BAYES ESTIMATION OF A QUEUE LENGTH

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Abstract: The paper deals with an application of Bayes for estimation of the queue length in junction arm. In Bayesian view the concept of probability is not interpreted in terms of limits of relative frequencies but more generally as a subjective measure of belief of a rationally and consistently reasoning person which is used to describe quantitatively the uncertain relationship between the statistician and the external world. This model splits controlled networks into microregions. The queue length and the occupancy of each junction approach are the basic state quantities for fully expressed traffic situation at given time instant. The occupancy determines relative time of the detector activation during sample period, i.e. the proportion of time when detector has been occupied and total time of measuring period. The optimization criterion for this attitude is minimization of the queue length. For clearness, the model is derived for simple junction.

Keywords: Bayes estimation; traffic flow; queue length

1. INTRODUCTION

One of the main symbols of advanced society is high transportation. A lot of people permanently commutes to work and for pleasure. A lot of commodities travel from one country to another, frequently through the continents. Increasing demand for transportation is the reason why the amount of vehicles increases very quickly. A lot of those vehicles means higher density of transportation and large amount of junctions and roads are congested. Very frequent cause is older transport network, which has not sufficient capacity for contemporary vast volume of traffic.

The most common approaches to solution of these transport network deficiencies are attempts to rebuild parts of the network in order to increase their capacity, reroute transit traffic by building large bypasses around affected locations or making drivers pay for entering central zones. In many cities it is impossible or ineffective to reconstruct existing street network due to their historical development. This is the reason why advanced traffic control is being applied.

And so different category of possible solutions is to maintain the present transport network and improve the traffic control. But this control can be based on different ideas. Majority of attitudes uses detectors for giving feedback. Those detectors are based on inductive electric coils placed under the road surface. Presence of huge metallic object above the coil changes its magnetic properties and thus individual cars are detected. Each detector signalizes the presence or absence of a car above it. From this signal, basic transportation quantities such occupancy, intensity, density, velocity, number of cars, and others are evaluated or estimated.

For example Michal Kutil focuses [4] on the urban traffic control, based on the model using difference state equations. Model is described by the number of vehicles in the queue and mainly by the mean value of waiting time which describes the queue dynamics. To make the appropriate non-linear model and to identify its parameters he uses real data measured during one day in Prague. The objective of his work is to balance the vehicle waiting time on different streets in one intersection.

The paper will present other way based on Bayes estimation [1, 6]. In Bayesian view the concept of probability is not interpreted in terms of limits of relative frequencies but more generally as a subjective measure of belief of a rationally and consistently reasoning person which is used to describe quantitatively the uncertain relationship between the statistician and the external world [5]. This model splits controlled networks into microregions [3]. The queue length and the occupancy of each junction approach are the basic state quantities for fully expressed traffic situation at given time instant. The occupancy determines relative time of the detector activation during sample period, i.e. the proportion of time when detector has been occupied and total time of measuring period. The optimization criterion for this attitude is minimization of the queue length. For clearness, the model is derived for simple junction.

2. NOTATIONS

2.1 Traffic data

Controlled networks are split into microregions. They are logically self-contained transportation areas of several crossroads with their adjoining roads. Their modeling and feedback control exploit data measured by detectors based on inductive electric coils placed under the road surface. Presence of a huge metallic object above the coil changes its magnetic properties and thus individual cars are detected. Each detector signalizes presence or absence of a car above it. From this signal, basic transportation quantities are evaluated [3,7]:

Occupancy: determines the relative time of the detector activation during the sample period, i.e. the proportion of time when the detector has been occupied and the total time of measuring period. The occupancy unit is [%]. This quantity has similar meaning as the density. The higher density decreases the vehicles velocity and intensities and queues are formed on the arms. That is why the value of occupancy of the detectors under the queue increases. Conversely, if the vehicles can go through faster in case of low traffic and they loose the minimum time by queueing, the occupancy decreases.

Intensity: denotes the number of vehicles which have passed a detector during the sample period. Usually, the value of this quantity is transformed into an hourly intensity of unit vehicles, i.e [uv/h]. This quantity captures the queue dynamics in a sense of the queue protraction but it does not fully determine the actual situation. The value of intensity can be low because of low traffic or high density which are two converse traffic situations. Intensity is very important information for considered models from a counting point of view.

Density: denotes the number of vehicles on some road segment and its meaning comes near to the occupancy one, on condition of low velocities particularly. Its unit is [uv/km].

Velocity: can be point or segmental. It determines the average speed of vehicles passing over a detector or a certain stage. Its unit is [km/h].

It is necessary to measure density and velocity to determine the actual traffic situation. The standard outputs of the measurement are values of the intensity and occupancy. Experience shows that detectors are fault-prone and their reparation is far from being easy. Therefore the filtration of their data is necessary. Here, outlier filtration and normalization to zero mean and unit standard deviation were applied on the raw data.

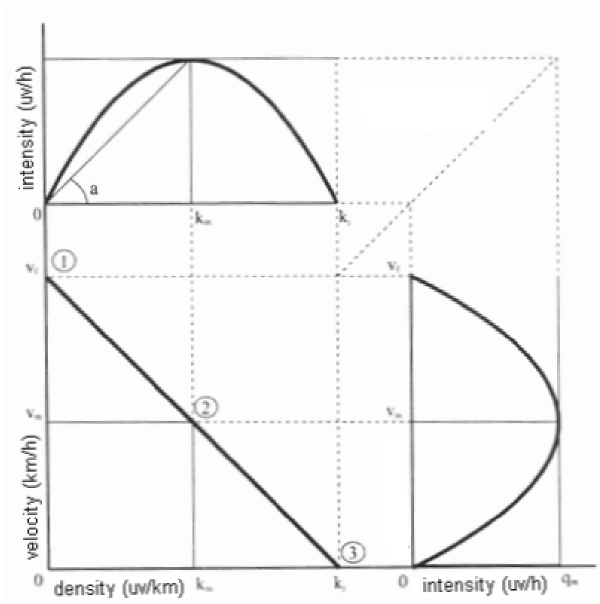

Fig. 1: Relations between basic traffic quantities

It is necessary to take into account intensity and density as, for example, from zero intensity it is not clear whether the road is empty or it is completely stuffed with standing cars (Fig. 1). Nevertheless, they are mutually dependent. It is well known that the relationship of intensity in dependence on density, has concave parabolic shape. It reflects the basic types of traffic states:

- *growth state:* The road is free, the speed of cars is determined by the upper allowed speed limit in cities. With increasing density, while the speed being still constant, the intensity grows linearly, up to a certain limit. Then the distances between cars become too small and safety reasons force the drivers to decrease speed.
- *stagnant state:* The further growth of density results in a continued decrease of the speed. The intensity is almost constant and the top of the reversed parabolic shape is reached.

- *decay state:* The drivers are afraid to go on and they slow down or stop when the distances between cars fall below a certain critical level. The cars stack up and become a jumping tail-back. This stage represents a collapse.

The parabolic shape (Fig.1) of the function is common but its precise course varies in dependence of the road monitored and other factors like the kind of a weekday, season etc. In any case, the measured data pairs $[d_{1,t}, d_{2,t}] = [density, intensity]_t$ represent points in an $intensity$ - $density$ plane. They cluster there and form regions of increased data density. Each detected region reflects a specific state of the traffic area. A description of these regions and their intermediate stages is a useful model of the investigated system.

2.2 Queue length

The queue represents amount of vehicles at the end of red time. The main problem is the fact that the queue length is not measurable in practice. The value of the occupancy is higher for the longer queue because the vehicles pass the detector more slowly near to the end of this queue. For the equation describing the queue length time course, the following equation is used:

$$\xi_{t+1} = \delta_t \xi_t - [\delta_t S + (1 - \delta_t) I_t] z_t + I_t, \tag{1}$$

where

ξ_t	is queue length,
t	time instant,
δ_t	indicates the possibility of queue creation, alternatively the type of the passage,
S	saturation flow,
I_t	input intensity,
z_t	green time.

Basic output equation is:

$$\eta_{t+1} = -\xi_{t+1} + \xi_t + I_t, \tag{2}$$

where:

η_{t+1} is passage (intensity on stop-line).

The linear relation between the occupancy and the queue length is:

$$O_{t+1} = \kappa \xi_t + \beta O_t + \lambda, \tag{3}$$

where:

O_{t+1}	is occupancy at time $t + 1$,
κ, β, λ	parameters (constants).

Constants κ, β and λ can be determined experimentally for each set of the subsequent detectors because distances between the detectors placed on one approach specify the minimum and maximum queue lengths and corresponding limit average values of the occupancy can be measured. Assuming the linear relation, then the constants can be specified. In this paper is supposed that the parameters of occupancy dependance are known and constant in time or they can be changed continuously (known, time-dependent) or estimated (unknown, variable). For measurements mentioned above is used a pair of remote and strategic detectors. The detectors on stop-lines are not suitable (Fig.2).

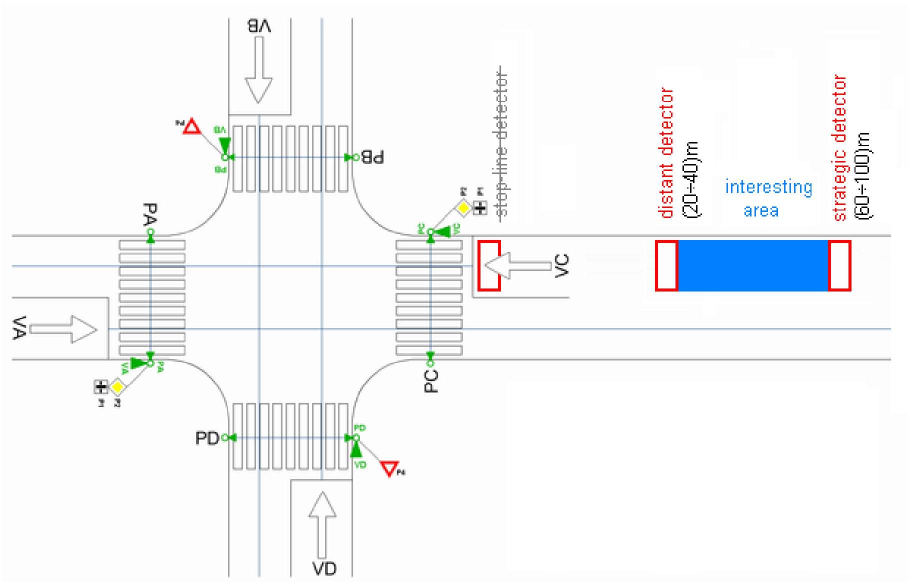

Fig. 2: Detectors

3. STATE SPACE MODEL OF JUNCTION WITH 4 ARMS

This situation is displayed in Figure 2. State of the model is composed of queue length, intensity of traffic flow and occupancy.

The state vector is:

$$x_t = [\xi_1, \xi_2, \xi_3, \xi_4, O_1, O_2, O_3, O_4]' \tag{4}$$

and output vector is:

$$y_t = [\eta_1, \eta_2, \eta_3, \eta_4, O_1, O_2, O_3, O_4]'. \tag{5}$$

The state model can be written as:

$$x_{t+1} = Ax_t + Bz_t + F, \tag{6}$$

$$y_{t+1} = Cx_{t+1} + G, \tag{7}$$

where:

A_t	is matrix of parameters,
B_t	matrix reflects the passage dependence of splits,
F_t	vector adds the typical courses of intensities,
C_t	matrix reflects directional relations of the junction,
G_t	vector adds the typical courses of intensities and densities.

Model matrixes are:

$$A = \begin{bmatrix} \delta_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \delta_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \delta_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \delta_4 & 0 & 0 & 0 & 0 \\ \kappa_1 & 0 & 0 & 0 & \beta_1 & 0 & 0 & 0 \\ 0 & \kappa_2 & 0 & 0 & 0 & \beta_2 & 0 & 0 \\ 0 & 0 & \kappa_3 & 0 & 0 & 0 & \beta_3 & 0 \\ 0 & 0 & 0 & \kappa_4 & 0 & 0 & 0 & \beta_4 \end{bmatrix}, B = \begin{bmatrix} -b_1 & 0 \\ 0 & -b_2 \\ -b_3 & 0 \\ 0 & -b_4 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, F = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix}, \quad (8)$$

where:

$$b_i = \delta_i S_i + (1 - \delta_i) I_i, \quad (9)$$

for i = 1, 2, 3, 4.

$$C = \begin{bmatrix} \alpha' C_\eta \\ C_O \end{bmatrix}, G = \begin{bmatrix} \alpha' G_\eta \\ G_O \end{bmatrix}, \quad (10)$$

where:

$\alpha_i j$ is ration of cars which go from arm i to arm j,

$$C_\eta = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}, G_\eta = \begin{bmatrix} \hat{\xi}_1 + I_1 \\ \hat{\xi}_2 + I_2 \\ \hat{\xi}_3 + I_3 \\ \hat{\xi}_4 + I_4 \end{bmatrix}, \quad (11)$$

$$C_O = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}, G_O = [\emptyset]. \quad (12)$$

Unknown parameters $(\beta, \kappa, \lambda, S, \alpha)$ and quantities (I) must be added to vector of estimated parameters θ.

4. ESTIMATION OF PARAMETERS

Linear Kalman filter

$$x_{t+1} = Ax_t + Bz_t + v_t, \tag{13}$$

$$y_{t+1} = Cx_{t+1} + e_t, \tag{14}$$

where

v_t	is the state noise,
e_t	is the output noise,
$cov(v_t) = R_v, cov(e_t) = R_e.$	

A priori estimation of state and covariant matrix are:

$$\hat{x}_{1|0} = E[x_1|D_0, P_{1|0} = E[(x_1 - \hat{x}_{1|0})(x_1 - \hat{x}_{1|0})'|D_0]. \tag{15}$$

Prediction of output - This step is using before data update:

$$\hat{y}_t = C\hat{x}_{t|t-1}, \tag{16}$$

$$Q_t = R_e + CP_{t|t-1}C'. \tag{17}$$

Data update - Estimation of state is repaired by new measured data y_t:

$$\hat{x}_{t|t} = \hat{x}_{t|t-1} + K_t(y_t - \hat{y}_t), \tag{18}$$

$$P_t = P_{t|t-1} - P_{t|t-1}CQ_tC'P_{t|t-1}, \tag{19}$$

where:

$K_t = Pt|tC'{Q_t}^{-1}$ currently measured data.

Prediction of state

$$\hat{x}_{t+1|t} = A\hat{x}_{t|t} + Bu_t, \tag{20}$$

$$P_{t+1|t} = AP_{t|t}A' + R_v. \tag{21}$$

5. CONCLUSIONS

In the Institute of Information Theory and Automation of the Academy of Science of the Czech Republic an algorithm for Bayes estimation of a queue length is developed by group which is lead by Dr. Nagy and Dr. Kárný. The algorithm is tested on real traffic data samples and the overall model estimation is done. The paper will present basic model for Bayes estimation.

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