Distributed Bayesian Decision Making: Early Experiments

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1 Introduction

Decision-making under uncertainty is a natural part of everyday life of every human being. In societal science, various aspects of decision-making were studied, mostly in the area of psychology. In technical science, the process was formalized using probability theory yielding so called Bayesian theory of decision making [1]. However, one of the key assumptions of this theory is that the decision-maker is the only entity that intentionally influences the system. This assumption is certainly violated in more complicated systems, such as human society or distributed control. Recently, a series of papers attempts to offer an extension of the Bayesian theory for many decision-makers [2], i.e. decentralized stochastic control. Since there are no proofs of optimality of the proposed Bayesian distributed decision making available in the literature, we study this approach via experimental simulation studies. In this paper we present the first experimental results of the approach.

2 Summary of Bayesian decision Making

In this Section, we briefly present the operations required in the approach:

Model Parametrization; Each decision-maker must have its own model of its neighbourhood, i.e. part of the environment. This model is describes of dependence the observed data, y_t, on the decisions, u_t, and uncertainty modelled by unknown quantities Θ_t. In Bayesian paradigm, all models have the form of probability density functions (pdf), i.e. $f\left(y^{1:t}, \Theta^{1:t}, u^{1:t}\right)$ is the most complete model of the system.

Learning; is an operation of probability calculus, which uses the observed data to improve the agents knowledge of uncertain parameters Θ_t. In other words, parameters of the model are inferred. In Bayesian paradigm, this operation provides posterior distribution of parameters $f\left(\Theta_t|d^{1:t}\right)$.

Aim of decision making; we aim to design decision-makers to achieve some desired performance. In Bayesian paradigm, it is possible to specify aims in the form of so-called ideal distribution, ${}^{I}f\left(y^{1:t}, u^{1:t}\right)$. Intuitively, maximum of this pdf is the true ideal of performance, while probabilities assigned to deviations from the maximum indicates how much is such deviation acceptable.

Design of DM strategy; the final aim is to design a rule how to choose actions u_t based on the history of observations $d^{1:t-1}$ and current observation y_t. Typically, this rule is deterministic, however, the technique of fully probabilistic approach yields probabilistic form of the controller, i.e. pdf $f\left(u_t|y_t, d^{1:t-1}\right)$.

Communication; in the current setup, each decision-maker has no explicit model for distant parts of the environment, and intentions of its neighbours. The lack of this information may result in undesired behaviour of the overall system, ant thus it must be compensated by communication between the decision-makers. The communication is done in terms of variables that are common to both decision-makers in the form of probability density functions.

3 Experiments

The first experiment is adaptive control of two-input one-output system by two controllers. The environment is an AutoRegressive model

$$y_t = ay_{t-1} + by_{t-2} + cu_{1,t} + du_{2,t} + e_t,$$

where a, b, c, d are scalar parameters and $e_t \sim \mathcal{N}(0, \sigma)$ is a realization of normally distributed noise with zero-mean and variance σ. The controllers are chosen in the following form

$$\begin{aligned} y_t &= \dot{a}y_{t-1} + \dot{b}y_{t-2} + \dot{c}u_{1,t} + 0u_{2,t} + \dot{e}_t, \\ y_t &= \ddot{a}y_{t-1} + \ddot{b}y_{t-2} + 0u_{1,t} + \ddot{d}u_{2,t} + \ddot{e}_t, \end{aligned}$$

where symbols with $\dot{}$ denote estimated parameters of the first controller, and $\ddot{}$ parameters of the second controller. Naturally, the presence of zeros in the above models indicates that the decision-makers are, by design, unaware of the actions of each other. In the full paper, we show that this lack of knowledge results in poor performance of the resulting controllers, and we will compare several strategies of communication that compensates this lack of knowledge and improve the performance.

References

[1] J.O. Berger. *Statistical Decision Theory and Bayesian Analysis.* Springer-Verlag, New York, 1985.

[2] M. Kárný and T.V. Guy. On dynamic decision-making scenarios with multiple participants. In J. Andrýsek, M. Kárný, and J. Kracík, editors, *Multiple Participant Decision Making*, pages 17–28, Adelaide, May 2004. Advanced Knowledge International.