

# On Time Parameterizations of User Demands in Mechatronics

10<sup>th</sup> PhD Workshop - Hluboká nad Vltavou

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**Pod Vodárenskou věží 4**

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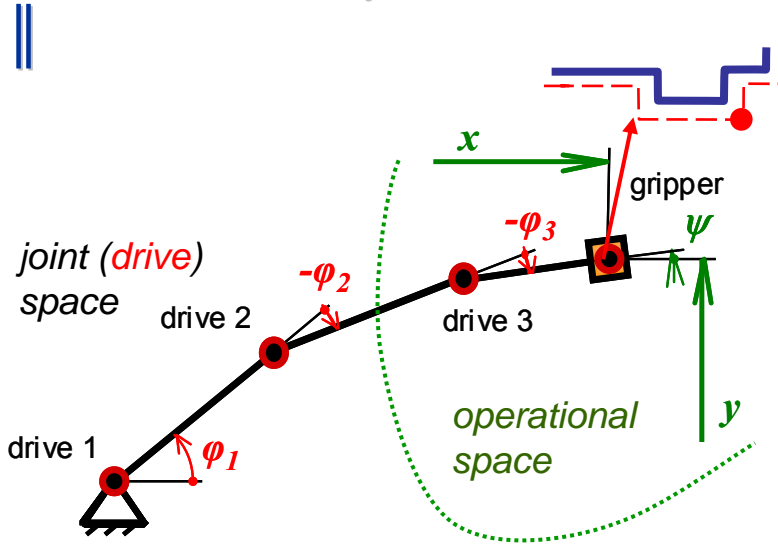
# On Time Parameterizations of User Demands in Mechatronics

## Outline:

- ⌊ User Demands in Mechatronics (Intro)
- ⌋ Concepts of Time Parameterization
- ⌋ Dynamical point of view of time parameterization
  - ⌋ Range-Space Modification - example
  - ⌋ End-Point Modification - example
- ⌋ Kinematical point of view of time parameterization
- ⌋ Several Notes as Conclusion

# User Demands in Mechatronics

- *mechatronic system*

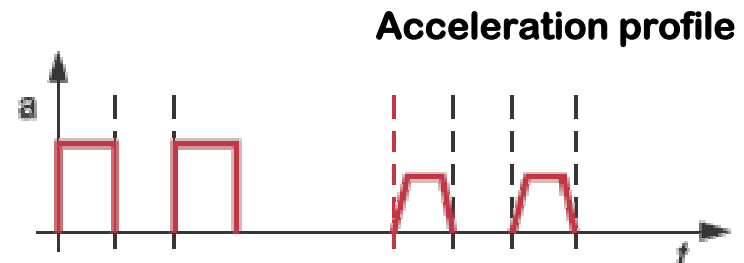
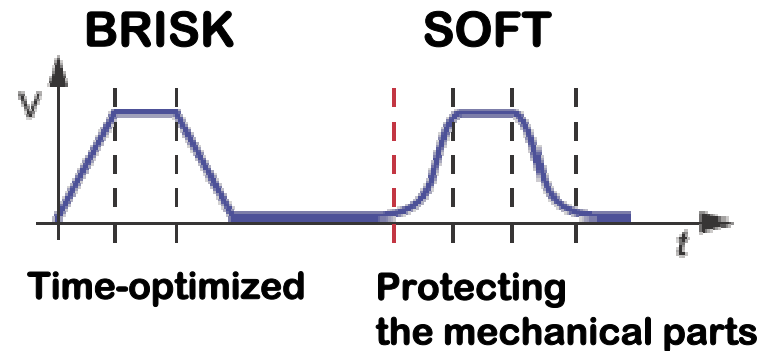


≡ *mechanical elements*  
(beams, joints, gears, grippers)

≡ *electro-mechanical*  
(drives, sensors)

≡ *electrical*  
(control units)

- *combination of technological demands and construction limits of the system:*  
*to accomplish assigned path*  
*or to reach predetermined key point*  
*with certain velocity*  
*and acceleration profiles . . .*





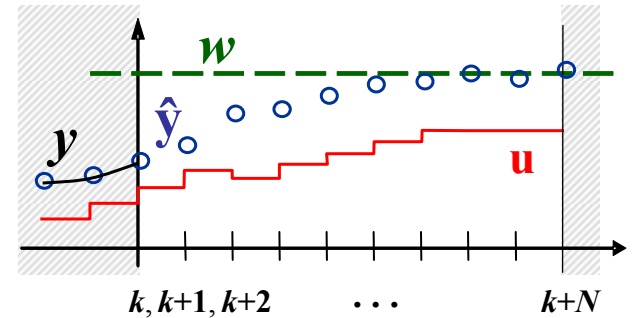
# Dynamical point of view of time parameterization via Predictive Control:

**Model + Cost function (criterion):**

$$\begin{aligned} \hat{\mathbf{x}}_{k+1} &= \mathbf{A} \mathbf{x}_k + \mathbf{B} \mathbf{u}_k \\ \hat{\mathbf{y}}_{k+1} &= \mathbf{C} \hat{\mathbf{x}}_{k+1} \end{aligned} \quad J = \sum_{j=k}^{k+N} \left( (\hat{\mathbf{y}}_{j+1} - \mathbf{w}_{j+1})^T \mathbf{Q}_y (\hat{\mathbf{y}}_{j+1} - \mathbf{w}_{j+1}) + (\mathbf{u}_j - \mathbf{u}_{j-1})^T \mathbf{Q}_u (\mathbf{u}_j - \mathbf{u}_{j-1}) \right)$$

**Equations of predictions  $\Rightarrow \hat{\mathbf{y}} = \mathbf{f} + \mathbf{G} \Delta \mathbf{u}$**

$$\hat{\mathbf{y}} = [\hat{\mathbf{y}}_{k+1}, \dots, \hat{\mathbf{y}}_{k+N+1}]^T, \quad \mathbf{u} = [\mathbf{u}_k, \dots, \mathbf{u}_{k+N}]^T$$



**Minimization of quadratic criterion:**

$$\min_{\mathbf{u}} J = \min_{\mathbf{u}} \mathbf{J} \mathbf{J}, \quad (\min_{\mathbf{u}} (\text{quadratic form})), \quad \mathbf{u}_{opt} = \arg \min_{\mathbf{u}} \mathbf{J}$$

**Range-Space Modification**

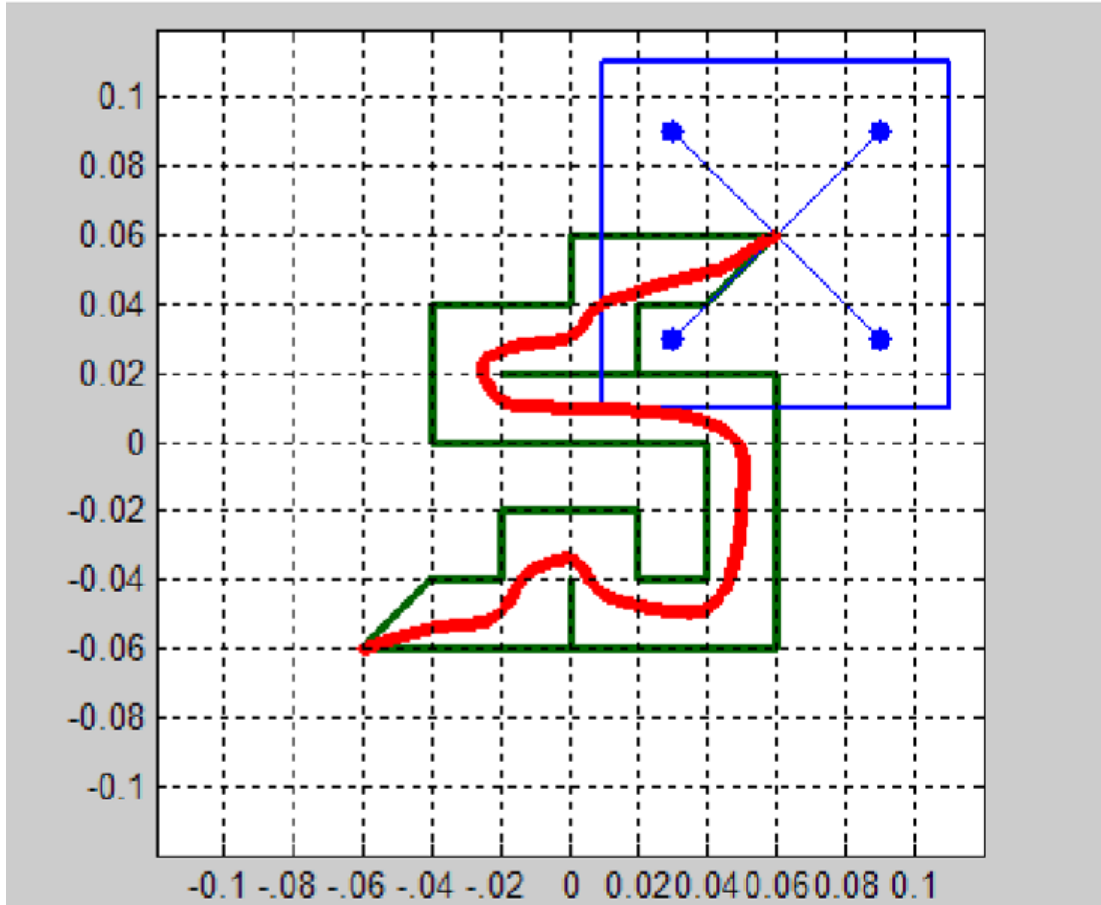
$$\begin{aligned} J_k &= \sum_{j=k}^{k+N} \{ \| (\hat{\mathbf{y}}_{j+1} - \mathbf{r}_{a,j+1}) \mathbf{Q}_{ra} \|^2 \\ &\quad + \| (\hat{\mathbf{y}}_{j+1} - \mathbf{r}_{b,j+1}) \mathbf{Q}_{rb} \|^2 + \| \mathbf{u}_j \mathbf{Q}_u \|^2 \} \end{aligned}$$

**End-Point Modification**

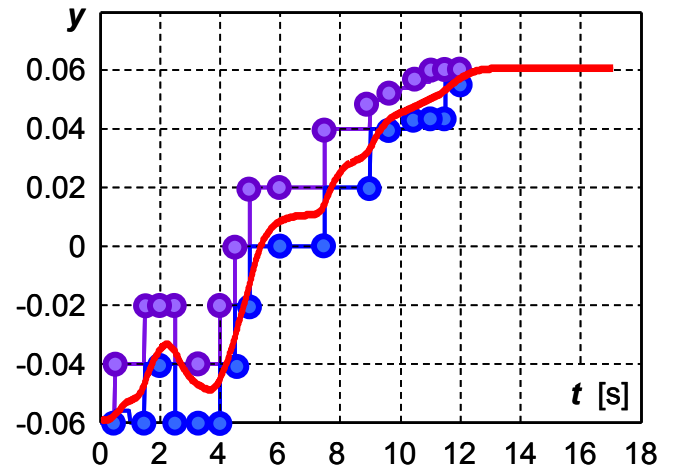
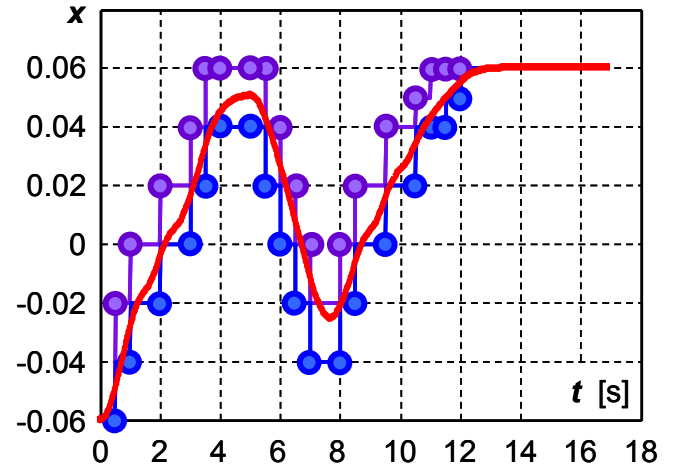
$$\begin{aligned} J_k &= \sum_{j=N_0+1}^{k+N} \{ \| (\hat{\mathbf{y}}_{k+j} - \mathbf{w}_{k+j}) \bar{\mathbf{Q}}_y \|^2 \\ &\quad + \| \mathbf{u}_{k+j-1} \bar{\mathbf{Q}}_u \|^2 \} \end{aligned} \quad \underbrace{\mathbf{w} = \text{const.}}$$

# Range-Space Modification - example

*xy*-graph: motion of a robot movable platform

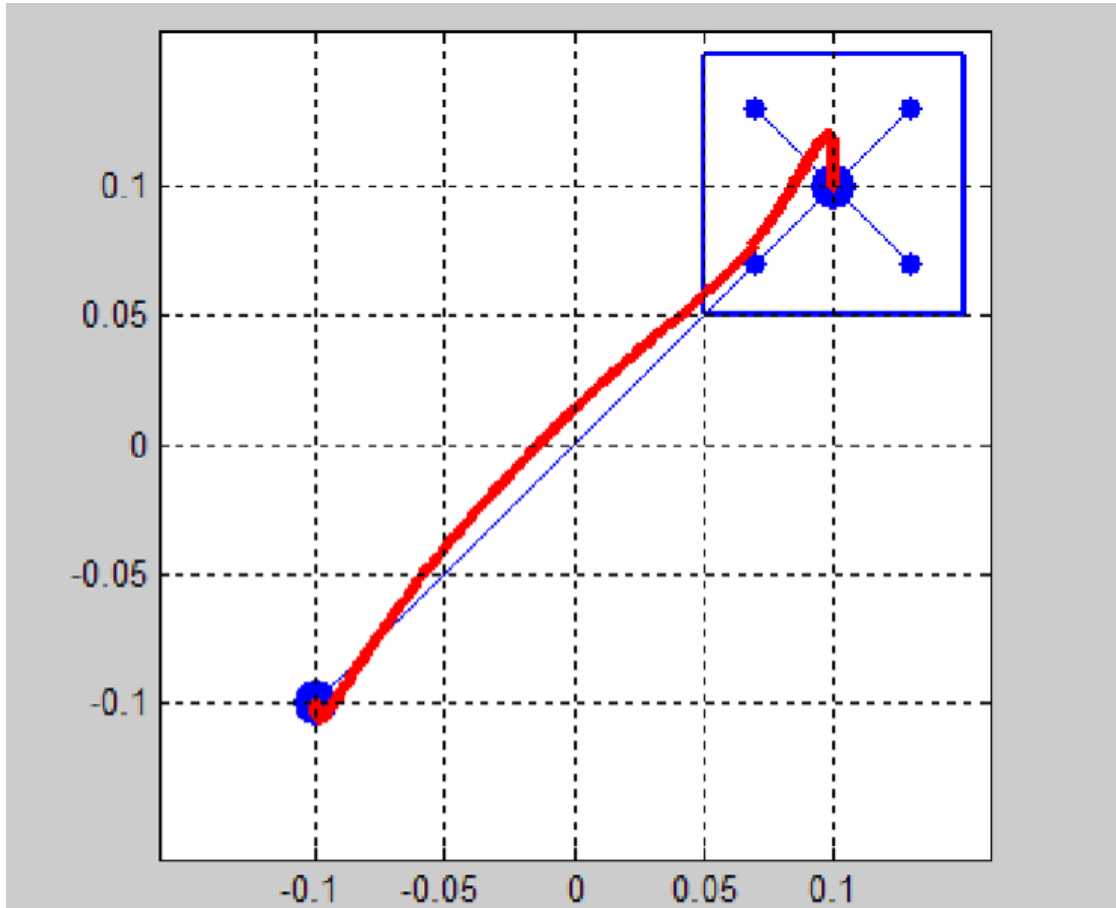


time – graphs:  $x(t)$ ,  $y(t)$

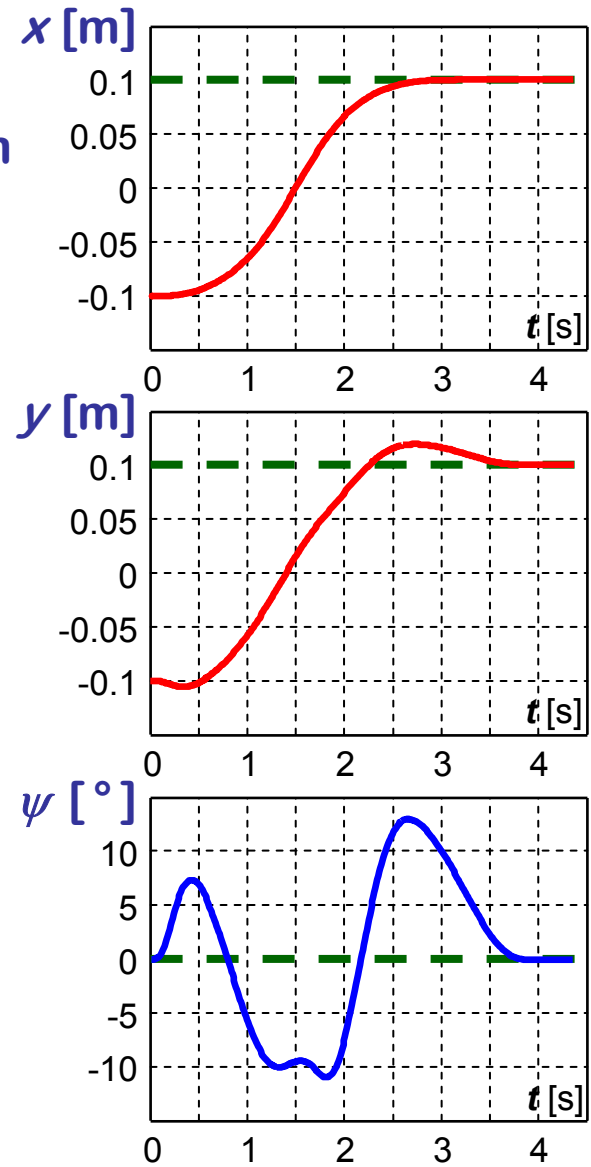


# End-Point Modification - example

*xy*-graph: vertical motion of a robot movable platform



time graphs



# Several Notes as Conclusion

- *Suitable time parameterization can provide:*
  - *safe utilization of machinery*
  - *optimal duration of the motion process*
  - *optimal path of the machinery motion*
- *In the presentation, there were addressed:*
  - *kinematical approach*  
*based on analytical geometry and kinematical laws*
  - *dynamical approach*  
*realized by specific control tasks*  
*and using appropriate dynamical models of systems*
- *The procedures of time parameterization together with path optimization are continuously under development due to continuous efforts to minimize production costs at keeping the profit from number of products*
  - ⇒ *minimize operational time*  
*(path optimization . . . )*
  - ⇒ *maximize machinery use*  
*(velocity and acceleration profile optimization)*



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Thank you  
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