Dual control: Benefits and Challenges

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Outline

Toy Problem – Control

Target position

Toy Problem – Control

Target position

- Trivial: put permanent magnet there,
- Smarter: rotating magnetic field,
- Difficult case: Original position is “N”.

Complications:

- the aim is to rotate at given rpm,
- constraint on smoothness of movement.

Toy problem – Estimation

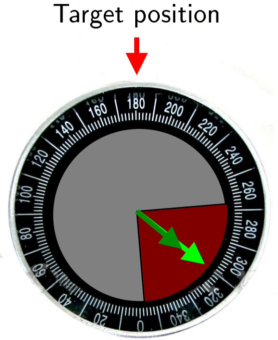
Light bar sensors

Toy problem – Estimation

Light bar sensors

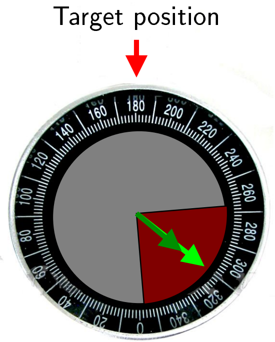
- Trivial: Which segment contains pointer,
  - Direction of the needle?
- Smarter: expected value + variance
- Full density: quite complex

Toy problem – Control & Estimation



- **Certainty equivalent adaptive:**
  estimation provides "best" point estimate.
  Deterministic control strategy design.
  Practice: Common ad-hoc technique.

Toy problem – Control & Estimation



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- **Robust control:** non-adaptive approach. Controller designed off-line to stabilize the system if the it stays within predefined limits.

Toy problem – Control & Estimation

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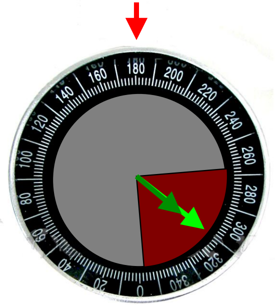

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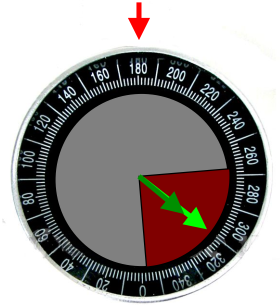
- **Dual control:** controller has "dual" tasks to provide
  - cautious guidance in case of uncertainty,
  - probing to actively learn the system.

  Practice: cautious controller + high frequency noise.

Toy problem – Control & Estimation

Target position



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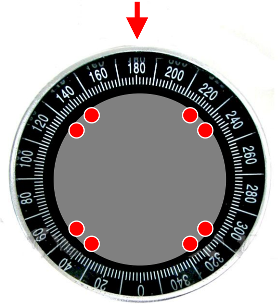
  Practice: cautious controller + high frequency noise.
- **Optimal control:** Theoretical solution to dual control.

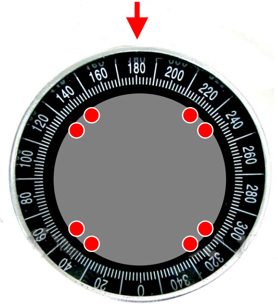
The challenge

Target position



The challenge

Target position

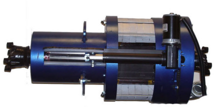


More complications:

- Observations of electrical variables only,
- Unknown load,
- Unreliable actuators (PWM)

Why is it important?

- Rotating engines are essential in today technology,



- Sensors are expensive or too big
- Can we control drives and engines with less sensors?
- Benefits:
  - Economical reasons
  - New designs due to smaller drives
  - More reliable control and fault tolerant control

History

Foundation: Feldbaum [1960, 1961] established the basic concepts.

Early years: e.g. Bar-Shalom and Tse [1974] or Bertsekas [2001]; first simulation examples and real-world applications,

Surveys: Filatov and Unbehauen [2000], Wittenmark [1995], Morozov

Recent development: Bayard and Schumitzky [2008], Simpkins, de Callafon, and Todorov [2008], Mathur and Morozov [2009]

- 13 papers on "dual control" & Bellman in 2008 on http://scholar.google.com
- 94 on "neuro-dynamic programming" & Bellman,
- 309 on "reinforcement learning" & Bellman,
- 898 on "optimal control" & Bellman.

Approximations of Optimal Control

Optimization problem in state-space formulation:

$$u_t = \arg \min_{u \in \mathrm{Supp}(u)} \left\{ \sum_{\tau=t}^{t_{max}} L(x_\tau, u_\tau) \right\}$$

given $p(x_{t+1}|x_t, u_t)$. Solution is given by the Bellman equation:

$$V(I_\tau) = \min_{u \in \mathrm{Supp}(u)} \mathrm{E}\left(L(x_\tau, u_\tau) + V(I_{\tau+1})|I_\tau\right)$$

where I_τ is information state (or sufficient statistics, or hyper-state).

Value function: (Bellman function, cost-to-go) $V(x_t)$ is given implicitly. Analytically intractable.

Uncertainty space: the information state may grow with time. Closely related to estimation procedures.

Approximate Dynemic Programming

Approximations of Value function:

- Neural networks, Bertsekas [2001]
- *Interpolated grid functions*, Thompson and Cluet [2005]
- Gaussian mixtures (kernels).

Approximations of uncertainty space:

- *importance sampling*, (particle filter) Thompson and Cluet [2005]
- expansions around selected trajectory, Simpkins et al. [2008]
- approximate sufficient statistics. (extended Kalman filter) Bar-Shalom and Tse [1974]

Practical applications

Pure theoretical algorithms:
Benchmark model (Åström and Helmersson 1986) revisited by Thompson and Cluet [2005]

$$y_t = y_{t-1} + bu_t + re_t.$$

1 unknown parameter (b), 2-dimensional information state (sufficient statistics). Time horizon 30, 64×64 grid, computed for 2122 min exactly and 1017 min approximately.

Typical practical induction engine 5–7 dimensional state space.
Approximate sufficient statistics from the extended Kalman filter 20–35 dimensional space.
Dual control **is** applied in practice – probing signals.

The state-space of the model has specific structure that is not considered in general-purpose algorithms.

- Can this be considered when choosing approximations?

Permanent Magnet Synchronous Machine

State $x_t = [i_{\alpha t}, i_{\beta t}, \omega_t, \theta_t, L_t, (u_{\alpha t}, u_{\beta t})]$

$$\text{Currents} \quad i_{\alpha t+1} = i_\alpha \left([i_{\alpha t}, i_{\beta t}, \omega_t, \theta_t, \quad , u_{\alpha t}, u_{\beta t}]\right)$$

$$\text{Speed} \quad \omega_{t+1} = \omega \left([i_{\alpha t}, i_{\beta t}, \omega_t, \theta_t, L_t, \quad , \quad]\right)$$

$$\text{Position} \quad \theta_{t+1} = \theta \left([\quad , \quad , \omega_t, \theta_t, \quad , \quad , \quad]\right)$$

$$\text{Voltages} \quad u_{\alpha t+1} = u_\alpha \left([i_{\alpha t}, i_{\beta t}, \omega_t, \quad , \quad , \quad , \quad]\right)$$

Empirical experience:

- key variables are ω_t when rotating and θ_t when in standstill.

Which approach to choose?

Conclusion

- Dual control is a classical concept with interesting recent development,
- It has great practical potential, the need for intelligent control will grow,
- It is rarely used in applications - issues with reliability and software implementation
  - major task is computation of the Bellman function,
  - on-line operation may be affordable,
- Implementation of algorithms in software package BDM: http://mys.utia.cas.cz:1800/trac/bdm

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