquad J = 1 \qquad F^{-1} = \begin{bmatrix} 1 & -\operatorname{tg}\gamma \\ 0 & 1 \end{bmatrix}$$

$$H = J F^{-1} h = \begin{bmatrix} 1 & -\operatorname{tg}\gamma \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ a \end{bmatrix} = \begin{bmatrix} -a \operatorname{tg}\gamma \\ a \end{bmatrix}$$

$$\|\vec{H}\| = |a| \sqrt{1 + \operatorname{tg}^2\gamma} = \|\vec{h}\| \sqrt{1 + \operatorname{tg}^2\gamma}$$

Vektor $\vec{H}$ zachovává orientaci vůči materiálním vláknům.

Vnitřní energie bude vztažena na jednotku hmoty

$$\mathcal{U} = \int_V \rho u \, dV = \int_{V_0} J \rho u \, dV_0 = \int_{V_0} \rho_0 u \, dV_0$$

u [J/kg] vzhledem k Ω_0 i Ω_0

$$\dot{\mathcal{U}} = \int_{V_0} \rho_0 \dot{u} \, dV_0 = \int_V \rho \dot{u} \, dV \qquad \text{materiálová derivace}$$

$$\boxed{\dot{Q} + \dot{W} = \dot{\mathcal{U}}}$$

Euler

$$\boxed{r - \operatorname{div} \vec{h} + \sigma_{ij} D_{ij} = \rho \dot{u}}$$

Lagrange

$$\boxed{R - \operatorname{Div} \vec{H} + S_{ij}^{(m)} \dot{E}_{ij}^{(m)} = \rho_0 \dot{u}}$$

Druhý zákon termodynamiky

$\exists\, \eta\ [J/kg\,K]$ hustota entropie = stavová veličina

CD:
$$\dot{S} \geq \int_V \frac{r}{T}\, dV + \int_S \frac{\vec{h}\cdot\vec{n}}{T}\, dS \qquad \text{Euler}$$

$$\dot{S} \geq \int_{V_0} \frac{R}{T}\, dV_0 + \int_{S_0} \frac{\vec{H}\cdot\vec{N}}{T}\, dS \qquad \text{Lagrange}$$

Lokální forma

$$\rho\dot{\eta} \geq \frac{1}{T}\left(r - \operatorname{div}\vec{h}\right) + \frac{1}{T^2}\,\vec{h}\cdot\operatorname{grad}T$$

$$\rho_0\dot{\eta} \geq \frac{1}{T}\left(R - \operatorname{Div}\vec{H}\right) + \frac{1}{T^2}\,\vec{H}\cdot\operatorname{Grad}T$$

Dissipační nerovnost

$$-\rho\eta\dot{T} + \sigma_{ij}D_{ij} - \frac{1}{T^2}\,\vec{h}\cdot\operatorname{grad}T \geq \rho\dot{\psi}$$

$$-\rho_0\eta\dot{T} + S_{ij}^{(m)}\dot{E}_{ij}^{(m)} - \frac{1}{T^2}\,\vec{H}\cdot\operatorname{grad}T \geq \rho_0\dot{\psi}$$

PEVNÉ LÁTKY

(1) $\rho_0 = J\rho$ zákon zachování hmotnosti

(2) $\dfrac{\partial \Sigma_{ij}}{\partial x_j} + F_i = \rho_0 \ddot{u}_i$ zákon zachování hybnosti

$\Sigma = J\sigma F^{-T}$, $\vec{F} = J\vec{f}$, $\ddot{u}_i = \dfrac{\partial^2 u_i}{\partial t^2}$ mat. derivace

(3) $\sigma = \sigma^T$, $\Sigma F^T = F\Sigma^T$ zákon zachování momentu h.

(4) $R - \mathrm{Div}\,\vec{H} + S_{ij}^{(m)} \dot{E}_{ij}^{(m)} = \rho_0 \dot{u}$ Z. zachování energie

$H = JF^{-1}h$, $R = Jr$

(5) $-\rho_0 \eta \dot{T} + S_{ij}^{(m)} \dot{E}_{ij}^{(m)} - \frac{1}{T}\vec{H}\cdot \mathrm{Grad}\,T \geq \rho_0 \dot{\Psi}$

dissipační nerovnost

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7) $S_{ij}^{(m)}(E_{kl}^{(m)}, T)$; $T \to \vec{H}$ konstitutivní vztahy

neznámé: u_1, u_2, u_3, T rovnice (2), (4)

Konstitutivní vztahy termoelasticity

$E_{ij}^{(m)}, T$: nezávislé stavové veličiny

$$S_{ij}^{(m)} = \rho_0 \frac{\partial \Psi}{\partial E_{ij}^{(m)}} \qquad \eta = -\frac{\partial \Psi}{\partial T} \qquad \vec{H} \cdot \mathrm{Grad}\, T \leq 0$$

Formulace nerovnost

$$H_i = J F_{ij}^{-1} h_j \qquad [\mathrm{Grad}\, T]_i = \frac{\partial T}{\partial X_i} = \frac{\partial T}{\partial \xi_k} \frac{\partial \xi_k}{\partial X_i} = \frac{\partial T}{\partial \xi_k} F_{ki}$$

$$\vec{H} \cdot \mathrm{Grad}\, T = J F_{ij}^{-1} h_j \frac{\partial T}{\partial \xi_k} F_{ki} = J \delta_{kj} h_j \frac{\partial T}{\partial \xi_k} = J \vec{h} \cdot \mathrm{grad}\, T$$

$$J \neq 0 \qquad \boxed{\vec{h} \cdot \mathrm{grad}\, T \leq 0}$$

Variační formulace

$\delta u_i \stackrel{\text{def}}{=} \tilde{u}_i - u_i$; u_i = skutečné řešení, $\tilde{u}_i$ = libovolná f.

označme $\quad u_{i,j} = \dfrac{\partial u_i}{\partial x_j}$

$$(\delta u_i)_{,j} = (\tilde{u}_i - u_i)_{,j} = \tilde{u}_{i,j} - u_{i,j} = \delta(u_{i,j}) = \delta u_{i,j}$$

$$e = \frac{1}{2}\left(Z + Z^T + Z^T Z \right) \qquad e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i} + u_{k,i} u_{k,j} \right)$$

$$\dot{e}_{ij} = \frac{1}{2}\left(\dot{u}_{i,j} + \dot{u}_{j,i} + \dot{u}_{k,i} u_{k,j} + u_{k,i} \dot{u}_{k,j} \right)$$

$$\delta e_{ij} \overset{def}{=} \frac{1}{2}\left(\delta u_{i,j} + \delta u_{j,i} + \delta u_{k,i} u_{k,j} + u_{k,i} \delta u_{k,j} \right)$$

Pozn: δe_{ij} není přírůstek

PlaW
$$\int_{V_0} S_{ij} \dot{e}_{ij} = \int_V f_i \dot{u}_i \, dV + \int_S t_i \dot{u}_i \, dS$$

$$\Rightarrow \quad \int_{V_0} S_{ij} \delta e_{ij} \, dV_0 = \int_V f_i \delta u_i \, dV + \int_S t_i \delta u_i \, dS$$

$$= \int_{V_0} F_i \delta u_i \, dV_0 + \int_{S_0} T_i \delta u_i \, dS_0$$

MKP
$$u = H q \qquad \delta e = B \delta q \qquad (e \neq B q)$$

$$\int_{V_0} B^T S \, dV_0 = R$$

$$R = \int_V H^T f \, dV + \int_S H^T t \, dS$$ sledující zatížení (tlak)

nebo

$$R = \int_{V_0} H^T F \, dV_0 + \int_{S_0} H^T T \, dS_0$$ konstantní zatížení (tíha)

Tečná matice

$$R = \int_{V_0} B^T S \, dV_0 \quad \Rightarrow \quad \delta q^T R = \int_{V_0} \delta e^T S \, dV_0 = \int_{V_0} \delta e_{ij} S_{ij} \, dV_0$$

$$\underline{\underline{u_i = H_i q}} \qquad \delta e_{ij} = \frac{1}{2} \left(H_{i,j} + H_{j,i} + H_{k,i} u_{k,j} + u_{k,i} H_{k,j} \right) \delta q$$

$$S_{ij} \delta e_{ij} = \left(S_{ij} H_{i,j} + S_{ij} H_{k,i} u_{k,j} \right) \delta q =$$

$$= \delta q^T \left(H^T_{i,j} + H^T_{k,i} u_{k,j} \right) S_{ij}$$

$$\delta q^T R = \delta q^T \underbrace{\int_{V_0} \left(H^T_{i,j} + H^T_{k,i} u_{k,j} \right) S_{ij} \, dV_0}_{R}$$

$$\dot{R} = \int_{V_0} \left(H^T_{i,j} + H^T_{k,i} u_{k,j} \right) \dot{S}_{ij} \, dV_0 + \int_{V_0} H^T_{k,i} \dot{u}_{k,j} S_{ij} \, dV_0$$

$$\dot{S}_{ij} = C_{ijpq} \dot{e}_{pq} = C_{ijpq} \left(H_{p,q} + H_{k,p} u_{k,q} \right) \dot{q}$$

$$\dot{u}_{k,j} = H_{k,j} \dot{q}$$

$$\dot{R} = \underbrace{\int_{V_0} \left(H^T_{i,j} + H^T_{k,i} u_{k,j} \right) C_{ijpq} \left(H_{p,q} + H_{k,p} u_{k,q} \right) dV_0}_{K_0 + K_L} \dot{q} \; +$$

$$+ \underbrace{\int_{V_0} H^T_{k,i} S_{ij} H_{k,j} \, dV_0}_{K_\sigma} \dot{q} = K_T \dot{q}$$

Stabilita

1) $u_{i,j} \ll 1$ 2) $S \simeq \sigma$

$$\dot{R} = (K_0 + K_L)\,\dot{q} + K_\sigma \dot{q} \simeq (K_0 + K_\sigma)\,\dot{q}$$

$$K_0 = \int_{V_0} H^T_{i,j}\, C_{ijpq}\, H_{p,q}\, dV_0$$ matice tuhosti z lineární pružnosti

$$K_\sigma \simeq \int_{V_0} H^T_{k,i}\, \sigma_{ij}\, H_{k,j}\, dV_0$$ matice počátečního napětí (geom. m.)

$$R(\lambda) = R_0 \lambda \Rightarrow K_\sigma = \lambda K_\sigma^0$$

$$K_T = (K_0 + K_\sigma^0 \lambda)$$

$\det |K_T| = 0 \Rightarrow$ problém vl. čísel

$$R_{krit} = \lambda_{krit} R_0$$

TEKUTINY

(1) $\dot{\rho} + \rho \operatorname{div} \vec{v} = 0$ z. zachování hmotnosti

(2) $\dfrac{\partial \sigma_{ij}}{\partial \xi_j} + f_i = \rho \dot{v}_i$ z. zachování hybnosti

(3) $\sigma^T = \sigma$ z. zachování momentu hybnosti

(4) $r - \operatorname{div} \vec{h} + \sigma_{ij} D_{ij} = \rho \dot{u}$ z. zachování energie

(5) $-\rho \eta \dot{T} + \sigma_{ij} D_{ij} - \frac{1}{T} \vec{h} \cdot \operatorname{grad} T \geq \rho \dot{\psi}$ disipační nerovnost

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$\sigma_{ij}(\rho, T, D_{kl})$, $T \to \vec{h}$

neznámé: v_1, v_2, v_3, ρ, T (1), (2), (4)

tekutiny
- nevazké (Euler)
- vazké (Newton)

Nevazké tekutiny

$$\boxed{\sigma_{ij} = -p\,\delta_{ij}} \qquad p = \text{dynamický tlak}$$

$$\sigma_{ij} D_{ij} = -p\,\delta_{ij} D_{ij} = -p\,D_{ii} = -p\,\frac{\partial v_i}{\partial \xi_i} = -p\,\operatorname{div}\vec{v}$$

z rovnice kontinuity $\quad \sigma_{ij} D_{ij} = \frac{p}{\rho}\dot{\rho}$

dissipační nerovnost : $\quad -\rho\eta\dot{T} + \frac{p}{\rho}\dot{\rho} - \frac{1}{T}\vec{h}\cdot\operatorname{grad}T \geq \rho\dot{\psi}$

ρ, T: nezávislé stavové veličiny

$$\dot{\psi} = \frac{\partial\psi}{\partial\rho}\dot{\rho} + \frac{\partial\psi}{\partial T}\dot{T}$$

$$-\rho\underbrace{\left(\eta + \frac{\partial\psi}{\partial T}\right)}_{0}\dot{T} + \underbrace{\left(\frac{p}{\rho} - \rho\frac{\partial\psi}{\partial\rho}\right)}_{0}\dot{\rho} - \frac{1}{T}\vec{h}\cdot\operatorname{grad}T \geq 0$$

$$\frac{p}{\rho} = \rho\frac{\partial\psi}{\partial\rho} \qquad \eta = -\frac{\partial\psi}{\partial T} \qquad \vec{h}\cdot\operatorname{grad}T \leq 0$$

$p(\rho, T)$ stavová rovnice $\Rightarrow \eta$

(libovolná)

řešení

(1)
$$\frac{\partial \varrho}{\partial t} + \frac{\partial \varrho}{\partial \xi_k} v_k + \varrho \frac{\partial v_i}{\partial \xi_i} = 0$$

$$\frac{\partial \varrho}{\partial t} + \frac{\partial}{\partial \xi_k}(\varrho v_k) = 0$$

$$\boxed{\frac{\partial \varrho}{\partial t} + \operatorname{div}(\varrho \vec{v}) = 0}$$ r. kontinuity

(2)
$$\frac{\partial \sigma_{ij}}{\partial \xi_j} = -\frac{\partial p}{\partial \xi_j}\delta_{ij} = -\frac{\partial p}{\partial \xi_i}$$

$$\boxed{-\operatorname{grad} p + \vec{f} = \varrho \dot{\vec{v}}}$$ Euler

(4)
$$\boxed{r - \operatorname{div} \vec{h} + \frac{p}{\varrho}\dot{\varrho} = \varrho \dot{u}}$$ vedení tepla

(5) $p(\varrho, T) \Rightarrow \Psi, \eta \Rightarrow u(\varrho, T) = \Psi + T\eta$

Pozn: Nestlačitelná kapalina $\varrho = \varrho_0 = \text{konst.}$

(1) $\operatorname{div} \vec{v} = 0$ (2) $-\operatorname{grad} p + \vec{f} = \varrho_0 \dot{\vec{v}} \Rightarrow p, \vec{v}$

(4) $r - \operatorname{div} \vec{h} = \varrho_0 \dot{u}$ vyřeší se nezávisle

Ideální plyn

vedení tepla: $\vec{h} = -\lambda \operatorname{grad} T \qquad \lambda\,[W/mK] = \text{konst.},\ r = \tilde{r}$
(označení)

stavová rovnice: $\dfrac{p}{\rho} = rT \qquad r\,[J/kg\,K]$.

$$\frac{p}{\rho} = \rho\frac{\partial \Psi}{\partial \rho} = rT \quad \Rightarrow \quad \Psi = rT\ln\frac{\rho}{\rho_0} + g(T)$$

$$\eta = -\frac{\partial \Psi}{\partial T} \qquad \eta = -r\ln\frac{\rho}{\rho_0} - g'(T)$$

$$u = \Psi + T\eta = rT\ln\frac{\rho}{\rho_0} + g(T) - rT\ln\frac{\rho}{\rho_0} - Tg'(T)$$

$$\boxed{u = \varepsilon(T)}$$

$$(4) \qquad \tilde{r} + \lambda\Delta T + \frac{p}{\rho}\dot{\rho} = \rho\frac{d\varepsilon}{dT}\dot{T}$$

$$c_v \overset{\text{def}}{=} \frac{d\varepsilon}{dT} \quad [J/kg\,K]$$

$$\boxed{\tilde{r} + \lambda\Delta T + rT\dot{\rho} = \rho c_v \dot{T}}$$

Př Rozložení statického tlaku v atmosféře

(2) $$-\frac{dp}{d\xi} - \rho g = 0 \qquad g = \text{grav. zrychlení}$$

(4) $$\Delta T = 0$$

$$\frac{d^2T}{d\xi^2} = 0 \Rightarrow T(\xi) = T_0 - \alpha\xi \qquad \alpha = \text{konst.}$$

$$\frac{dp}{d\xi} = -\rho g = -\frac{gp}{rT}$$

$$\ln\frac{p}{p_0} = -\frac{g}{r}\int_0^{\xi}\frac{d\xi}{T_0 - \alpha\xi} = \frac{g}{\alpha r}\ln\frac{T_0 - \alpha\xi}{T_0}$$

$$\boxed{p(\xi) = p_0\left(\frac{T}{T_0}\right)^{g/\alpha r}}$$

Pozn: $\alpha \to 0$ $$\lim_{\alpha\to 0} p(\xi) = p_0 \exp\left[-\frac{g\xi}{rT_0}\right]$$

což plyne z integrace $$\ln\frac{p}{p_0} = -\frac{g}{r}\int_0^{\xi}\frac{d\xi}{T_0}$$

Př.

Šíření rázové vlny
(rovinná podélná vlna)

$v_1(\xi_1 - ct)$ $\qquad$ $v(\xi - ct)$

$v_2 = 0$ $\qquad$ $\rho(\xi - ct)$

$v_3 = 0$ $\qquad$ $p(\xi - ct)$

$$\frac{\partial g}{\partial t} = -c\frac{\partial g}{\partial \xi}$$

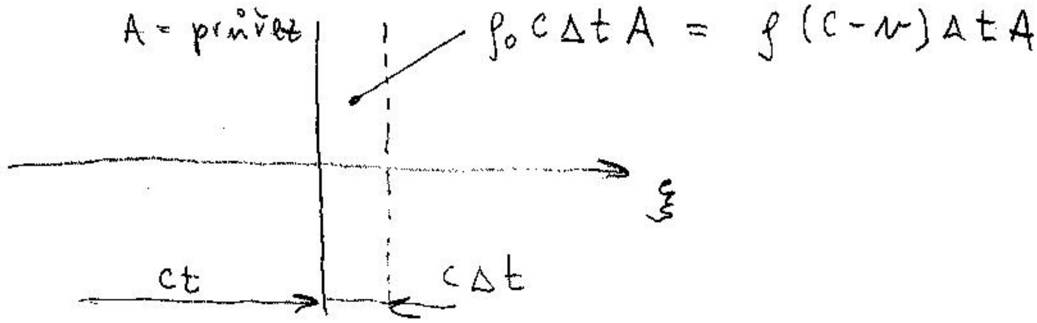
$$(1) \qquad \frac{\partial \rho}{\partial t} + \frac{\partial \rho}{\partial \xi} v + \rho \frac{\partial v}{\partial \xi} = 0$$

$$-c\frac{\partial \rho}{\partial \xi} + \frac{\partial}{\partial \xi}(\rho v) = 0$$

$$-c\rho + \rho v = \alpha(T) = \text{konst.}$$

počáteční podmínka $\quad \underline{\xi > ct}: \quad -c\rho_0 + 0 = \alpha$

$$\boxed{\rho(c - v) = \rho_0 c}$$



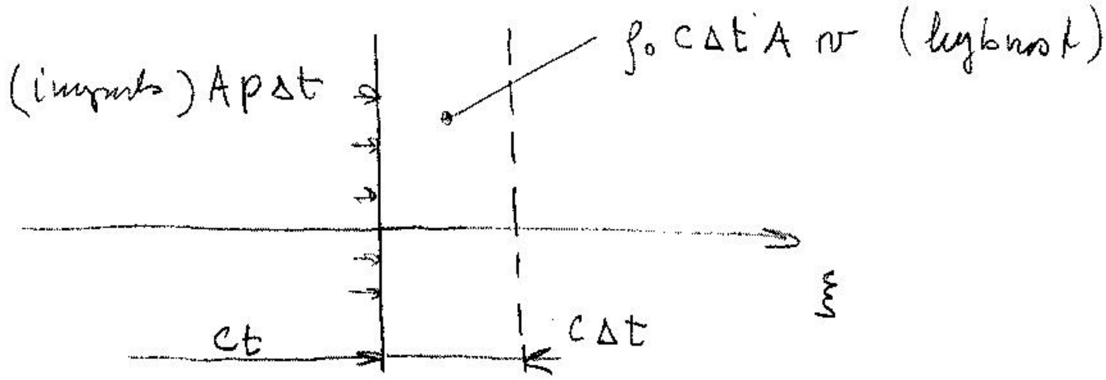
(2) $$-\frac{\partial p}{\partial \xi} = \rho\left(\frac{\partial v}{\partial t} + \frac{\partial v}{\partial \xi} v\right) = \rho \frac{\partial v}{\partial \xi}(-c + v)$$

z r. kontinuity $$-\frac{\partial p}{\partial \xi} = -\rho_0 c \frac{\partial v}{\partial \xi}$$

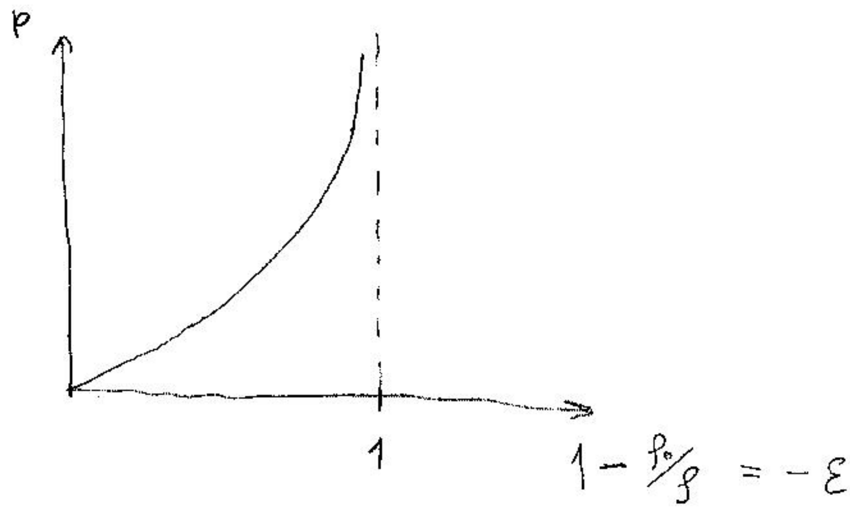
$$p = \rho_0 c v + \beta, \qquad \beta = \text{konst.}$$

$\xi > ct$: $\quad p = 0 \quad \Rightarrow \quad \beta = 0$

$$\boxed{p = \rho_0 c v}$$



adiabatický děj $\quad p = p(\rho) \quad$ Hugoniot



Vazké tekutiny

$$\boxed{\sigma_{ij} = -p\,\delta_{ij} + \tau_{ij}}$$

Viskózní část

$$\tau_{ij} = C_{ijkl}\, D_{kl} \qquad \text{(Newton - Stokes)}$$

Pro splnění disipační nerovnosti je nezbytné, aby

$$\forall D_{ij}: \qquad \tau_{ij} D_{ij} \geq 0$$

Dosazením

$$C_{ijkl}\, D_{ij}\, D_{kl} \geq 0 \quad \Rightarrow \quad + \text{def.}$$

Pozn. Zavedením potenciálu $\tau_{ij} = \dfrac{\partial \phi}{\partial D_{ij}}$, kde

$\phi =$ disipační funkce

Isotropní tekutina:

$$\tau_{ij} = \lambda D_{kk}\, \delta_{ij} + 2\mu\, D_{ij}$$

$\lambda, \mu =$ koeficienty viskozity

$$\boxed{\tau = \lambda \operatorname{div} \vec{v}\, I + 2\mu\, D}$$

Navier – Stokes

$$\frac{\partial \tau_{ij}}{\partial \xi_j} = \lambda \frac{\partial}{\partial \xi_j}(\operatorname{div} \vec{v})\,\delta_{ij} + 2\mu \frac{\partial}{\partial \xi_j} D_{ij} =$$

$$= \lambda [\operatorname{grad}(\operatorname{div} \vec{v})]_i + 2\mu \frac{1}{2}\left(\frac{\partial^2 v_i}{\partial \xi_j \partial \xi_j} + \frac{\partial^2 v_j}{\partial \xi_i \partial \xi_j} \right) =$$

$$= \lambda [\operatorname{grad}(\operatorname{div} \vec{v})]_i + \mu [\Delta \vec{v}]_i + \mu [\operatorname{grad}(\operatorname{div} \vec{v})]_i$$

$$\boxed{-\operatorname{grad} p + (\lambda + \mu)\operatorname{grad}(\operatorname{div} \vec{v}) + \mu \Delta \vec{v} + \vec{f} = \rho \dot{\vec{v}}}$$

Pozn: konvektivní složka $\dot{\vec{v}}$ činí rovnice nelineární

Reynoldsovo číslo

$\nu = \frac{\mu}{\rho}$ $[m^2/s]$ kinematická viskozita

nestlač. kapalina $-\frac{1}{\rho}\operatorname{grad} p + \nu \Delta \vec{v} + \frac{1}{\rho}\vec{f} = \dot{\vec{v}}$

$\underbrace{\frac{\partial \vec{v}}{\partial \xi_i} v_i}_{\frac{V^2}{L}}$ porovnáme s $\underbrace{\nu \Delta \vec{v}}_{\nu \frac{V}{L^2}}$

V, L = charakteristické veličiny

$$Re = \frac{\text{konvektivní}}{\text{viskózní}} = \frac{VL}{\nu} \quad [\text{bez rozměru}]$$

$Re \to 0$ linearizace $\qquad Re \to \infty$ Eulerovy rovnice