Improved Estimation Method of Region of Stability for Nonlinear Autonomous Systems

(Abstract)

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The stability and the stability region of (controlled) industrial systems is an important property to be determined. The reason for that is that it is highly desirable to have a system that is globally stable; that is why stability theory is continuously in the focus of both theoretical researchers and industrial partitioner. A substantial part of the different methods described in the literature is based on the classical results of Lefschetz and La Salle using a suitably chosen Lyapunov function, see for example [1], [2], [3], [4] and [5].

The subject of this paper is to use the idea of Vanelli and Vidyasagar [4] to develop a practically useful algorithm to estimate the stability region (or domain of attraction, DOA) of autonomous nonlinear systems by improving the original method. Vanelli and Vidyasagar proved that there exists a sequence of special kind of Lyapunov functions V_m that can be used to estimate the domain of attraction (DOA). Based on this idea, an iterative method has been implemented in a Mathematica-package to find the appropriate approximating functions V_m.

Estimation method of the domain of attraction Consider a nonlinear autonomous system $\dot{x}(t) = f(x(t)$ supposing that the origin is its asymptotically stable equilibrium point. In our context DOA of the origin is defined by the set

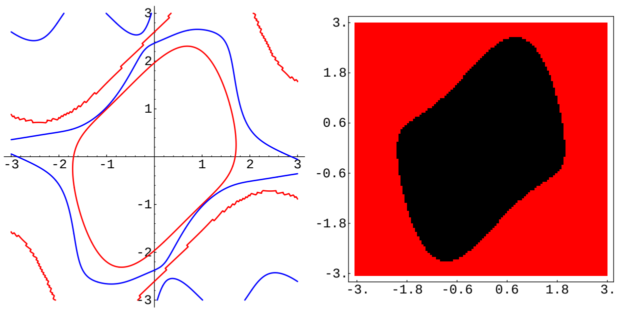
$$S(0) = \{x_0 : x(t, x_0) \to 0, t \to \infty\}$$

where $x(t, x_0)$ denotes the solution of the system corresponding to the initial condition $x(0) = x_0$. Vanelli and Vidyasagar established a result [4] concerning the existence of a so-called maximal Lyapunov function which can be used for approximating the DOA.

An open set S which contains the origin coincides with the DOA of the asymptotically stable steady state $x = 0$, if and only if there exists a continuous function $V : S \to \mathbb{R}_+$ and a positive definite function ψ on S which fulfills the following properties:

- $V(0) = 0$ and $V(x) > 0$, for any $x \in S \setminus \{0\}$
- $D_r V(x_0) = \lim\limits_{t \to 0^+} \frac{V(x(t,x_0)) - V(x_0)}{t} = -\psi(x), \forall x_0 \in S$
- $V(x) \to \infty$ as $x \to \partial S$ and/or $\|x\| \to \infty$.

For the case of $\mathbb{R}$-analytic function f for which the real parts of the eigenvalues of matrix $\frac{\partial f}{\partial x} f(0)$ are negative a second theorem has been established which provides a sequence of Lyapunov functions (V_m), which are not necessarily maximal, however can be used in order to approximate



(a) DOA by the algorithm of Vanelli and Vidyasagar

(b) DOA by direct scanning of the domain

Figure 1: DOA of the Van der Pole system

the DOA. The method is an iterative approach which in each step finds a better estimate of the maximal Lyapunov function in the form of

$$V_m(x) = \frac{\sum_{i=2}^{m} R_i(x)}{1 + \sum_{i=1}^{m} Q_i(x)}, m \in \mathbb{N}$$

where R_i and Q_i are homogenous polynomials of degree i. The construction of V_m is relatively complex involving the solution of a LP problem, but one should not know the solution of the examined system.

Example To show the efficiency of the algorithm the DOA of a Van der Pole system is examined:

$$\dot{x_1} = -x_2 \quad , \quad \dot{x_2} = x_1 - x_2 + {x_1}^2 x_2$$

The result of the algorithm after 8 iterations can be seen in Fig. 1(a) where the DOA is the region bounded by the innermost closed red curve within the blue one. To verify this result, direct scanning has been applied to the examined region $[-3, -3] \times [-3, -3]$. Black points in Fig. 1(b) are those from which the system once has been started then the trajectory converges to the origin. It is seen that the domain found by Vanelli's algorithm is the subset of the full domain and they almost are equal.

The detailed results and further examples will be presented in the final paper.

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