TRAFFIC FLOW CONTROL – OPTIMIZATION ON HORIZON

Jitka Homolová

Department of Adaptive Systems
Institute of Information Theory and Automation
Academy of Sciences of the Czech Republic
Prague, CZECH REPUBLIC

E-mail: jkratoch@utia.cas.cz

Abstract: The paper introduces the concept of a linear programming approach to traffic flows control on a finite time horizon in an urban traffic region. The control problem is to determine relative lengths of traffic lights signals, so called green splits, due to actual traffic conditions to improve the traffic situation in the traffic area. A control criterion of this optimization task is to minimize a weighted sum of queue lengths that are formed on intersections arms due to traffic lights control.
The task in a static form can fail due to a traffic flow characteristic course during a day. Therefore the time horizon approach is introduced as a one-shot optimization on finite horizon that uses point estimates of intensities predicted by a dynamic regression model and predicted queue lengths. In the end, few experiments on real data are described. The results proved that the sequential modeling helps to predict better control actions.

Keywords: Linear Programming, Traffic Flow Control, Control on a Horizon

1. INTRODUCTION

The basic principle of traffic flow control is traditionally minimization of the lost times, the passage times and the number of stopping during a journey, which are proportional to queue lengths [Diakaki (2002)]. Such criteria are usually quadratic and therefore LQ control method is often used. Another approach is to suggest the mentioned problem in a linear way [Homolová, Nagy (2005)]. The proposed local model counts and estimates the queue length using maximum available traffic information. This task is trivial in the case of complete knowledge of all measured traffic quantities for all junction arms. Then, the model simply counts the queue length from known input and output intensities. However, the net of all needed detectors is not usually complete and some significant traffic flows (parking cars, etc.) are not measurable in practice. In this case, the model estimates the queue length relative to modeled and estimated traffic characteristics. The control problem sets itself a task to determine lengths of traffic lights signals, green splits in point of fact, due to actual traffic conditions to improve the traffic situation in the urban traffic area. This problem has been solved as a classical sequential linear programming task using the mentioned linear model of traffic microregions. The model computes lengths of queues that can be created on intersections' arms due to traffic lights. A control criterion of this optimization task is to minimize a weighted sum of queue lengths. All restrictions of states and control variables come from relations between them, from the model and a traffic nature of the task [Homolová (2005)].

It is quite evident that a static task of the optimization can easily fail. For this reason, the time horizon approach to linear programming is introduced as one-shot optimization on finite horizon. In this paper, the model for estimating queue lengths on a given horizon is explained. This model gives restrictions for the linear programming task. The model counts queue lengths from the future values of traffic flow intensities that need to be predicted by a regression model. The optimized lengths of green signals are derived for the whole horizon but only the first time step is realized in the way of an incremental change of the actual values. Then next prediction of intensities is performed and model restrictions are applied before the next one-shot optimization on the horizon.

2. TRAFFIC AREA MODEL

The presented optimization task is based on the linear model of a traffic microregion that is a part of the whole traffic region consisting from several intersections with traffic lights. It is also assumed that the microregion is delimited by strategic detectors which are placed far from a stop line enough not to be influenced by queue phenomena. The model consists of two basic equations for each arm in a microregion where vehicle queuing is possible. The first one is important in view of the control on horizon. It is a hydrodynamic analogy for a queue length and traffic inflows and passages:

$$\xi_{t+1} = \delta_t \xi_t + I_t - \left[(1 - \delta_t) I_t + \delta_t S \right] z_t \qquad (1)$$

where

ξ_{t+1}	is a queue length at time $t + 1$ [veh/T_C];
δ_t	is a queue indicator (0 in case of no vehicle to be served in the end of green time, 1 otherwise)
It	is an input intensity [veh/T_C];
S	is a saturation flow [veh/T_C];
z_t	is a relative green (the proportion of the green signal to the cycle time)
T_C	cycle time [h].

As it can be seen, no queue length can be counted without prior information. However, it is easy to assume that there are zero queues at night. Queue indicators are determined by the comparison of an actual passage demand with maximum capacity of a given intersection arm or lane.

3. MODEL OF INTENSITIES

The aim of the control on a horizon is not to optimize the greens only for actual queue lengths but also for queues on a given horizon. As it can be seen from the model equation (1), it is necessary to predict future values of the input intensities on arms. For this reason, the suitable auto-regression model of input intensities has to be established.

Several regression models have been tested with respect to a model order (from 2 to 10) and the length of the prediction horizon (2 and 3) on real data. The simplicity of such model and the accuracy of predicted values of intensities for each day of a weak is mainly stressed. These requirements led us to choose the regression model of order 2:

$$I_{t+1} = p_1 I_t + p_2 I_{t-1} \, . \qquad (2)$$

This equation models a course of intensity during a given day for a given intersection arm. Prior information of intensities is given by assumption of no traffic during nights (as it has been noticed it is possible find such time instant in the real traffic when there is no traffic or intensity is almost zero). The intensities can vary not only in view of days but also in places or seasons; thus the continuous parameter estimation is needed. The regression model (2) allows predicting intensities with maximum divergence of 5 vehicles for each day on both horizons.

4. OPTIMIZATION TASK

The classical optimization method of the linear programming minimizes a linear criterion in finite number of steps. The variables are restricted by a system of constraints in the form of inequalities and equalities. The criterion for our optimization task is supposed to be the weighted sum of the queue lengths over the whole microregion and given horizon:

$$J_t = \sum_{\tau=t+1}^{t+h+1} \omega_\tau \left(\sum_{i=1}^{n} w_{i;\tau} \hat{\xi}_{i;\tau} \right). \qquad (7)$$

where

- J_t is a value of the criterion at time t;
- h is a horizon of the intensity prediction (*h=1,2 or 3*);
- ω_τ is a horizon weight for a given step;
- n is a number of arms in the microregion;
- $w_{i;\tau}$ is a weight for the queue on arm i at time τ;
- $\hat{\xi}_{i;\tau}$ is a point estimate of the queue length on the arm i at time τ.

The weights wi;τ can be chosen experimentally according to the very good knowledge of the traffic situation in a microregion which is, unfortunately, sometimes impossible or quite difficult. Moreover, the fixed weights can even make the situation worse in specific cases like unexpected or temporary increased traffic volume. Due to these facts, it is necessary to minimize not only the queue lengths but also the differences between them. The weights are supposed to be changed continuously according to the point estimates of the queue lengths in the traffic microregion at the time t+1. It represents just the proportion of one queue to the total sum of the queues.

The weights are constant on a given horizon. These queue weights are multiplied again by horizon weights ω_τ which ensure an optimization preference on the whole horizon. The three basic types of the preference are studied in our task: (i) no preference, (ii) the preference of time-nearest queues and (iii) the preference of time-furthest queues to be minimized. In the first case, the horizon weights are set uniformly on the horizon; in other cases, they are increasing or decreasing with horizon time.

The constraints for the solved optimization task come from the traffic nature of the task. They are given by the traffic model, by non-negativity, minimal and maximal values of all variables. The resultant values of the greens are used for the corresponding incremental change of actual greens. This approach prevents from an over-oscillation of the greens and it follows from a psychological influence on the drivers. The control increment for the green signal is 1 second. The control criterion is minimizing the weighted sum of the queues.

5. EXPERIMENTS

The tested traffic microregion consists of two four-arm intersections with the traffic lights. Each arm has one input and one output lane to all directions. Measuring devices are placed on all entries and exits of the microregion that is on six arms. The detectors are missing on the arms interconnecting these two intersections. The parameters of the microregion are known and constant during the day. The signal schemes have two phases for both intersections; one phase gives the right of way to the horizontal lanes and the second phase to the vertical lanes. The cycle time is fixed and the same for both intersections. Moreover, it is also constant during the experiment. The real measurements of the intensities are used as the input.

The horizon of the optimization is chosen from 1 to 3 time steps and the predictions of the future intensities are counted by the regression model of the second order (2) with on-line parameters estimation. Each study case is combined with different options of the weights. The queue weights w_i can be uniform or preset according to (9). The horizon weights ω_τ have three possibilities for (i) no preference, (ii) the preference of time-nearest queues and (iii) the preference of time-furthest queues to be minimized. The results of the experiment are expressed by the average queues on the arms and their total average and variance. Their values are written in the following tables 1, 2 and 3. The arms of the first intersection are numbered from 1 to 4 and for the second intersection, from 5 to 8. The arms are numbered anticlockwise from the left.

The first experiment deals with the case when the greens are fixed during the whole time. It can be seen that there are very big queues on arms 1, 3, 5 and 7 and on the remaining arms, the queues are of the half length. The control with static linear programming optimization with no queue preference worsens the situation without any control – the queues are longer and the differences between then are bigger. The presetting of the weights helps to balance queues (the variance is almost two times lower) but the total average queue did not decrease.

Horizon 1											
		average queue length on arms								total average	variance
weights		arms of left intersection				arms of right intersection					
w	ω	1	2	3	4	5	6	7	8		
no control	no control	24	10	23	11	23	12	22	9	16	6.341
uniform	[1]	11	33	19	31	22	33	14	18	23	8.124
preset	[1]	15	23	19	21	14	18	16	10	17	3.876

Tab. 1: Control on horizon of the length 1.

The next two tables show that the control with horizon optimization and uniform queue weights runs into the same total average queue like for the static task with optimal weights but the differences between the queues are lower then in case of no control but bigger then in case of the static task with preset weights. The presetting of the weights again helps to balance queues (the variance is more then two times lower) and moreover, the total average queue decreased. The strong differences can not be noticed for the time preference realized by the horizon weights in both cases of longer optimization horizon. In spite of it, it can be said that the best results are received for the preference of time-furthest queues to be minimized.

Horizon 2											
weights		average queue length on arms								total average	variance
		arms of left intersection				arms of right intersection					
w	ω	1	2	3	4	5	6	7	8		
uniform	[1, 1]	11	22	11	21	23	18	21	10	17	5.069
uniform	[10, 1]	11	23	10	23	23	19	20	11	18	5.440
uniform	[1, 10]	12	21	11	20	21	16	20	10	16	4.576
preset	[1, 1]	16	14	14	17	14	19	16	10	15	2.353
preset	[10, 1]	16	15	13	16	14	18	16	10	15	2.375
preset	[1, 10]	15	14	15	16	14	17	15	10	15	1.859

Tab. 2: Control on horizon of the length 2.

Horizon 3											
weights		average queue length on arms								total average	variance
		arms of left intersection				arms of right intersection					
w	ω	1	2	3	4	5	6	7	8		
uniform	[1, 1, 1]	11	24	12	23	20	17	19	10	17	5.211
uniform	[10, 5, 1]	11	25	11	23	22	18	21	10	18	5.700
uniform	[1, 5, 10]	11	24	13	22	19	16	18	10	17	4.841
preset	[1, 1, 1]	17	16	17	17	14	17	16	10	15	2.212
preset	[10, 5, 1]	17	16	17	18	14	17	16	10	16	2.306
preset	[1, 5, 10]	16	15	15	16	15	16	17	10	15	2.014

Tab. 3: Control on horizon of the length 3.

The horizon prolongation from two to three steps also does not bring any noticeable effect. The results for the horizon of the length 2 are little bit better than longer one by one step. This phenomenon is probably caused by the less accurate predictions of the input intensities.

6. CONCLUSIONS

The paper introduces a finite time horizon concept of a linear programming approach to traffic flows control in an urban traffic region. The point estimates of the input intensities are predicted by a dynamic linear regression model of the second order. The control problem is to determine lengths of green signals and the criterion is minimizing a weighted sum of queue lengths. All restrictions of variables come from the linear model of the traffic microregion and a traffic nature of the task. The aim of the control is not only minimizing but also balancing the queues on the individual intersection arms in the microregion. That is why the weights are very important and they can be set experimentally or preset according to the actual traffic situation.

The results positively prove the big positive effect of the dynamic setting of the weights for the requirement of the balancing queues in the microregion. The experiments also show that a static task can really fail and moreover, it can worsen the situation. The profit from the finite time horizon concept of a linear programming approach is also noticeable.

ACKNOWLEDGEMENTS

This research has been partially supported by Ministry of Transportation, Czech Republic, National programme of research 2004 - 2009, Project num. 1F43A/003/120.

REFERENCES

Homolová J., Nagy I.: Traffic model of a microregion. In: *Preprints of the 16th World Congress of the International Federation of Automatic Control*. (Horáček P., Šimandl M., Zítek P. eds.). IFAC, Prague 2005, pp. 1-6

Homolová J.: Hierarchical regulator of traffic flows. In: *Proceedings of the 6th International PhD Workshop on Systems and Control, Young generation Viewpoint.* (Tinta D., Benko U. eds.). Jozef Stefan Institute, Ljubljana 2005, pp. 1-6

Homolová J.: Traffic flow control. In: *Proceedings of the 15th International Conference on Process Control 2005*. Slovak University of Technology, Bratislava 2005, pp. 1-5