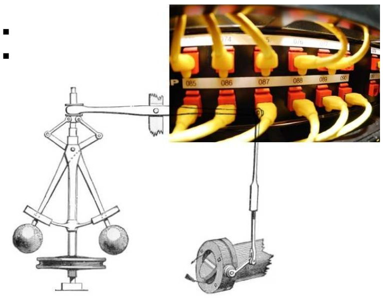
Communication Networks in Control: New Dimensions of Complexity



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Hluboka nad Vlavou
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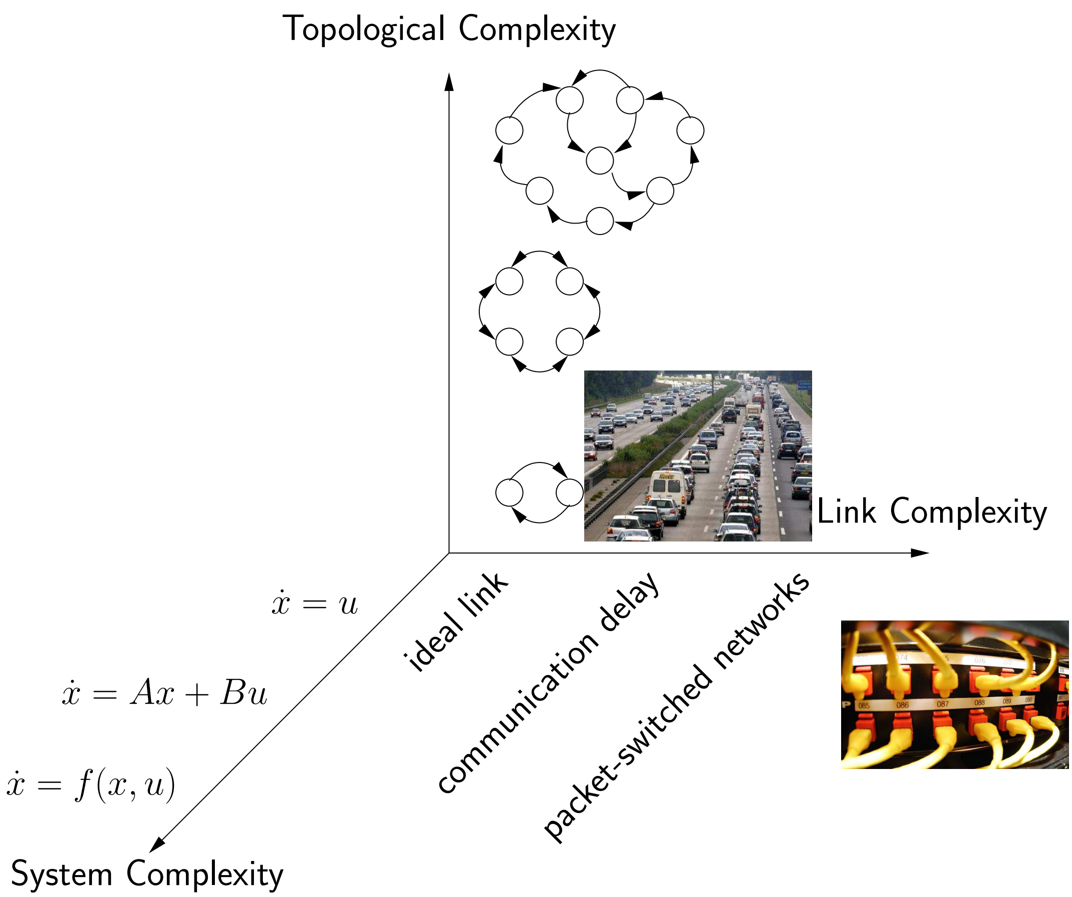
Dimensions of Complexity



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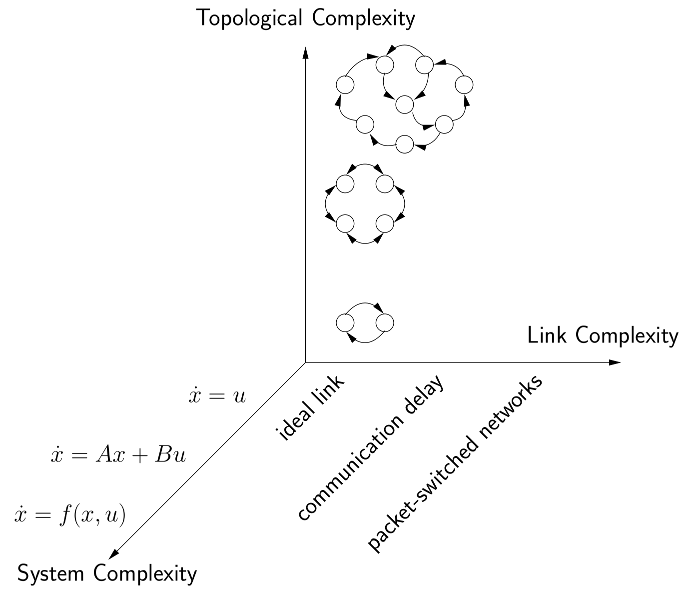

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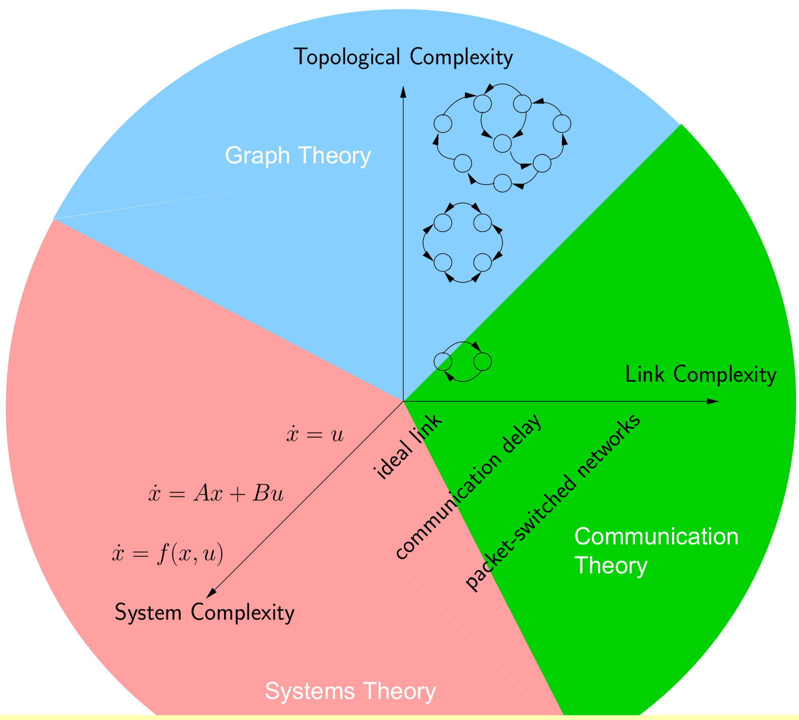
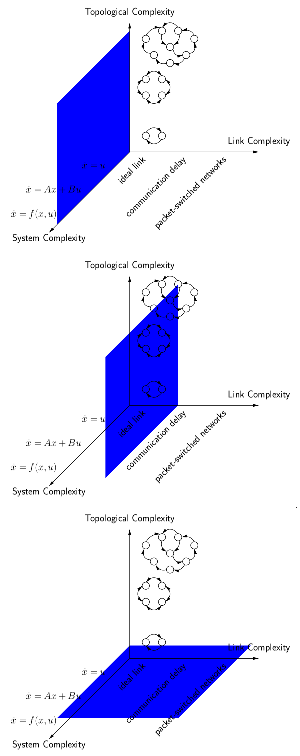


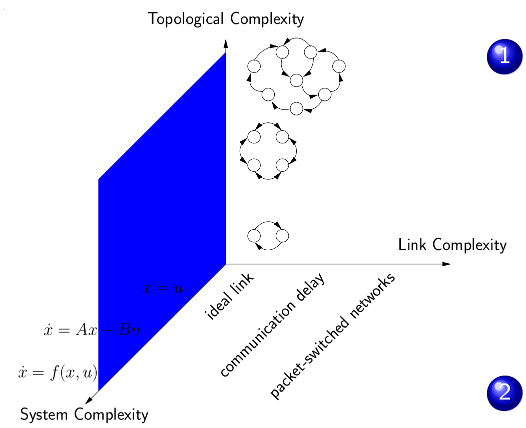
Challenges for Control over Communication Networks:
Combine systems, graph and communication theory

Overview



1. Consensus in linear Multi-Agent Systems with ideal links
2. Consensus in Multi-Agent Systems with Delays
3. Control via digital networks

Overview



1. **Consensus in linear Multi-Agent Systems with ideal links**
2. Consensus in Multi-Agent Systems with Delays
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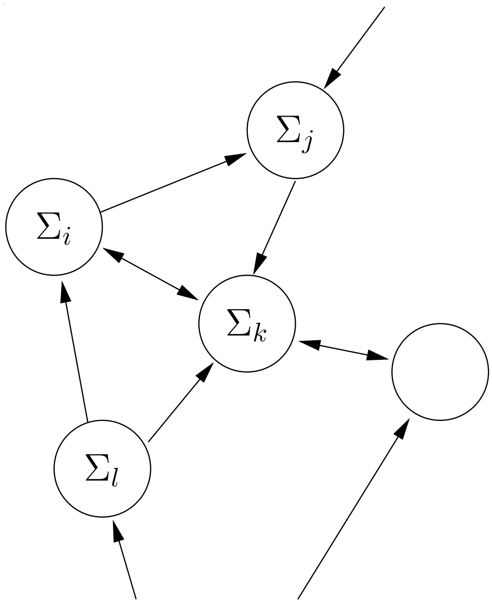
Consensus in Multi-Agent Systems (MAS)
Motivation



- Networks of dynamical agents occur in a huge variety of applications such as
  - Unmanned vehicles
  - Mobile robots
  - Formation control
  - Synchronization problems

- Key players are *individual agents* and *interconnection topology*
- So-called *consensus problems* form the basis of most of the challanges appearing in these applications

Blend systems and graph theory $\Rightarrow$ methods for *analysis* and *design*

History of Consensus Problems

Early related work

- Synchronization of coupled oscillators goes back to Huygens (1657) and is still an active field of research.

History of Consensus Problems

Graph Theory

- Fiedler 1973: seminal work on algebraic connectivity of graphs
- Since then many extensions to more general classes of graphs (Ren et al. 2004; Wu 2005; Wieland et al. 2008; ...)

MAS Consensus

- kinematic agents: Jadbabaie et al. 2003; Olfati-Saber & Murray 2004; Ren et al. 2007; ...
- second order agents: Ren & Atkins 2005; Ren 2008; ...
- general LTI systems: Fax & Murray 2004; Tuna 2008; Wieland et al. 2008; ...

Problem Setup

MAS model

- We consider N identical linear agents

$$\dot{x}_i = Ax_i + Bu_i, \quad x_i \in \mathbb{R}^n, \ u_i \in \mathbb{R}$$

- The interconnections between the agents are represented by a weighted and directed graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, W\}$.

Consensus

(State-)Consensus is achieved if

$$x_i(t) - x_j(t) = 0 \text{ for } t \to \infty \text{ for all } i, j = 1, \dots, N.$$

Objectives

1. for given topology and control u_i, *analyse* consensus
2. for given topology, *design* u_i such that consensus is achieved

Graph Basics

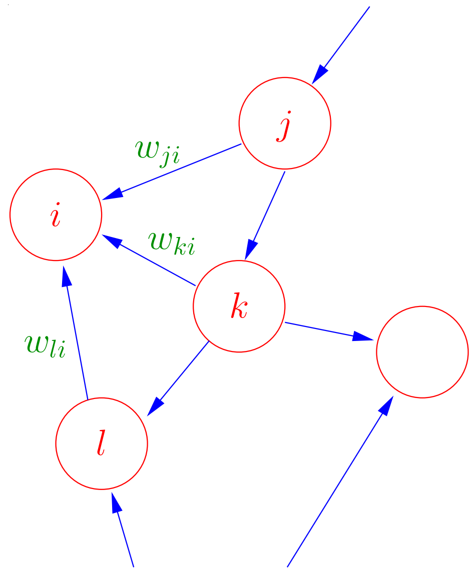
Algebraic graph theory

Graphs $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, W\}$ are represented by matrices such as

- the adjacency matrix $A \in \mathbb{R}^{|\mathcal{V}|} : [a_{ij}] = w_{ji}$
- the Laplacian matrix $L \in \mathbb{R}^{|\mathcal{V}|} : L = \mathrm{diag}(A\mathbf{1}) - A$

Goal

Use algebraic properties of graphs to characterize graph connectivity properties necessary/sufficient for consensus.



$\mathcal{V}$ – vertices
$\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ – edges
$W : \mathcal{V} \times \mathcal{V} \to \mathbb{R}_+$ – edge weights

Structure of Consensus Algorithm

Single agent consensus algorithm

We use the state feedback

$$u_i = -K \sum_{j=1}^{N} w_{ji} (x_i - x_j), \qquad i = 1, \dots, N$$

where w_{ji}, $i, j = 1, \dots, N$ reflect the interconnection topology and $K \in \mathbb{R}^{1\times n}$ is the design parameter.

Consensus algorithm of complete MAS

$$u = -(L \otimes K)\mathbf{x} \qquad \begin{array}{l} u = (u_1, \dots, u_N)^T \\ \mathbf{x} = (x_1^T, \dots, x_N^T)^T \end{array}$$

Laplacian matrix L appears naturally in consensus algorithm.

Local state feedback leads to *simple global representation* involving topology through Laplacian L.

Consensus Analysis

Theorem (Necessary and sufficient condition)

Convergence to consensus is achieved if and only if the polynomial

$$P(s) := \prod_{j=2}^{N} \det(sI - A - \lambda_j(L)BK)$$

is Hurwitz. Wieland et al. 2008, Fax & Murray 2004

Theorem (Dynamic evolution at consensus)

$$x_i(t) \to e^{At}\left(\frac{w_1(L)}{\|w_1(L)\|_1} \otimes I\right)\mathbf{x}(0), \quad i = 1, \ldots, N$$

- $\lambda_j(L)$ are eigenvalues of L counting multiplicities, $\lambda_1(L) = 0$.
- $w_1(L)$ is left-eigenvector of L s.t. $w_1(L)L = 0$ and $w_1(L) \neq 0$.

Consensus problem for *general identical LTI systems* reduced to stability problem with different feedback gains.
Vast linear systems theory applies.

Consensus Design

Idea

$$\prod_{j=2}^{N} \det(sI - A - \lambda_j(L)BK) \quad \text{Hurwitz}$$

is equivalent to

$u = Kx$ asymptotically stabilizes $\dot{x} = Ax + \lambda_j(L)Bu$ for $j = 2, \ldots, N$

If $\lambda_j = \sigma_j + \mathrm{j}\omega_j$ with $\omega_j \neq 0$, choose K such that

$v = \begin{pmatrix} K & 0 \\ 0 & K \end{pmatrix} z$ asymptotically stabilizes

$\dot{z} = \begin{pmatrix} A & 0 \\ 0 & A \end{pmatrix} z + \begin{pmatrix} \sigma_j B & \omega_j B \\ -\omega_j B & \sigma_j B \end{pmatrix} v$ for $j = 2, \ldots, N$.

Solve design problem as simple simultaneous stabilization problem.

Consensus Design

Theorem (LMI based design with guaranteedconvergence rate)

If there exists a scalar $\nu \geq 0$, a matrix $Q = Q^T \succ 0$, and a vector $\kappa \in \mathbb{R}^{1\times n}$ such that (with $\lambda_i(L) = \sigma_i + j\omega_i$)

$$C_0(Q,\nu) + \sigma_i C_R(\kappa) + \omega_i C_I(\kappa) \prec 0, \quad i = 2,\dots,N$$

and $K = \kappa Q^{-1}$ then all roots s_i of $P(s)$ satisfy $\mathrm{Re}(s_i) \leq -\nu$.

Wieland et al. 2008

$$C_0 = \begin{pmatrix} QA^T + AQ + 2\nu Q & 0 \\ 0 & QA^T + AQ + 2\nu Q \end{pmatrix},$$

$$C_R = -\begin{pmatrix} B\kappa + \kappa^T B^T & 0 \\ 0 & B\kappa + \kappa^T B^T \end{pmatrix},$$

$$C_I = \begin{pmatrix} 0 & \kappa^T B^T - B\kappa \\ B\kappa - \kappa^T B^T & 0 \end{pmatrix}$$

Design problem can be posed and efficiently solved using LMIs

Example: Formation Control

Vehicle model

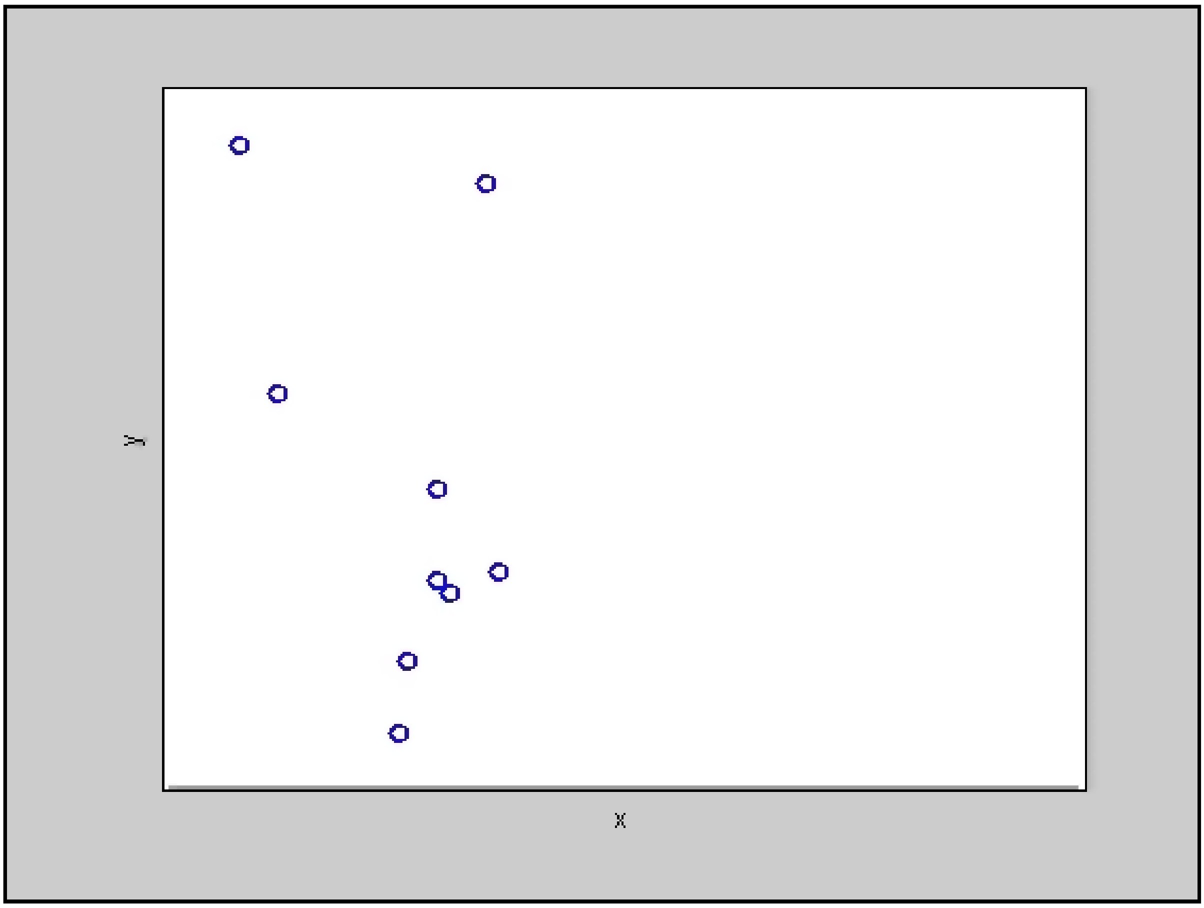
Consider N identical holonomic vehicles. The ith vehicle is modeled by two independent systems

$$\dot{z}_{i,q} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -a_1 & -a_0 \end{pmatrix} z_{i,q} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u_{i,q}, \quad q = x, y$$

with position, speed, and an actuator state as states $z_{i,q}$.

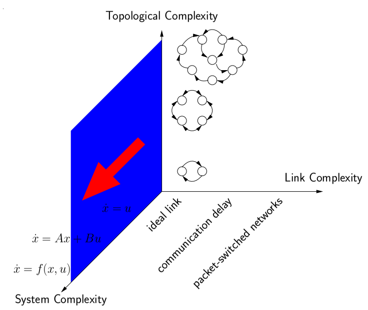
- The vehicles shall reach and keep a prespecified formation
- The consensus algorithm is used to correct formation errors
- While the shape of the formation is part of the design, its evolution in space depends on the initial conditions of the agents.

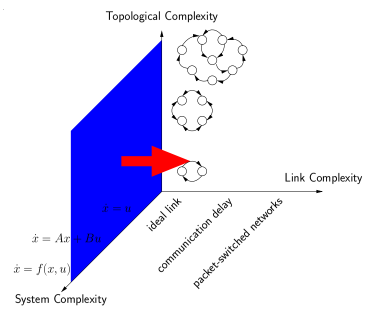
Example: Formation Control



Existing Extensions

- System Class Extensions
  - Passive/Lagrangian Systems (Chopra et al. 2006, 2008; Münz et al. 2009)
  - Polynomial Systems (Kim & Allgöwer 2007, 2008)
- Allow Changes in Interconnection Topology
  - Proximity Graphs (Jadbabaie et al. 2003, Tanner et al. 2003)
  - Switching Topology (Ren & Beard 2007)





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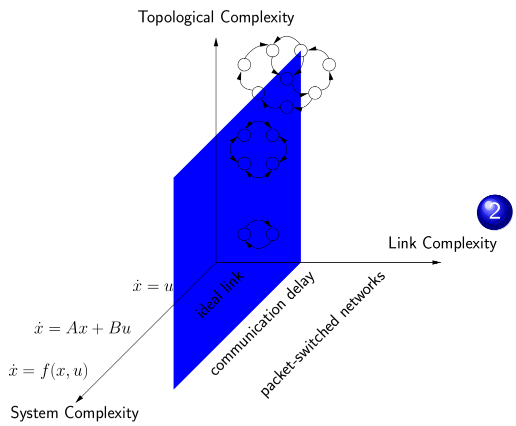

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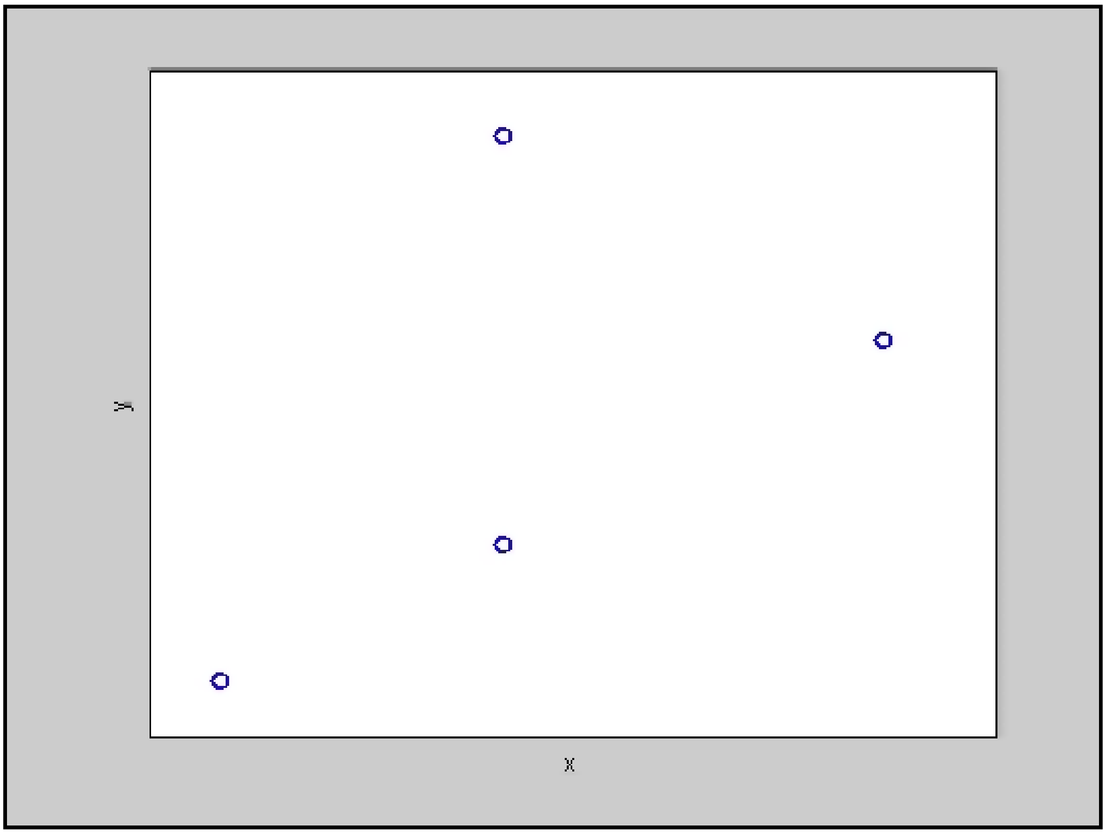
Delay Sources in Cooperative Control Problems

reaction time

comm. delay

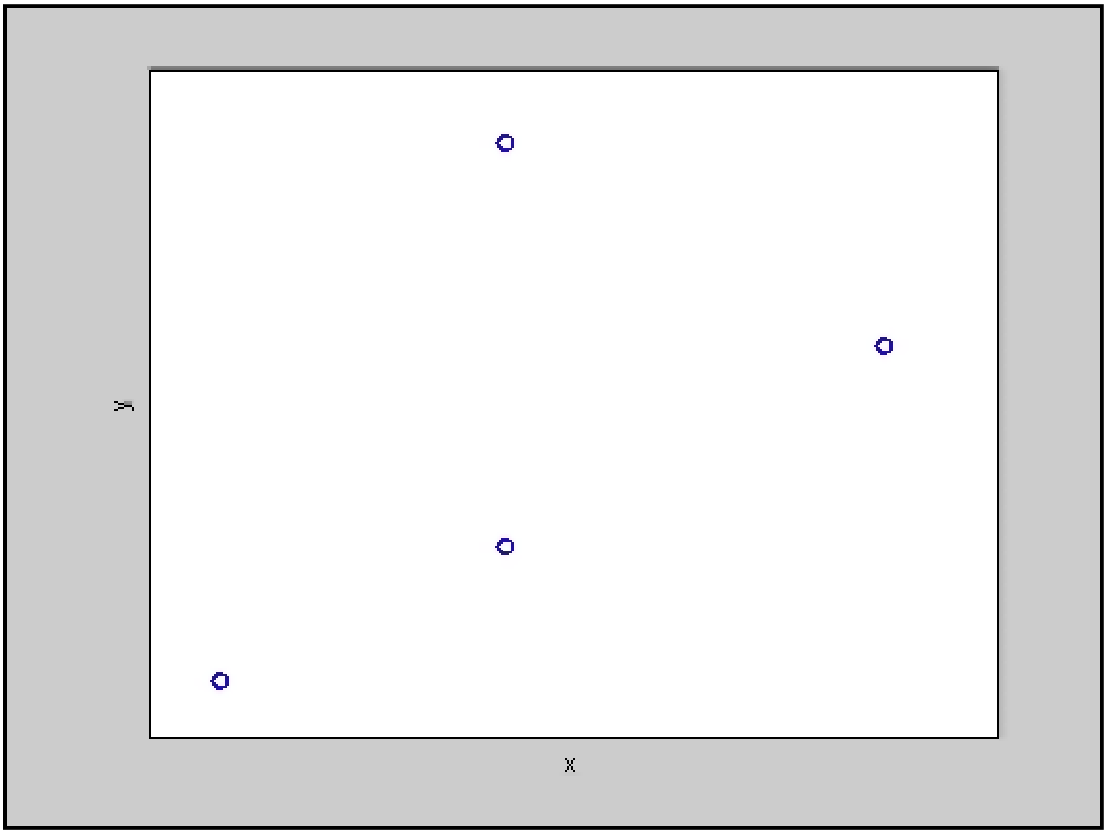
Do delays corrupt consensus?

Delays Corrupt Consensus!



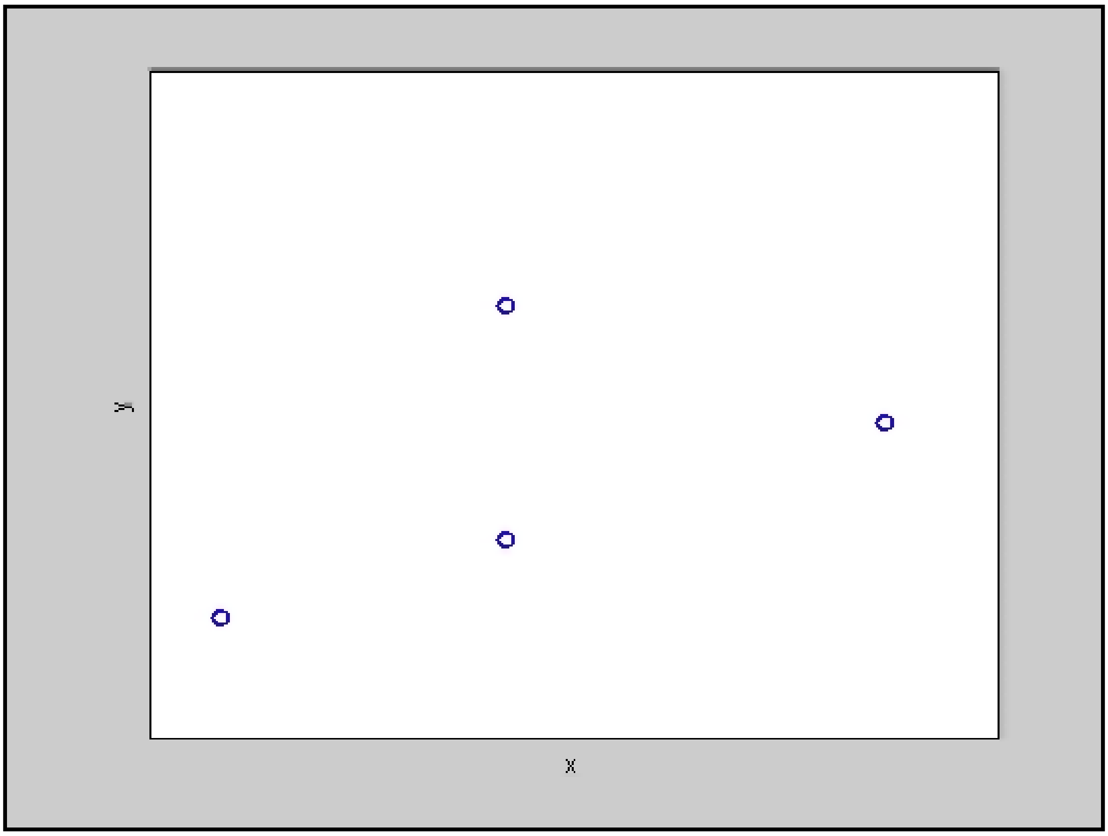
Second order linear MAS with communication delay $\tau = 0.01$.

Delays Corrupt Consensus!



Second order linear MAS with communication delay $\tau = 0.3$.

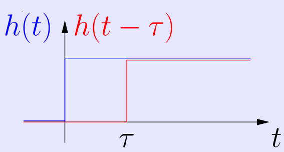
Delays Corrupt Consensus!



Second order linear MAS with communication delay $\tau = 1$.

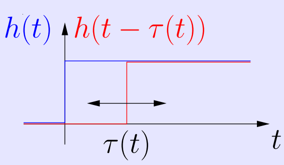
Delay Models for Communication Networks

Constant delay



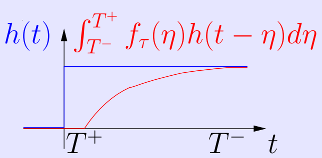
- easiest delay model
- approximation for reaction delay

Time-varying delay



- accurate description with discontinuities
- proofs often require continuous τ or even upper bound on $\dot{\tau}$

Distributed delay

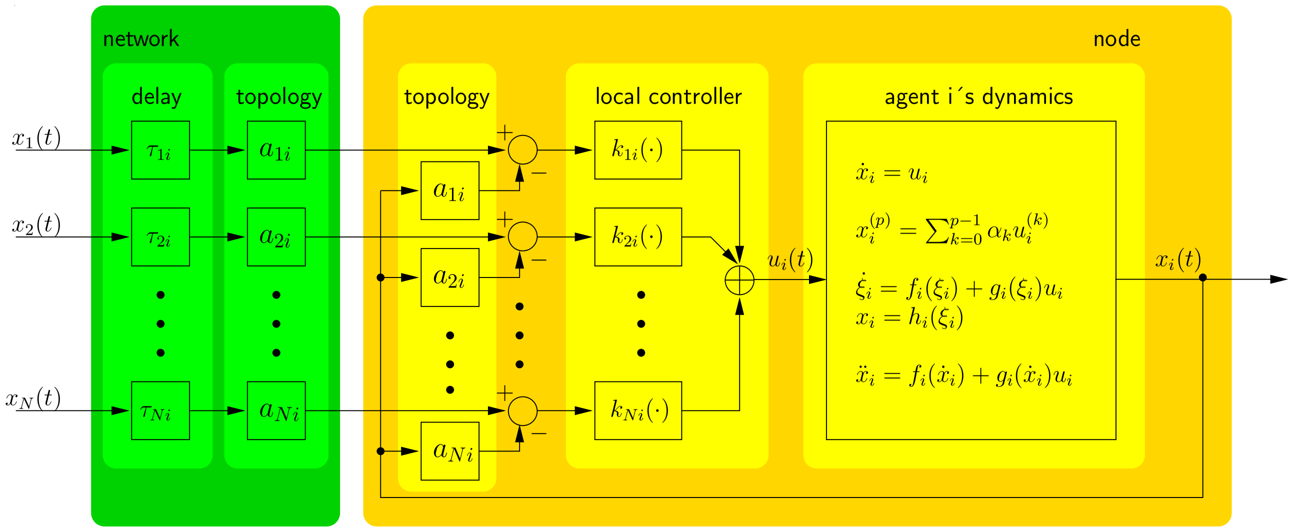


- approximation for packet-switched channel (Münz et al. 2007, 2009)
- f_τ models packet delay probability

delay-independent if consensus is guaranteed for all $\tau \geq 0$

delay-dependent if consensus is guaranteed for all $\tau \in [0, \overline{\tau}]$

State of the Art: Delayed MAS

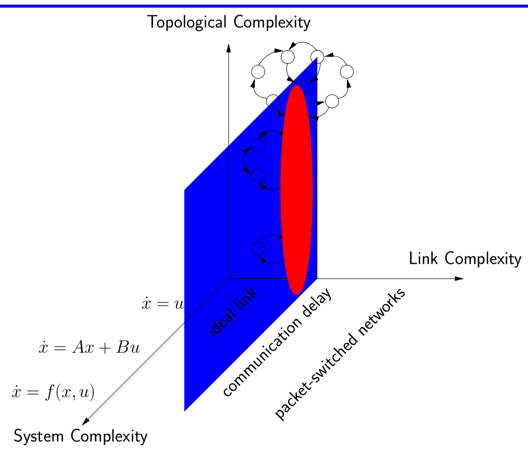


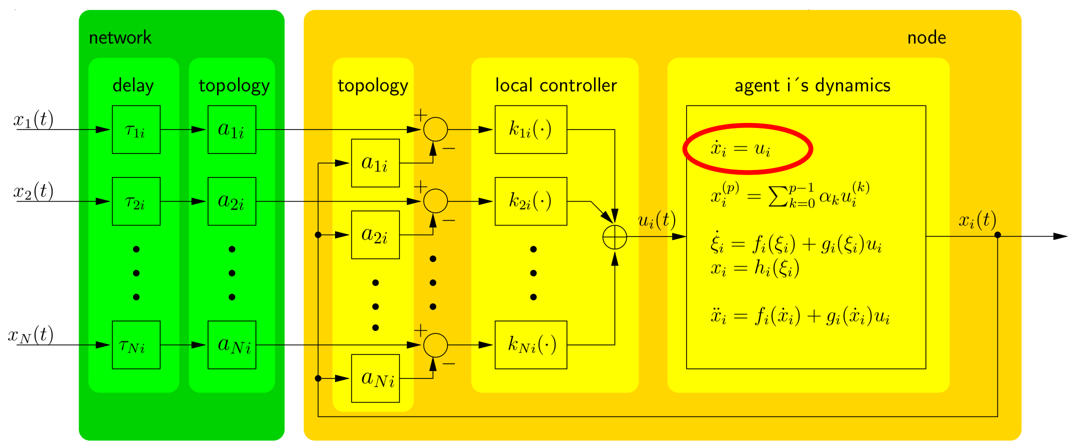
Papachristodoulou & Jadbabaie, 2005, 2006;
Chopra & Spong, 2006, 2008;
Münz, Papachristodoulou, Allgöwer, 2007, 2008, 2009;
Schmidt, Münz, Allgöwer, 2009

own state also delayed:

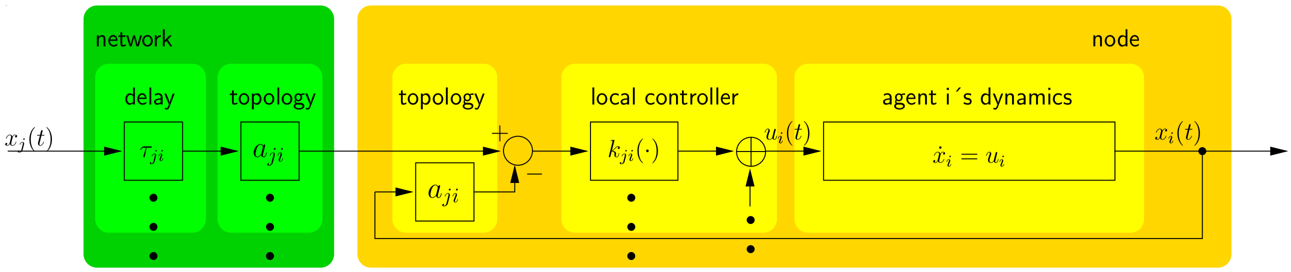
Olfati-Saber & Murray, 2004; Lestas & Vinnicombe, 2007;
Bliman & Ferrari-Trecate, 2008

Consensus in Nonlinear Delayed Single Integrator MAS





Consensus in Nonlinear Delayed Single Integrator MAS

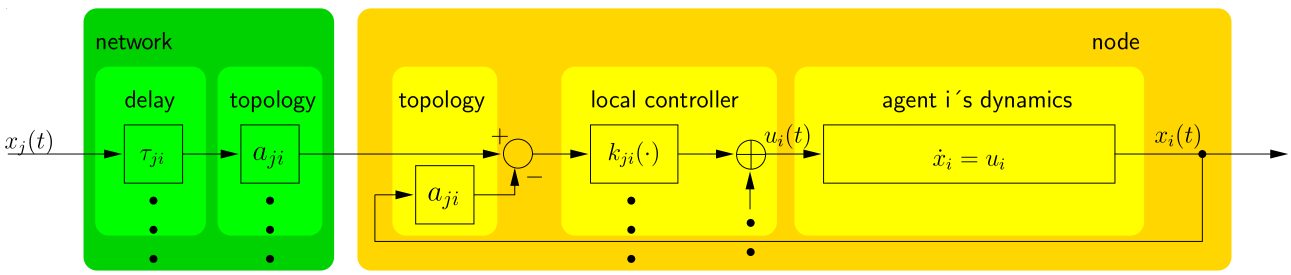


Theorem (Papchristodoulou & Jadbabaie, 2006; Münz, Papachristodoulou & Allgöwer, 2007, 2008, 2009)

Consensus is reached in directed, switching graphs with constant, time-varying, or distributed delays for any nonlinear, locally passive controller, i.e. $\eta k_{ji}(\eta) > 0, \forall \eta \in [-\gamma_{ji}^-, \gamma_{ji}^+] \setminus \{0\}$ if the initial condition satisfies

$$|x_i(\theta)| \leq \frac{\min_{i,j}\{\gamma_{ji}^-, \gamma_{ji}^+\}}{2}, \quad \forall \theta \in [-T, 0], i = 1, \ldots, N.$$

Consensus in Nonlinear Delayed Single Integrator MAS



Theorem (Papchristodoulou & Jadbabaie, 2006; Münz, Papachristodoulou & Allgöwer, 2007, 2008, 2009)

Consensus is reached in directed, switching graphs with

Delayed single integrator MAS ...

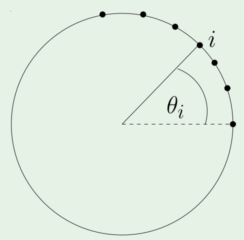
- always reach consensus with linear or nonlinear, globally passive controllers,
- reach consensus locally with nonlinear, locally passive controllers,

with minimal assumptions on delays and graph topology.

Example: Kuramoto Oscillator

Kuramoto oscillator (Kuramoto, 1984)

$$\dot{\theta}_i(t) = \omega_i + K \sum_{j=1}^{N} \frac{a_{ji}}{d_i} \sin(\theta_j(t - \tau_{ji}) - \theta_i(t))$$

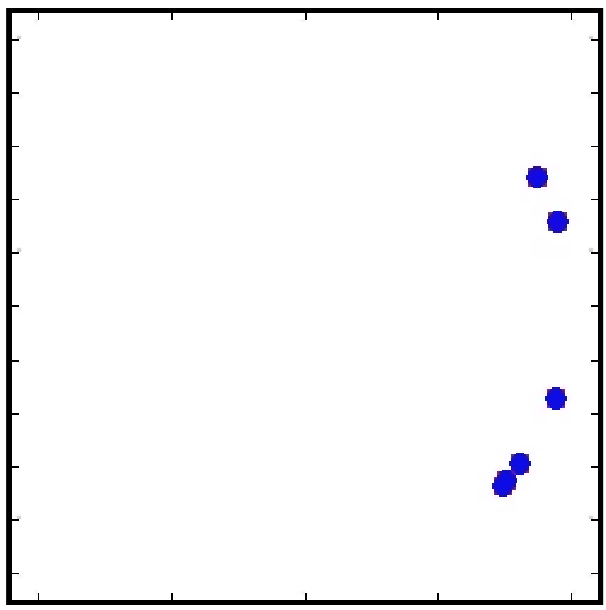


- pacemaker cells in the heart
- arrays of lasers
- microwave oscillations

Theorem (Papachristodoulou & Jadbabaie, 2006; Münz et al., 2009; Schmidt, Münz & Allgöwer 2009)

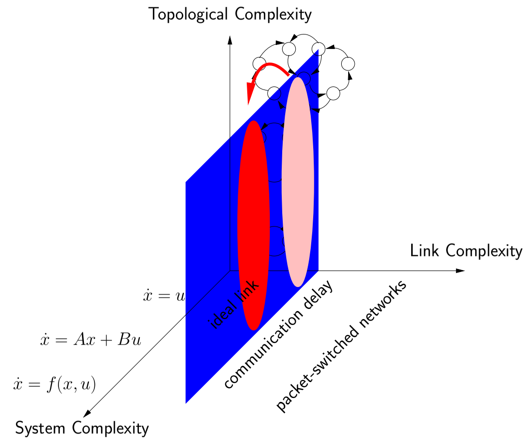
- phase synchronization if $\omega_i = \omega, \forall i$
- frequency synchronization if $\omega_i \in [\underline{\omega}, \overline{\omega}], \forall i$
- phase synchronization if $\omega_i \in [\underline{\omega}, \overline{\omega}], \forall i$ and if delays τ_{ji} are chosen appropriately (not possible without delays!)

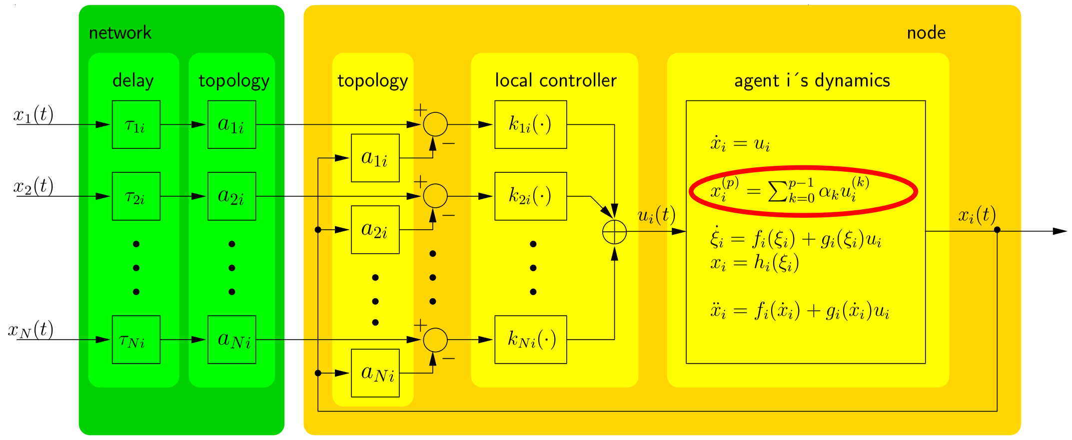
Phase Synchronization in Heterogeneous Kuramoto Oscillators with and without Delays



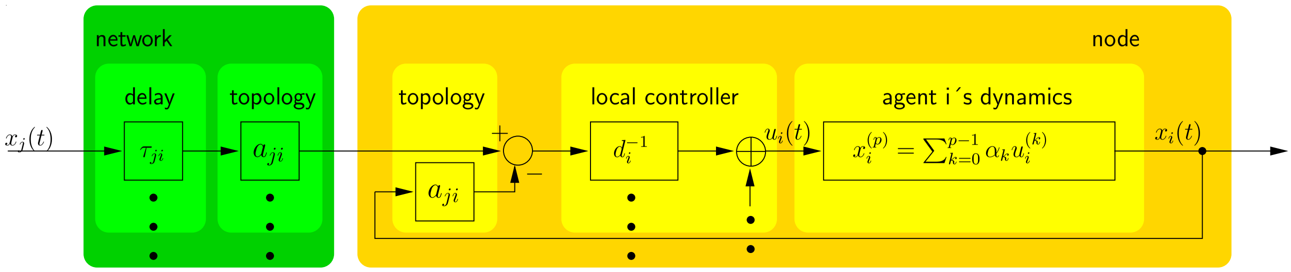
- Kuramoto oscillators synchronize in networks with delays
- Delays achieve phase synchronization, which is not possible without delays

Consensus in Linear Delayed Multi-Integrator MAS





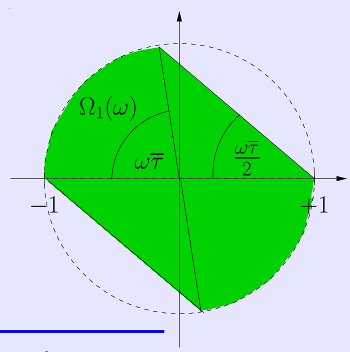
Consensus in Linear Delayed Multi-Integrator MAS



Theorem (Münz, Papachristodoulou, Allgöwer, 2009)

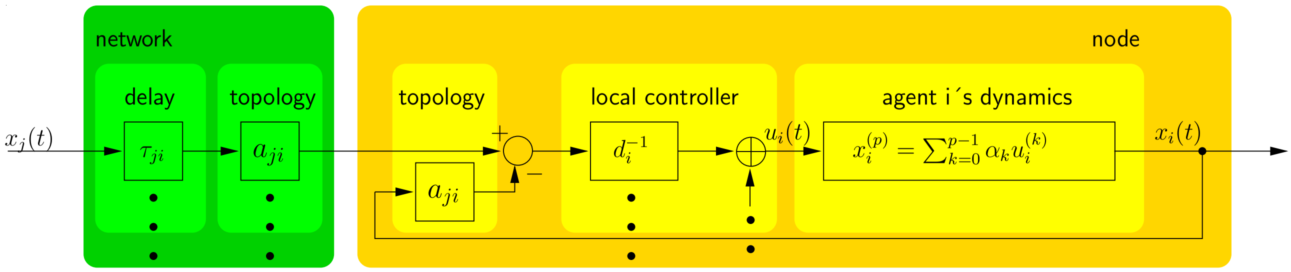
Consider linear delayed multi-integrator MAS in undirected graphs with constant, symmetric delays $\tau_{ji} = \tau_{ij} \leq \overline{\tau}$ and degree normalizing controllers $k_{ji}(\eta) = \frac{1}{d_i}\eta$, where $d_i = \sum_{j=1}^{N} a_{ji}$. Consensus is reached if and only if

$$\frac{(j\omega)^p + \sum_{k=1}^{p-1} \alpha_k (j\omega)^k}{\sum_{k=1}^{p-1} \alpha_k (j\omega)^k} \notin \Omega_1(\omega)$$



for all $\omega \neq 0$.

Consensus in Linear Delayed Multi-Integrator MAS



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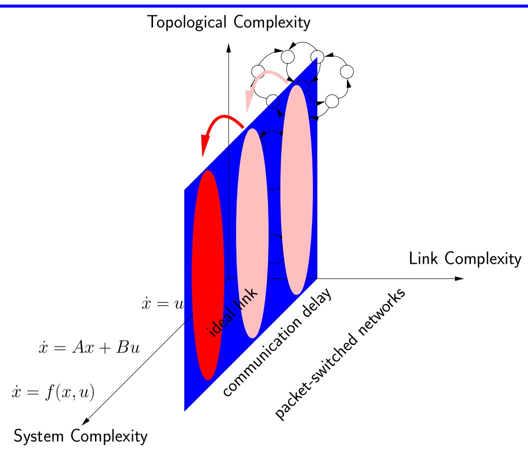
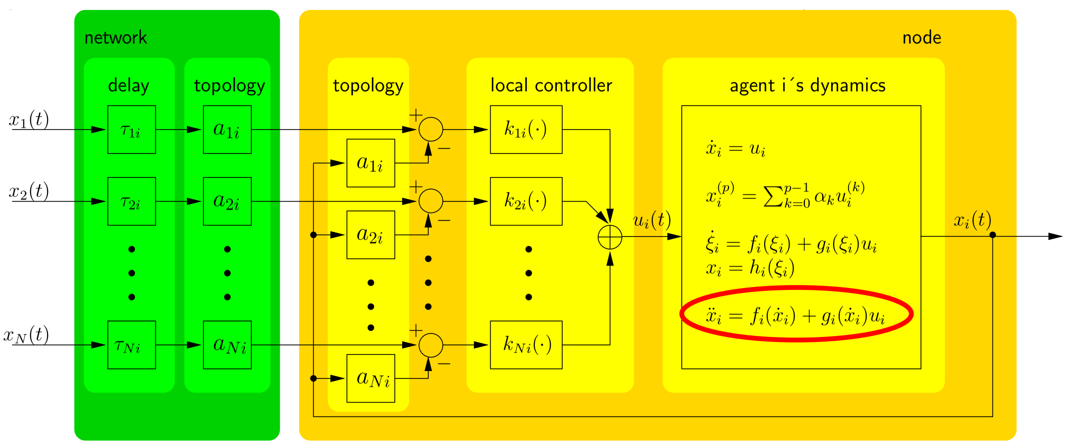
Linear delayed multi-integrator MAS

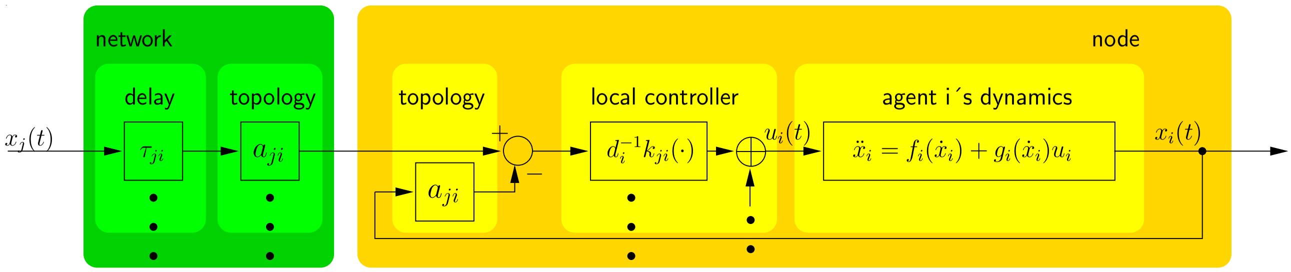
- necessary and sufficient set-valued condition
- analytical results for first and second order MAS
- delay-dependent convergence rate condition for first order MAS

Consensus in Nonlinear MAS with Relative Degree Two





Consensus in Nonlinear MAS with Relative Degree Two

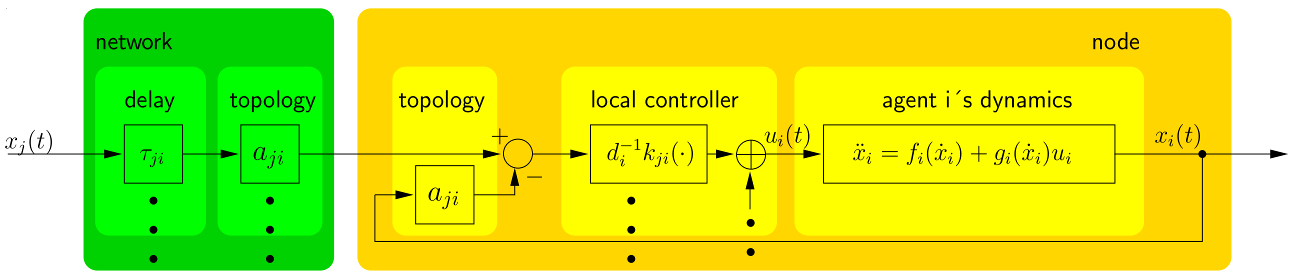


Theorem (Münz, Papachristodoulou, Allgöwer, 2009)

Assume f_i are globally sector bounded, i.e. $\eta f_i(\eta) \geq \alpha_i \eta^2$ and g_i are globally positive and bounded, i.e. $g_i(\eta) \in (0, \beta_i)$, $\forall \eta$.
Consensus is reached in undirected graphs with constant delays for any nonlinear, globally Lipschitz controller, i.e.
$|k_{ji}(\eta_1) - k_{ji}(\eta_2)| \leq \kappa_{ji}|\eta_1 - \eta_2|, \forall \eta_1, \eta_2$, if

$$\kappa_{ji} < \frac{\alpha_i}{\beta_i \max\{\tau_{ji}, \tau_{ij}\}}, \quad \forall i, j.$$

Consensus in Nonlinear MAS with Relative Degree Two



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Nonlinear delayed MAS with relative degree two

- first result for relative degree two agents
- delay-dependent decentralized design for heterogeneous agents

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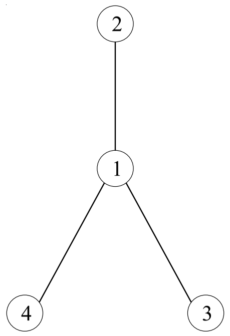
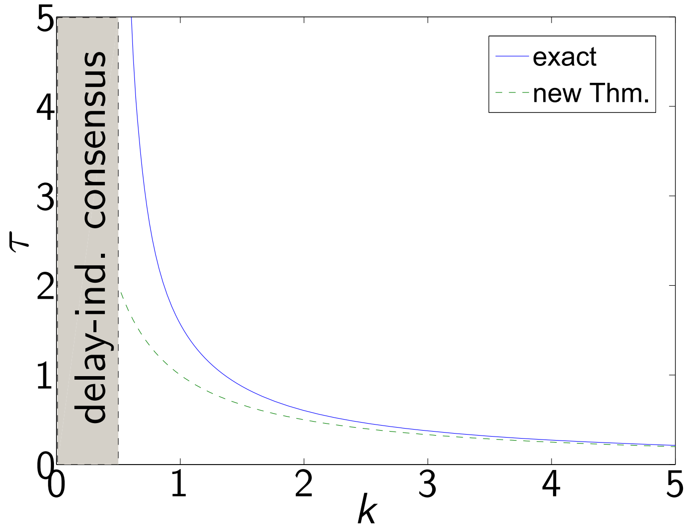
Motivating Example: Consensus of 4 Agents



$$\ddot{x}_i(t) = -\dot{x}_i(t) - \sum_{j=1}^{N} k \frac{a_{ji}}{d_i} (x_i(t) - x_j(t-\tau))$$

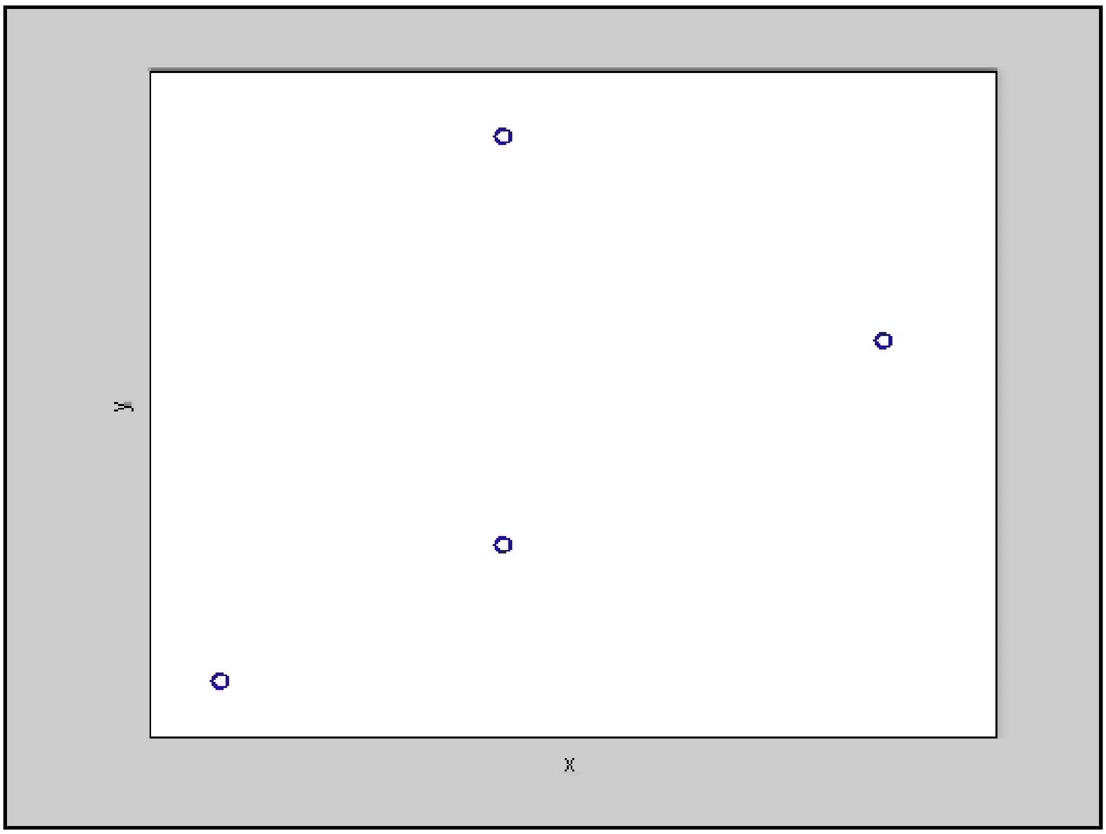
$k < \frac{1}{\tau} \Longrightarrow$ consensus



Simulation parameters:

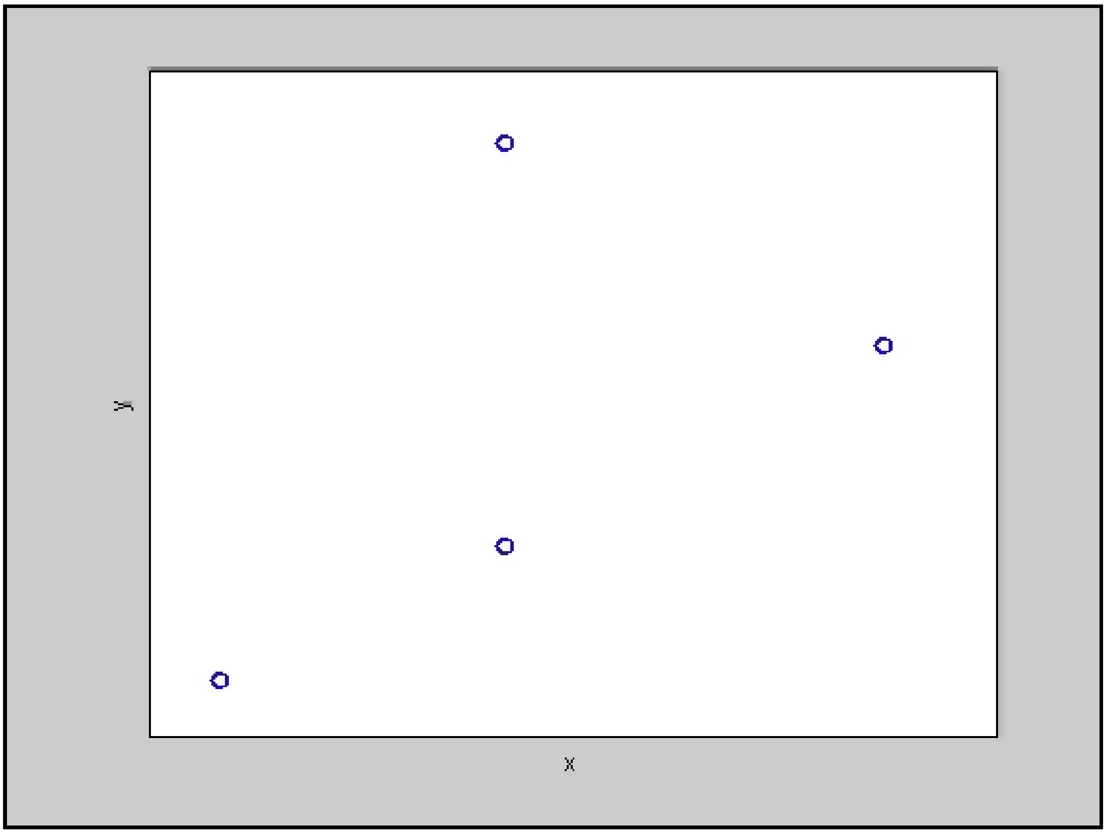
- chosen gain: $k = 2$
- exact bound: $\tau < 0.6046$
- new condition: $\tau < 0.5$

Simulation



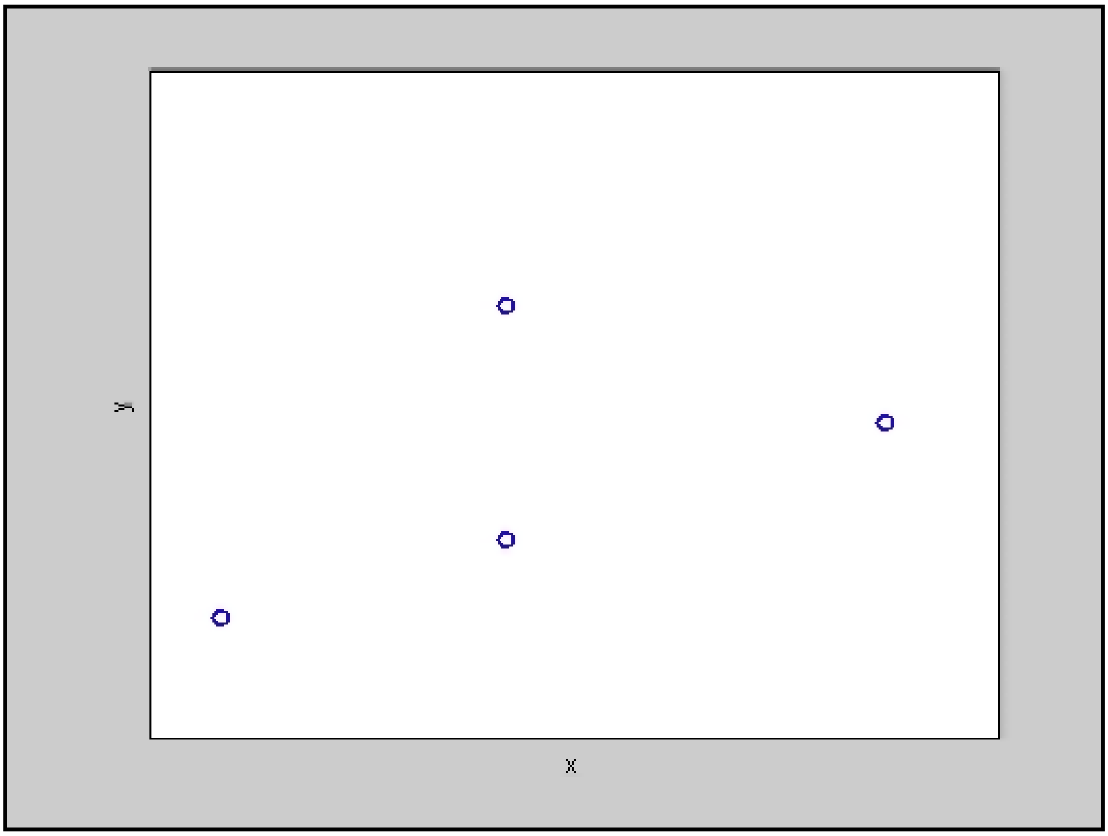
Second order linear MAS with communication delay $\tau = 0.01$.

Simulation



Second order linear MAS with communication delay $\tau = 0.3$.

Simulation

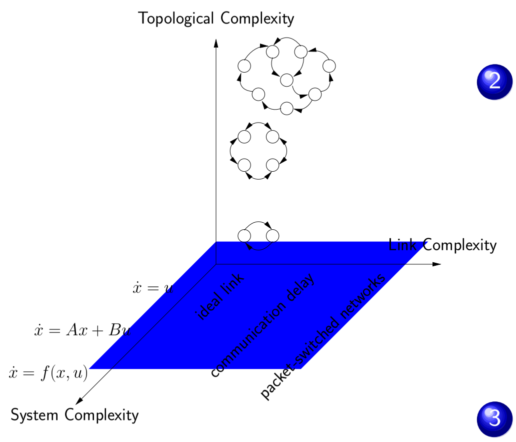


Second order linear MAS with communication delay $\tau = 1$.

Summary – Consensus of Multi-Agent Systems

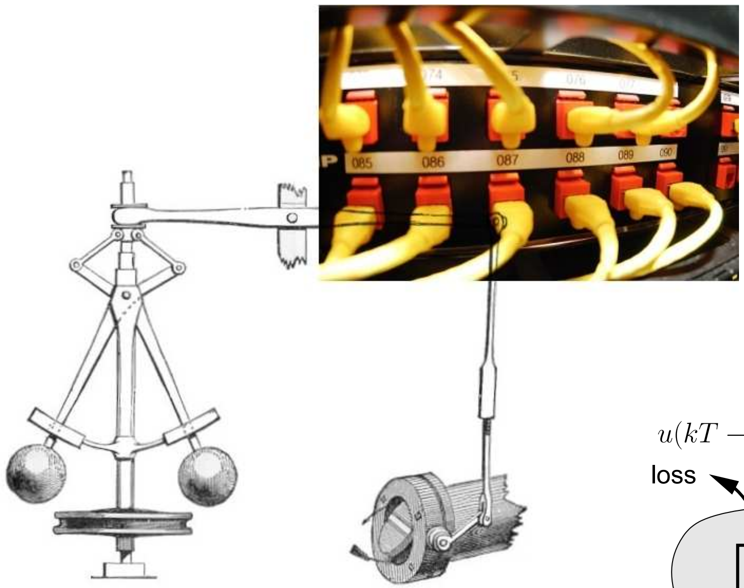
- ✓ consensus for general identical LTI systems without delays
- ✓ delays may corrupt consensus
- ✓ delay-independent consensus for first order MAS and MAS with relative degree one, but convergence rate is delay-dependent
- ✓ delay-dependent consensus for second order MAS and MAS with relative degree two
- ✗ more complex dynamics require more restrictions on delays and topology

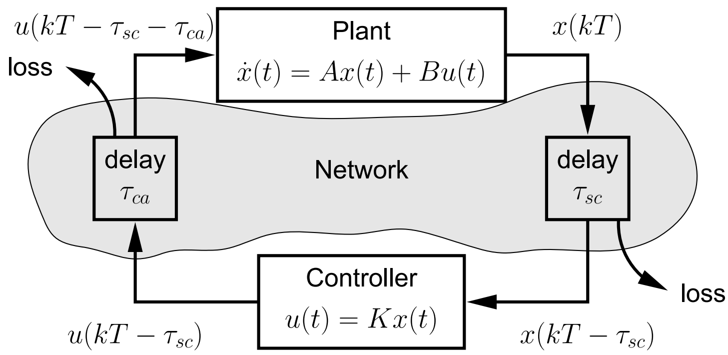
Overview



1. Consensus in linear Multi-Agent Systems with ideal links
2. Consensus in Multi-Agent Systems with communication delays
3. **Control via digital networks**

Networked Control Systems





Physical interconnections are replaced by digital networks.

Link Complexity

ideal link → delay → loss → detailed link model

ideal link
- no delay
- no loss
- no bandwidth limitations

delay

constant
- known in advance
- unknown, in interval

time varying
- known
- unknown
- random, with known distribution

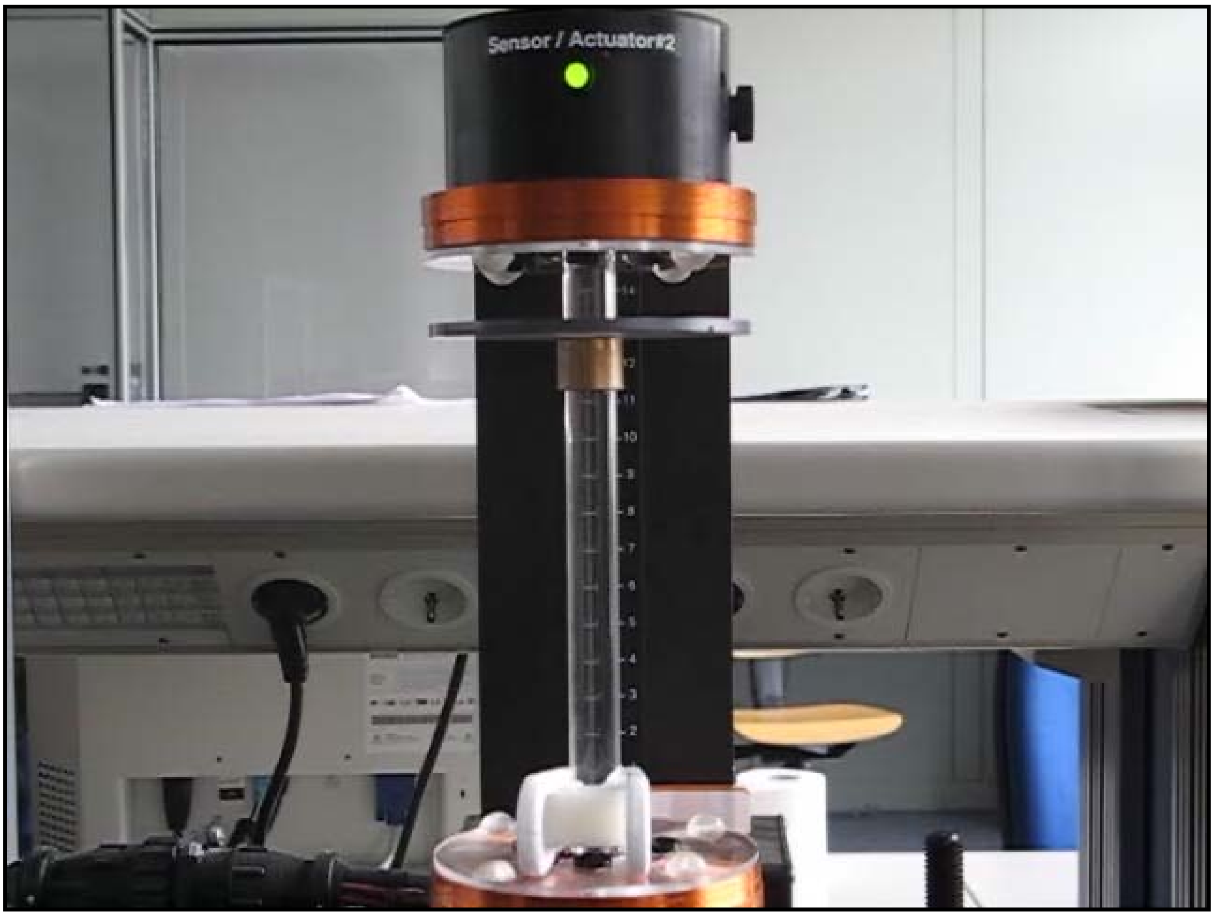
loss

where?
- sensor → controller
- controller → actuator
- ...

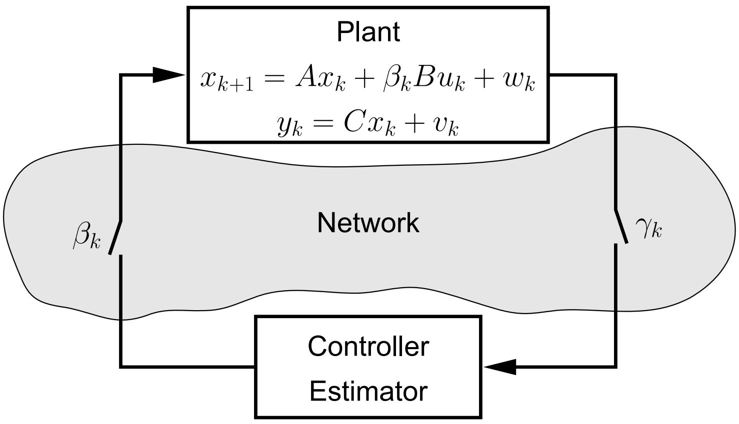
random process
- iid
- Markov

detailed link model
- interaction of loss, delay and data rate
- e.g. queuing system

The Negative Effects of Loss



Loss of Control or Measurement Packets



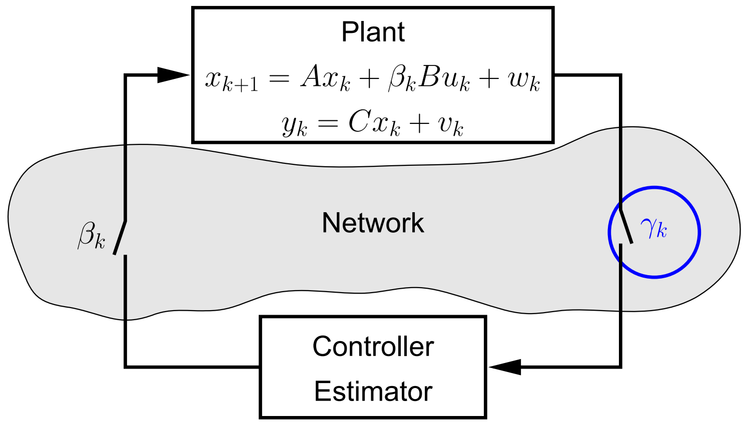
Loss of measurement packets:

- The estimator can only simulate, no correction step.

Loss of control packets:

- The system runs open loop.
- The input to the plant is unknown to the controller/estimator.

Loss of Control or Measurement Packets



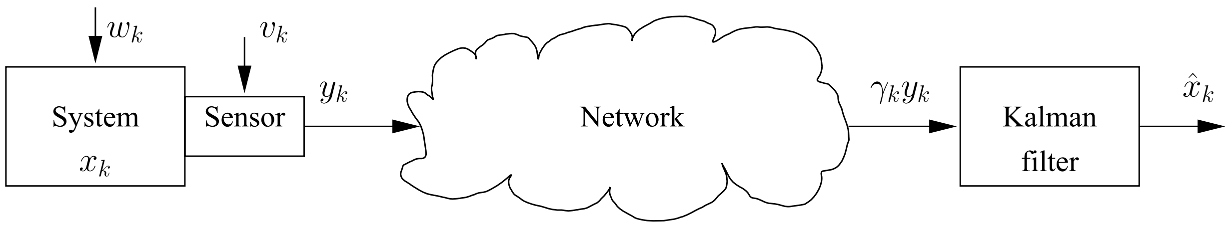
Loss of measurement packets:

- The estimator can only simulate, no correction step.

We want to find suitable methods to

- analyze the effects of the loss.
- take these effects into account.
- compensate these effects.

Kalman Filtering with Intermittent Observations



When do we get $x_k \approx \hat{x}_k$?

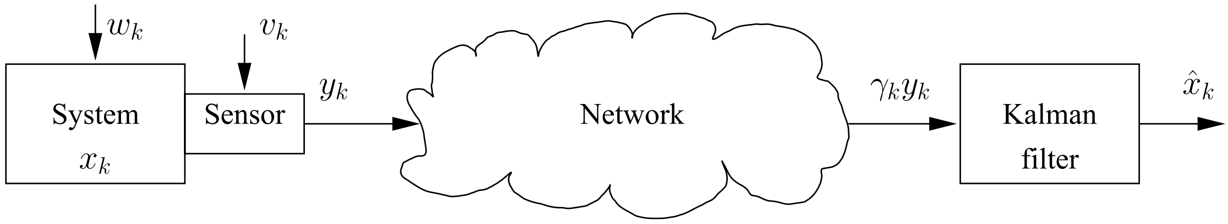
Theorem [Sinopoli et al. 04]

If $(A, Q^{1/2})$ is controllable, (A, C) is detectable, and A is unstable, then there exists a $\lambda_c \in [0, 1)$ such that

$$\lim_{k\to\infty} \mathrm{E}[P_k] = +\infty, \quad \text{for } 0 \leq \lambda \leq \lambda_c \text{ and } \exists P_0 \geq 0$$
$$\mathrm{E}[P_k] \leq M_{P_0} \forall k, \quad \text{for } \lambda_c < \lambda \leq 1 \text{ and } \forall P_0 \geq 0,$$

where $\lambda := E[\gamma_k]$ and $P_k = \mathrm{E}[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T | y_{k-1}, \gamma_{k-1}]$.
For λ_c a lower and upper bound can be given: $\underline{\lambda} \leq \lambda_c \leq \bar{\lambda}$.

Kalman Filtering with Intermittent Observations



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The system can be observed $(x_k \approx \hat{x}_k)$ if not too many packets are lost.

Kalman Filtering with Intermittent Observations

Theorem [Sinopoli et al. 04]

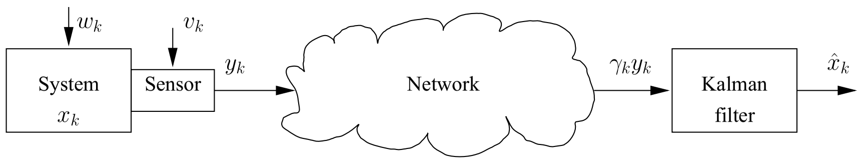
The upper bound $\bar{\lambda}$ is given by the solution of the following optimization problem:

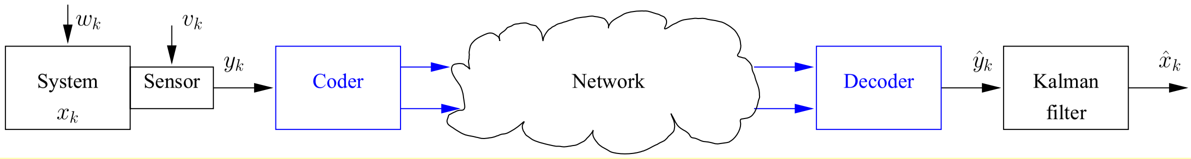
$$\bar{\lambda} = \arg\min_{\lambda} \Psi_\lambda(Y, Z) > 0, \qquad 0 \le Y \le I.$$

where

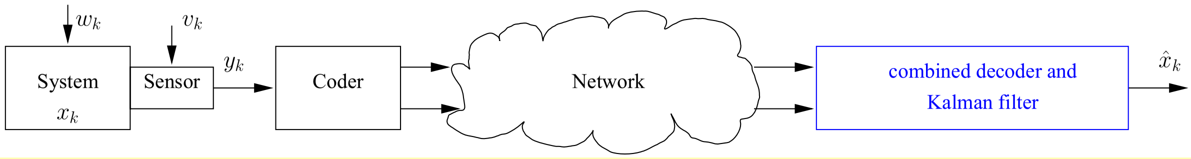
$$\Psi_\lambda(Y, Z) = \begin{bmatrix} Y & \sqrt{\lambda}(YA + ZC) & \sqrt{1-\lambda}YA \\ \star & Y & 0 \\ \star & \star & Y \end{bmatrix}$$

Coding to Improve the Kalman Filtering



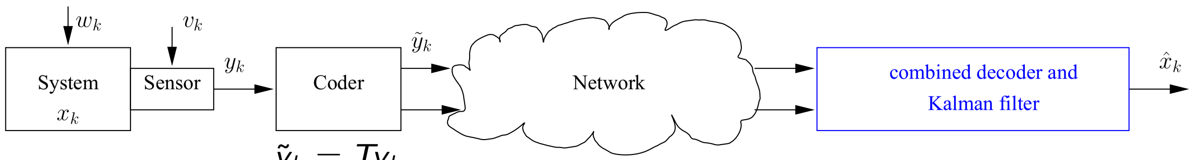


coder and decoder are designed such that $y_k \approx \hat{y}_k$



combined decoder and Kalman filter such that $x_k \approx \hat{x}_k$

Coding to Improve the Kalman Filtering



$\tilde{y}_k = Ty_k$

$$x_{k+1} = Ax_k + w_k$$
$$y_k = Cx_k + v_k$$

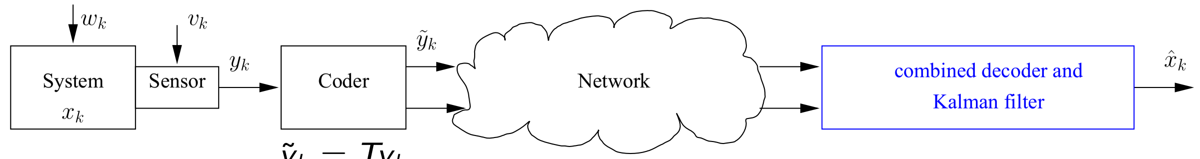
Motivating example

Consider the system

$$A = \begin{bmatrix} 2.5 & 0 \\ 0 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- The system is not observable if one measurement is missing.
- Send virtual measurement $\tilde{y} = Ty$, where T is invertible. E.g. $\tilde{y}_1 = y_1 + y_2$ and $\tilde{y}_2 = y_1 - y_2$.
- If one of the virtual measurements is lost, then the system is still observable.

Coding to Improve the Kalman Filtering



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- The system is not observable if one measurement is missing.
- Send virtual measurement $\tilde{y}_k = Ty_k$ where T is invertible

Choose T such that either

- the Kalman filter can tolerate a higher packet loss rate, or
- we get better estimates for a fixed loss rate.

Coding to Improve the Kalman Filtering

Theorem [Blind et al.]

The upper bound $\bar{\lambda}$ is given by the solution of the following optimization problem:

$$\bar{\lambda} = \arg\min_{\lambda} \Psi_\lambda(Y, Z_1, \ldots, Z_E, T) > 0, \quad 0 \le Y \le I,$$

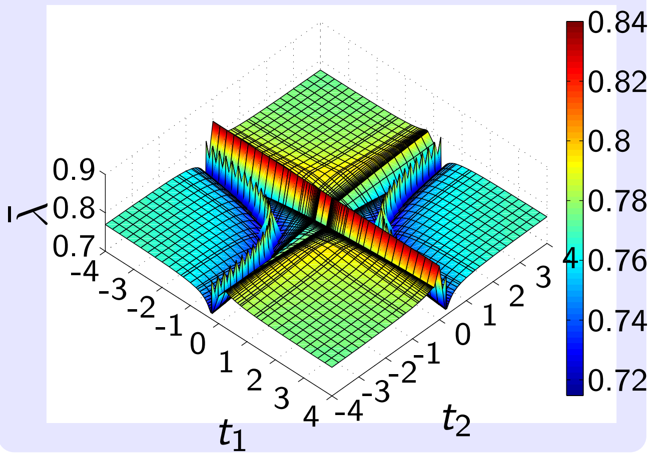
where

$$\Psi_\lambda(Y, Z_1, \ldots, Z_E, T) =$$
$$\begin{bmatrix} Y & \sqrt{w_1(\lambda)}(YA + Z_1\tilde{L}_1TC) & \cdots & \sqrt{w_E(\lambda)}(YA + Z_E\tilde{L}_ETC) \\ \star & Y & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \star & \star & \cdots & Y \end{bmatrix}.$$

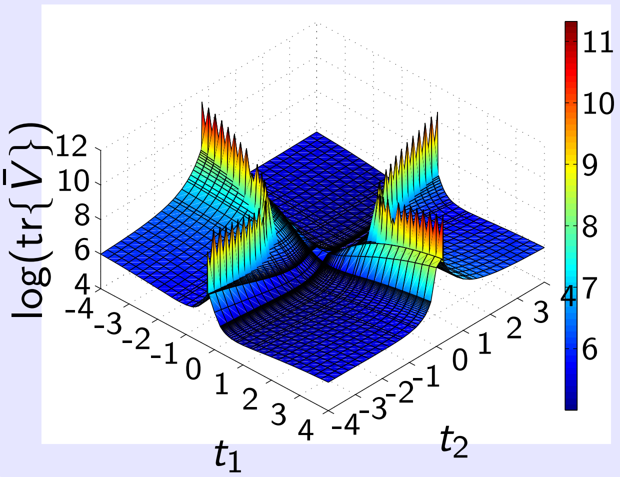
Coding to Improve the Kalman Filtering

T can be normalized to $T = \begin{bmatrix} 1 & t_1 \\ t_2 & 1 \end{bmatrix}$.

T to reduce $\bar{\lambda}$



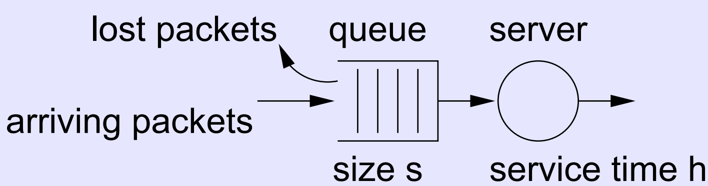
T to get better estimates

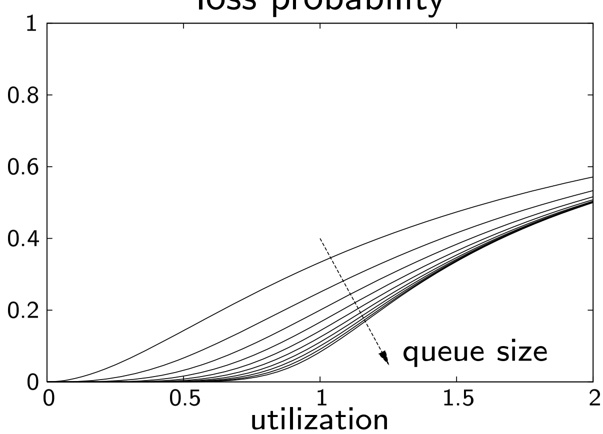


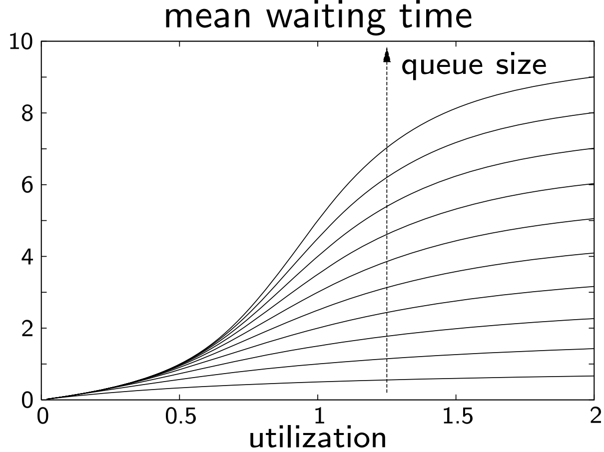
By a suitable choice of T, we can tolerate a higher loss rate or get better estimates for a fixed loss rate.

Detailed Link Model: Queueing System

Standard link model in communication networks:



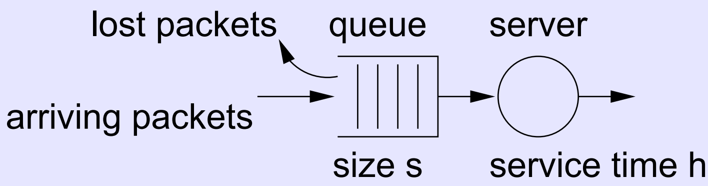




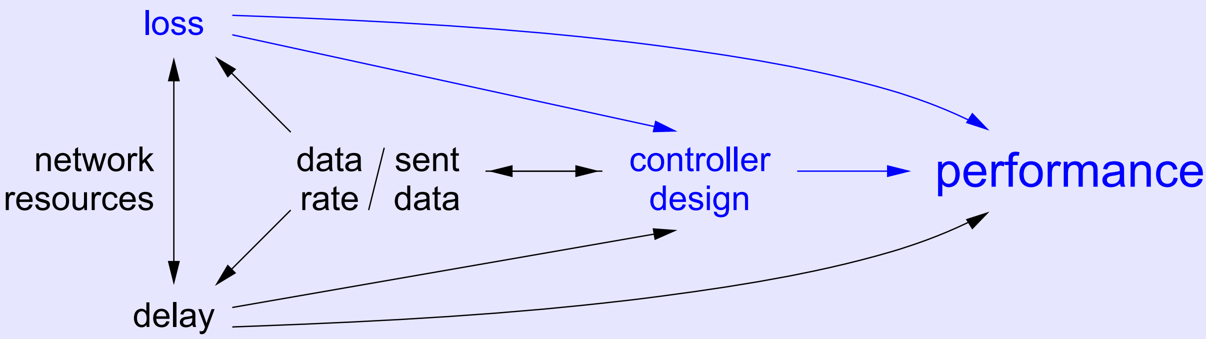
Loss and delay depends on the network resources.

How it All Interacts

Standard link model in communication networks:



How it all interacts



Summary – Control via Digital Networks

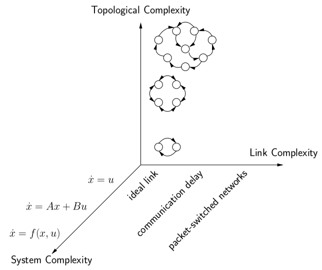
Summary

- Physical interconnections are replaced by digital networks ⇒ loss and delay of packets.
- Loss and delay of packets is considered individually.
- Methods to analyze the effects of loss.
- Methods to compensate these effects, e.g. coding.

Outlook

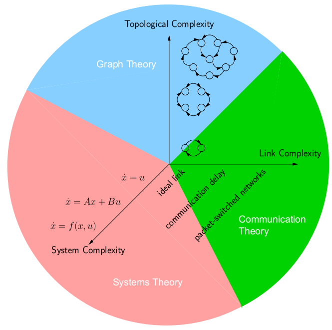
Use more complex network models, which model the interaction between loss and delay.

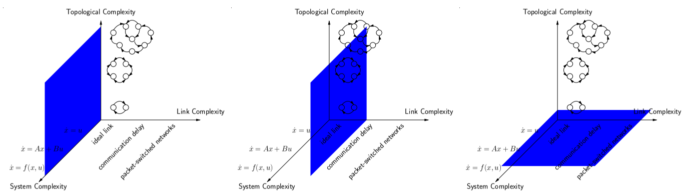
Conclusions



current trends in control systems introduce new dimensions of complexity

control over communication networks combines systems, graph, and communication theory





The story just started!
We need to exploit all dimensions of complexity.

Acknowledgements

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Control Theory of Digitally Networked Dynamical Systems
German Research Foundation (Deutsche Forschungsgemeinschaft)

Symposium and Workshop Invitation

NE{S|T}CoC Sep. 28/29, 2009

Symposium on
Recent Trends in Networked Systems and Cooperative Control
Monday, September 28, 2009, Stuttgart, Germany

Workshop on
Network-Induced Constraints in Control
Tuesday, September 29, 2009, Stuttgart, Germany