The Minority Game with Heterogenous Agents

10th International PhD Workshop, Hluboká nad Vltavou

Miroslav Pištěk
Department of Adaptive Systems, UTIA CAS, Prague
e-mail: miroslav.pistek@gmail.com

Supervisor: RNDr. František Slanina, CSc.
Institute of Physics, CAS, Prague

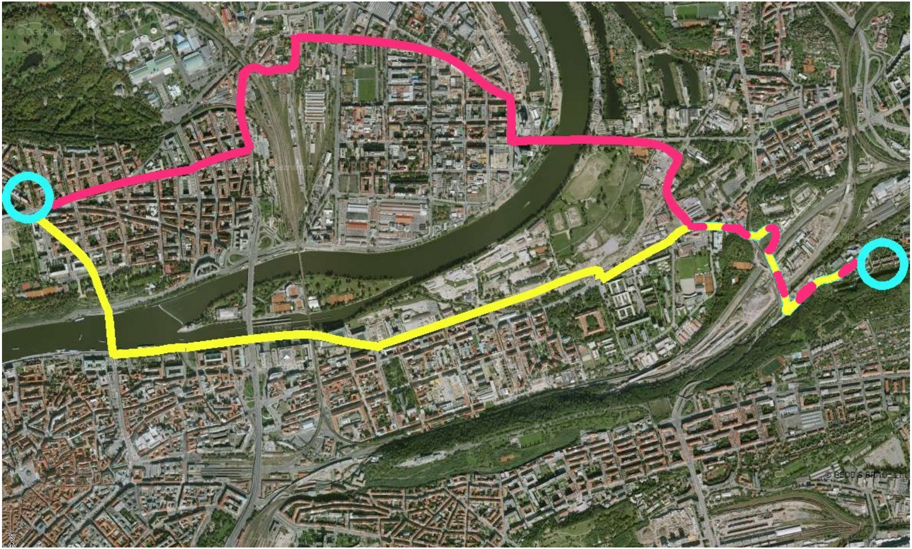
Outline

- Motivation for the Minority Game: traffic jams
- Local view, model of a driver
- Global view, traffic imbalance
- Introduction of heterogenous vehicles

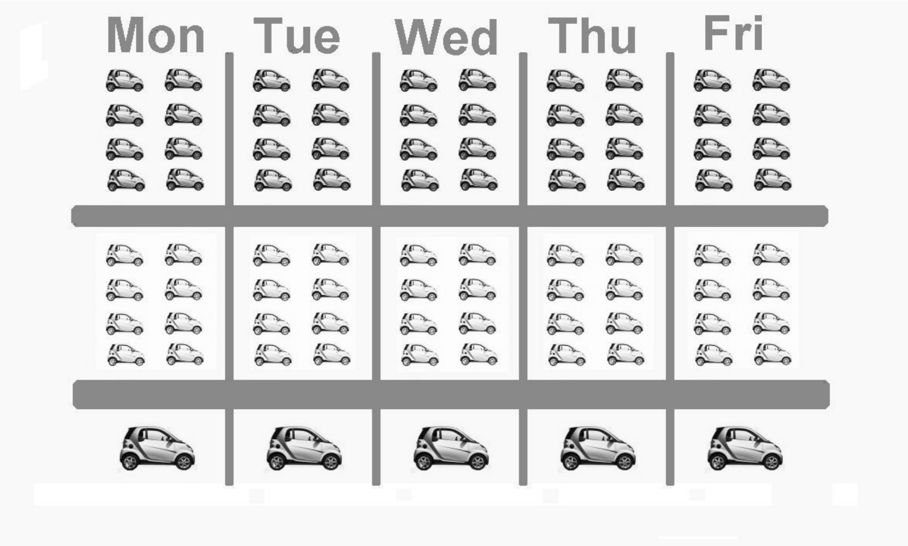
Life in the city = drive in the city

How to select an optimal road?

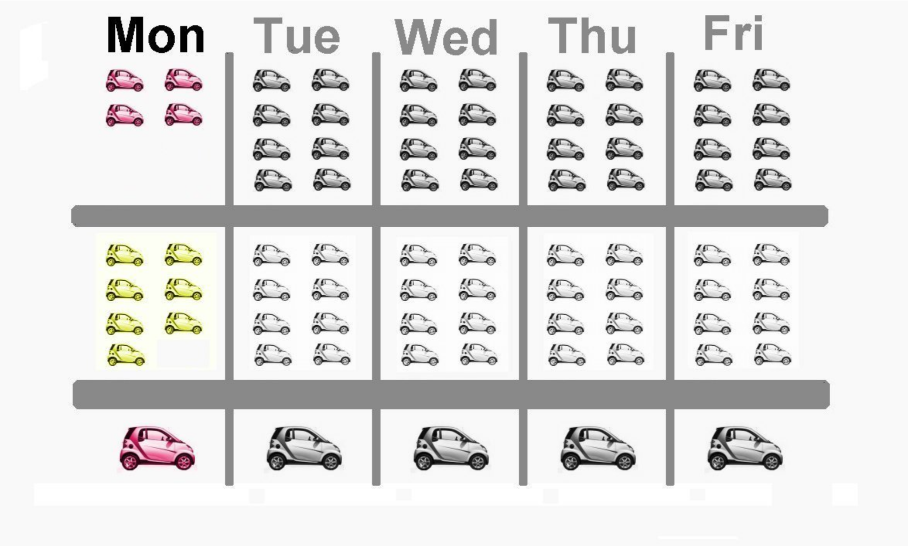
Not only length does matter!



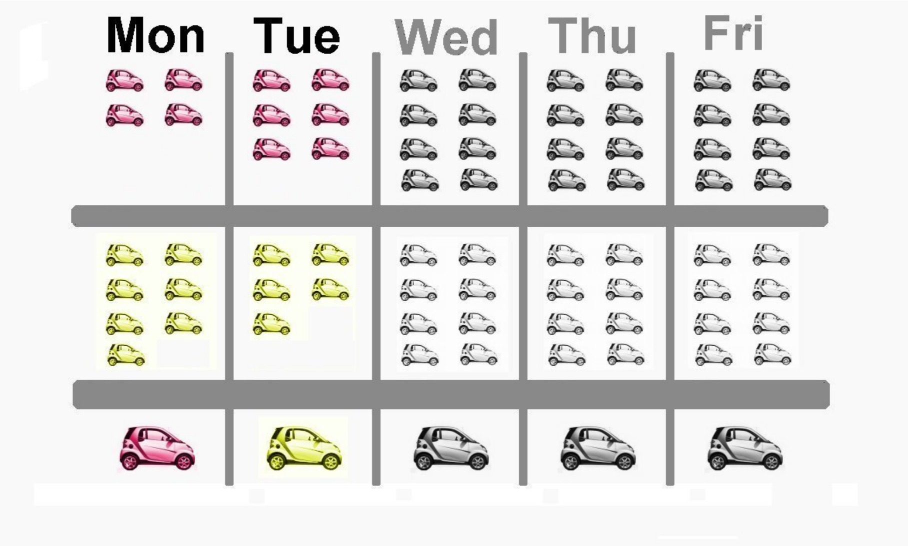
Key idea is to join the minority of drivers.



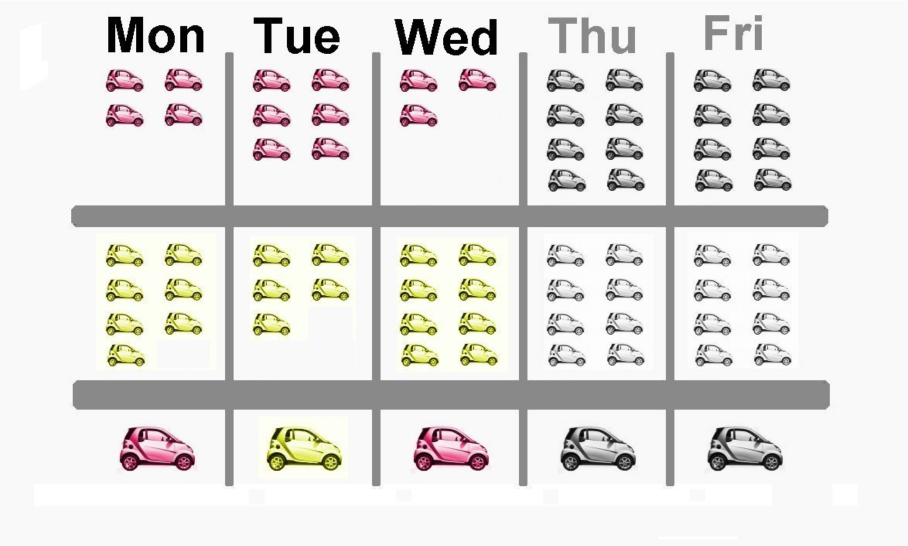
Key idea is to join the minority of drivers.



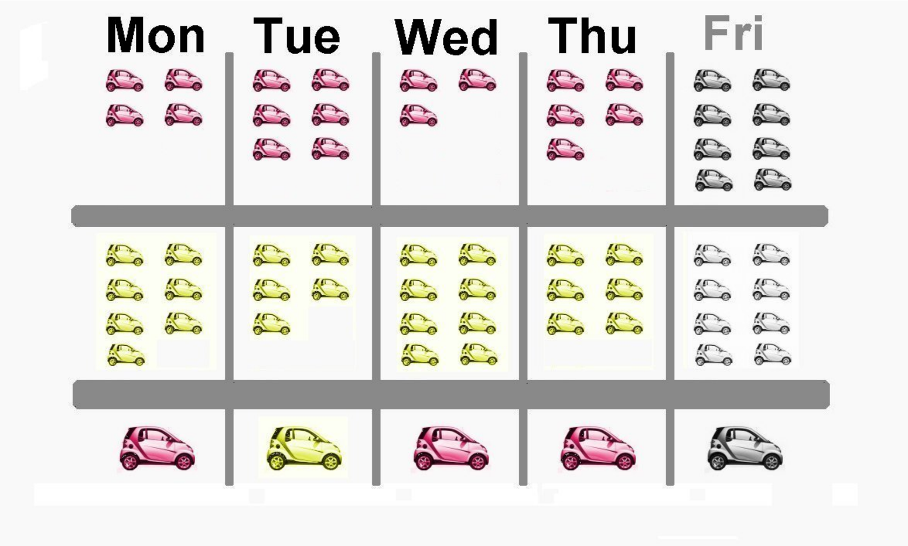
Key idea is to join the minority of drivers.



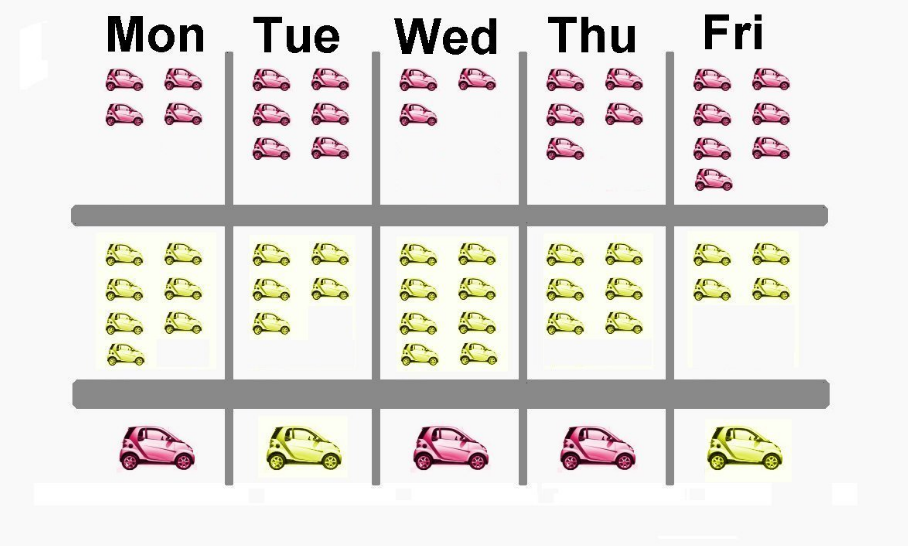
Key idea is to join the minority of drivers.



Key idea is to join the minority of drivers.



Key idea is to join the minority of drivers.



Model of a driver

- each driver has two predictors of the next day's optimal road

Model of a driver

- each driver has two predictors of the next day's optimal road
- predictors are randomly selected at the beginning, but then fixed

Model of a driver

- each driver has two predictors of the next day's optimal road
- predictors are randomly selected at the beginning, but then fixed
- each driver is able to memorize traffic situation of the previous M days

Model of a driver

- each driver has two predictors of the next day's optimal road
- predictors are randomly selected at the beginning, but then fixed
- each driver is able to memorize traffic situation of the previous M days
- all the driver can do is to follow the currently better predictor

Model of a driver

- each driver has two predictors of the next day's optimal road
- predictors are randomly selected at the beginning, but then fixed
- each driver is able to memorize traffic situation of the previous M days
- all the driver can do is to follow the currently better predictor

Example of predictors, $M = 1$

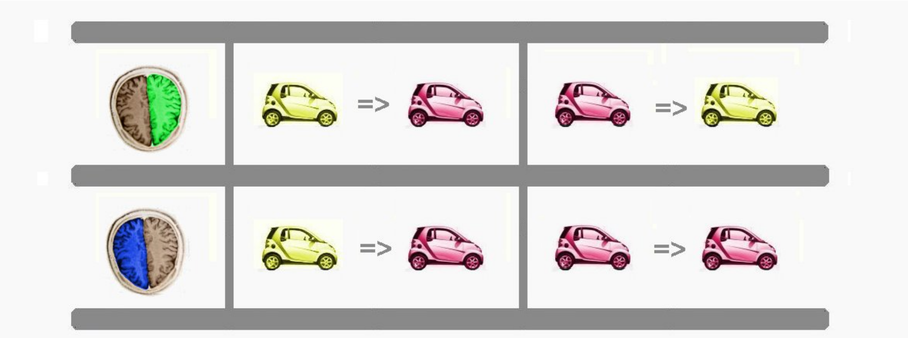


Illustration of driver's adaption, M = 1

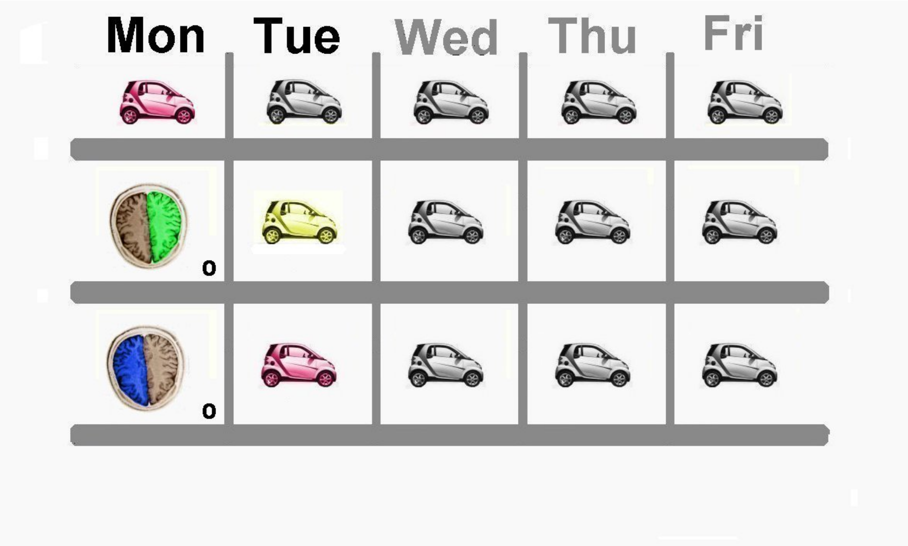


Illustration of driver's adaption, M = 1

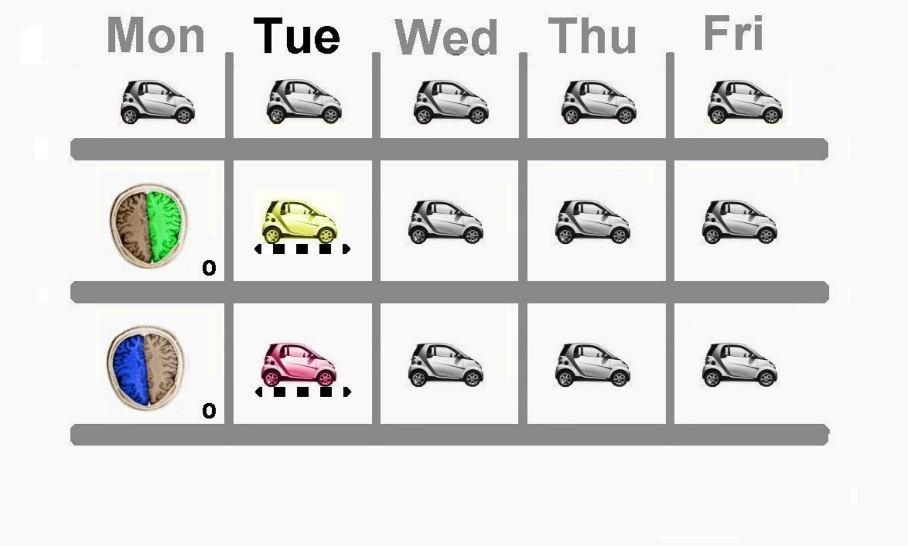


Illustration of driver's adaption, M = 1

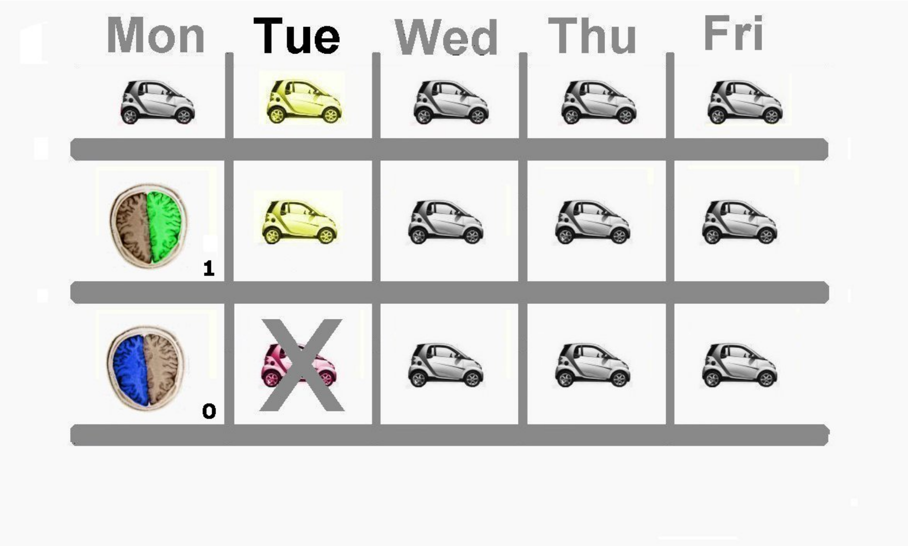


Illustration of driver's adaption, M = 1

Illustration of driver's adaption, M = 1

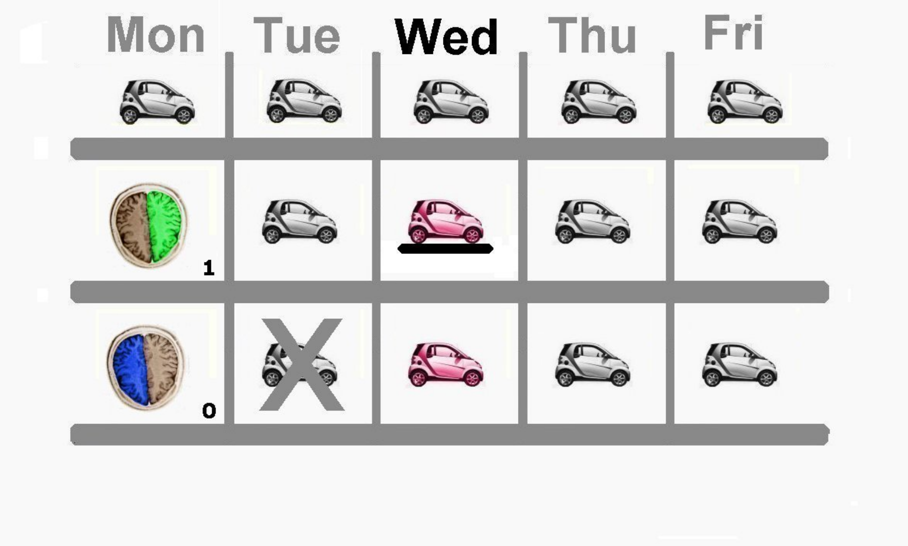


Illustration of driver's adaption, M = 1

Illustration of driver's adaption, M = 1

Illustration of driver's adaption, $M = 1$

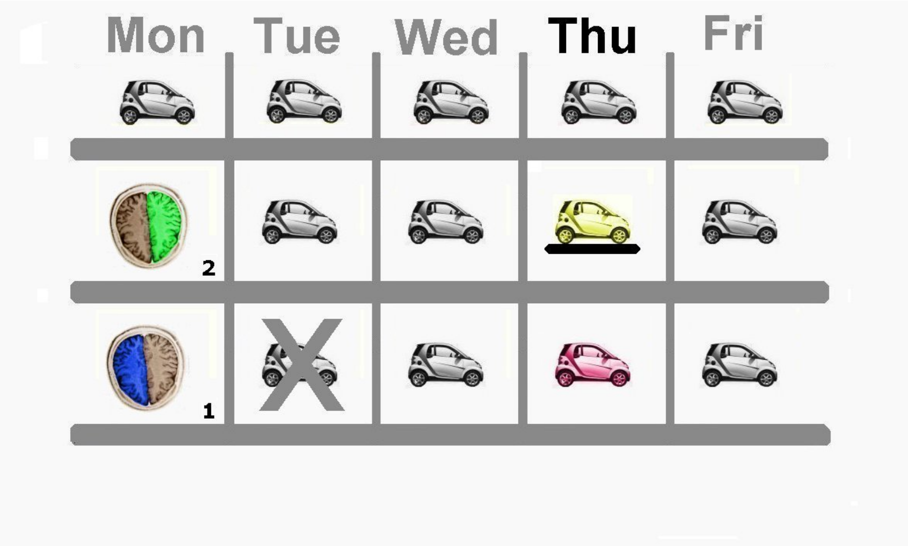


Illustration of driver's adaption, M = 1

Illustration of driver's adaption, M = 1

Illustration of driver's adaption, M = 1

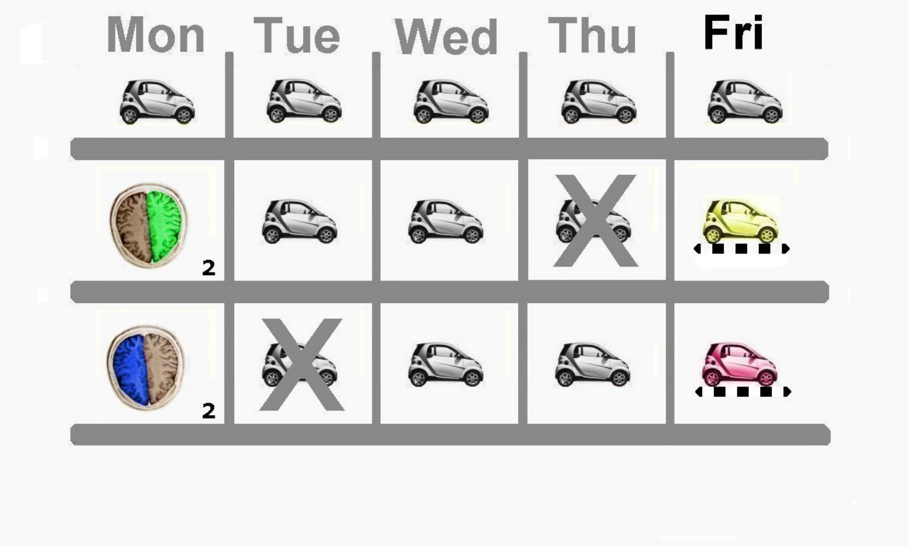
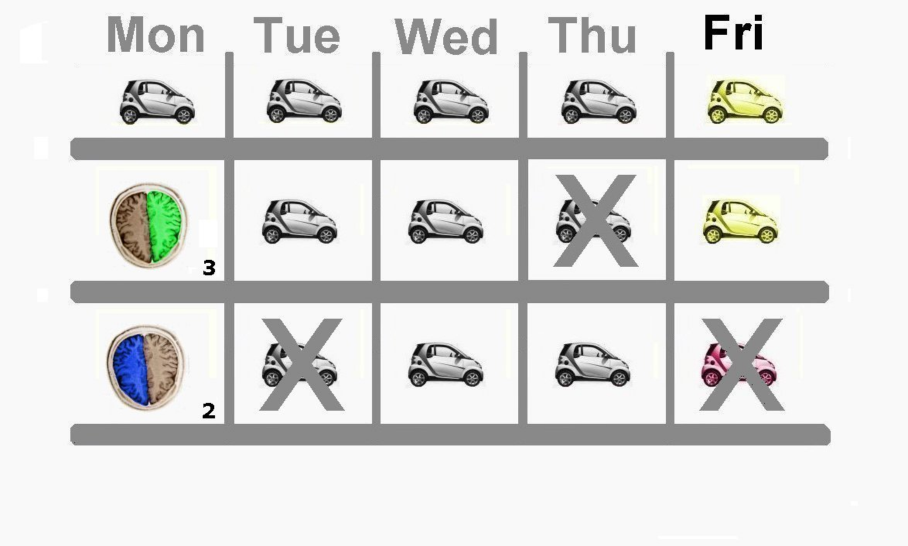


Illustration of driver's adaption, M = 1



Aim of a driver

Driver's only aim is to avoid the majority of vehicles.

Aim of a driver

Driver's only aim is to avoid the majority of vehicles.

- denote the number of drivers by N

Aim of a driver

Driver's only aim is to avoid the majority of vehicles.

- denote the number of drivers by N
- denote possible roads by -1, resp. $+1$

Aim of a driver

Driver's only aim is to avoid the majority of vehicles.

- denote the number of drivers by N
- denote possible roads by -1, resp. $+1$
- denote the current choice of the i-th driver by $a_i(t) \in \{-1, +1\}$

Aim of a driver

Driver's only aim is to avoid the majority of vehicles.

- denote the number of drivers by N
- denote possible roads by -1, resp. $+1$
- denote the current choice of the i-th driver by $a_i(t) \in \{-1, +1\}$

Then the current traffic imbalance reads

$$\Delta(t) \equiv \sum_{i=1}^{N} a_i(t)$$

Aim of a driver

Driver's only aim is to avoid the majority of vehicles.

- denote the number of drivers by N
- denote possible roads by -1, resp. $+1$
- denote the current choice of the i-th driver by $a_i(t) \in \{-1, +1\}$

Then the current traffic imbalance reads

$$\Delta(t) \equiv \sum_{i=1}^{N} a_i(t)$$

and for driver's loss function we get

$$z_i \equiv \frac{1}{T} \sum_{t=1}^{T} a_i(t)\, \Delta(t)$$

Frustration of drivers

Ideal of balanced traffic: $\Delta(t) = 0$

Frustration of drivers

Ideal of balanced traffic: $\Delta(t) = 0$

Average traffic imbalance reads:

$$\sigma^2 \equiv \frac{1}{T} \sum_{t=1}^{T} \Delta(t)^2$$

Frustration of drivers

Ideal of balanced traffic: $\Delta(t) = 0$

Average traffic imbalance reads:

$$\sigma^2 \equiv \frac{1}{T} \sum_{t=1}^{T} \Delta(t)^2$$

The loss of an average driver is:

$$\bar{z} \equiv \frac{1}{N} \sum_{i=1}^{N} z_i$$

Frustration of drivers

Ideal of balanced traffic: $\Delta(t) = 0$

Average traffic imbalance reads:

$$\sigma^2 \equiv \frac{1}{T} \sum_{t=1}^{T} \Delta(t)^2$$

The loss of an average driver is:

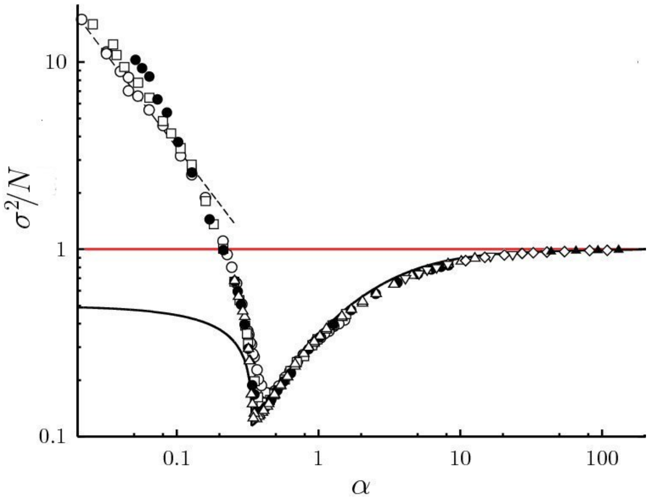
$$\overline{z} \equiv \frac{1}{N} \sum_{i=1}^{N} z_i$$

Principal frustration: $\overline{z} = \frac{\sigma^2}{N} > 0$

The majority of drivers takes a bad decision.

How do these selfish drivers perform in global view?

The average traffic imbalance depends only on $\alpha = \frac{2^M}{N}$



But the real vehicles are heterogenous!

Consider G groups of different vehicles, $g \in \{1, .., G\}$

But the real vehicles are heterogenous!

Consider G groups of different vehicles, $g \in \{1, .., G\}$

- L_g is the length of vehicles in group g

But the real vehicles are heterogenous!

Consider G groups of different vehicles, $g \in \{1, .., G\}$

- L_g is the length of vehicles in group g
- N_g is the number of vehicles in group g

But the real vehicles are heterogenous!

Consider G groups of different vehicles, $g \in \{1, .., G\}$

- L_g is the length of vehicles in group g
- N_g is the number of vehicles in group g
- $\lambda_g \equiv \frac{N_g}{N}$ is the relative ratio of group g

But the real vehicles are heterogenous!

Consider G groups of different vehicles, $g \in \{1, .., G\}$

- L_g is the length of vehicles in group g
- N_g is the number of vehicles in group g
- $\lambda_g \equiv \frac{N_g}{N}$ is the relative ratio of group g

Extended definition of the traffic imbalance reads:

$$\Delta(t) \equiv \frac{\sum_{g=1}^{G} L_g \sum_{i=1}^{N_g} a_{g,i}(t)}{\sum_{g=1}^{G} \lambda_g L_g}$$

But the real vehicles are heterogenous!

Consider G groups of different vehicles, $g \in \{1, .., G\}$

- L_g is the length of vehicles in group g
- N_g is the number of vehicles in group g
- $\lambda_g \equiv \frac{N_g}{N}$ is the relative ratio of group g

Extended definition of the traffic imbalance reads:

$$\Delta(t) \equiv \frac{\sum_{g=1}^{G} L_g \sum_{i=1}^{N_g} a_{g,i}(t)}{\sum_{g=1}^{G} \lambda_g L_g}$$

Now, average traffic imbalance differs from the loss of an average driver!

Examples of heterogenous vehicles

Personal transporter, $L = 0.6m$

Examples of heterogenous vehicles

Scooter, $L = 1.8m$

Examples of heterogenous vehicles

Smart car, $L = 2.5m$

Examples of heterogenous vehicles

Road train, $L = 50m$

Results for various ratios ($\alpha = 0.4$)

0.6m 	1.8m 	2.5m 	50m 	$\overline{Z}$	$\frac{\sigma^2}{N}$
0%	0%	100%	0%	0.13	0.13
0%	20%	80%	0%	0.13	0.14
0%	20%	78%	2%	-0.2	3.16
66%	20%	12%	2%	-0.9	9.55

Results for various ratios ($\alpha = 0.4$)

0.6m 	1.8m 	2.5m 	50m 	$\bar{Z}$	$\frac{\sigma^2}{N}$
0%	0%	100%	0%	0.13	0.13
0%	20%	80%	0%	0.13	0.14
0%	20%	78%	2%	-0.2	3.16
66%	20%	12%	2%	-0.9	9.55

Under some circumstances, the majority of drivers has bypassed traffic jam!

Thank you for being at tension.

Any questions?