Study on disturbance-rejection magnitude optimum method decay ratios

Satja Lumbar[1], Damir Vrančić[1]

[1]Jožef Stefan Institute, Ljubljana Slovenia

E-mail: satja.lumbar@ijs.si

Abstract: One of the important parameters for tuning the response of a system in closed loop is the decay ratio. In this paper three tuning methods are compared: the DRMO tuning method [15,16], the KT tuning method [1] and the non-convex tuning method [3,17]. The latter method is based on setting the value of maximum sensitivity function M_s. The results have shown, that the DRMO method sets a closed-loop response such, that the decay ratio is within a relatively tight interval with regard to other two methods, despite the fact that the DRMO method isn't based on optimization procedure (as is the case with non-convex method).

Keywords: disturbance rejection, decay ratio, controller tuning, PI controller

1. Introduction

PID controllers are the most widely used controllers in the process industry. It has been acknowledged that more than 95% of the control loops used in the process control is of the PID type, of which most are the PI type [1].

Today, the most often applied tuning rules for PID controllers are those based either on the measurement of process step response or on the detection of a particular point on the Nyquist curve of the process (usually one related to the ultimate magnitude and frequency of the process by using relay excitation).

Apart from standard tuning rules, such as Ziegler-Nichols, Cohen-Coon, Chien-Hrones-Reswick, or refined Ziegler-Nichols rules, more sophisticated tuning approaches have been suggested. They are usually based on more demanding process identification algorithms or tuning procedures, like non-convex optimization, gain and phase specification, IMC controller design, or identification of multiple points in frequency domain [3,4,5,6,7,8,17].

One of the frequent demands in the time domain when dealing with closed-loop regulation is, amongst other requirements, the closed-loop response decay ratio. In order to satisfy the prescribed decay ratio the closed-loop response has to be optimized in time domain which is a relatively demanding procedure. Some popular methods [3,17] tend to make use of an indirect approach that should guarantee acceptable decay ratios by optimizing the minimal distance of the open-loop transfer function to the critical point in the polar plot. The mentioned approaches result in stable closed-loop responses for different process models. However, the values of the decay ratios, despite being within acceptable range, may vary considerably.

On the other hand, while using the disturbance rejection magnitude optimum (DRMO) tuning method [9,10,15,16,23], it came to our attention that the decay ratios have quite similar values for a diversity of process models. Furthermore, the DRMO method is relatively simple to apply since, in its basic form, it does not require any form of optimization (i.e. retuning). In this paper the DRMO method will be tested on some process models often encountered in process and chemical industry, decay ratios will be analyzed and compared with two other modern tuning methods based on frequency-domain optimization.

This paper is set out as follows. Section 2 provides some basic definitions. A study on uniformity of the decay ratios of the DRMO method is given in Section 3. Section 4 provides a comparison of the decay ratios obtained with the DRMO method and two other modern tuning methods [1,3,17]. Lastly, the conclusions are provided in Section 5.

2. Basic definitions

2.1 Decay Ratio

In the time domain a noteworthy characteristic of a closed-loop response on a step input disturbance is its decay ratio, which is defined by the following relation:

$$dr = \frac{B}{A}, \tag{1}$$

where B is the difference between the second peak and the second valley of the closed-loop response and A is the difference between the first peak and the first valley, as depicted in Fig. 1 [22].

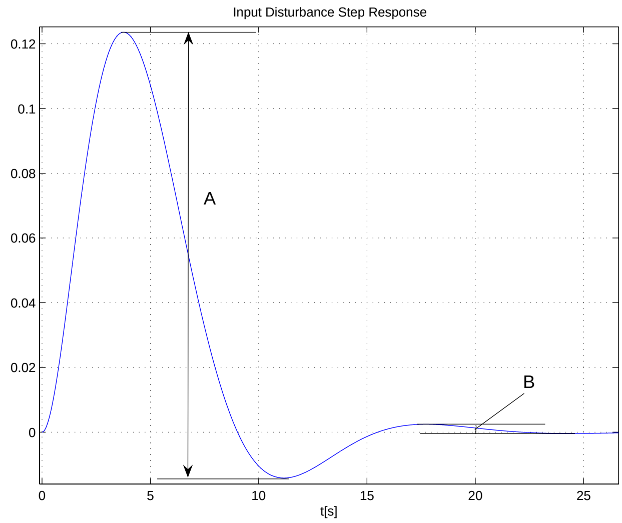

Fig. 1. Definition of decay ratio

2.2 Maximum Sensitivity

Maximum sensitivity M_s is defined as an inverse of the minimum distance of the open-loop transfer function to the critical point (-1+0*j) in the polar plot:

$$M_s = \max_{\omega} \left| \frac{1}{1+G_p(i\omega)G_c(i\omega)} \right|, \qquad (2)$$

where $G_p(i\omega)$ and $G_c(i\omega)$ are the process transfer function and the controller transfer function, respectively.

2.3 Magnitude optimum method

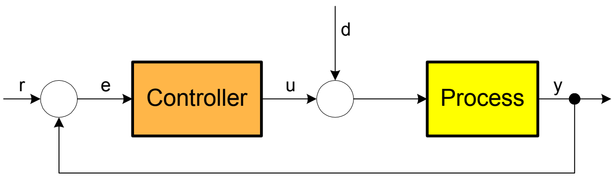

Fig. 2. Typical closed-loop configuration with the 1DOF controller.

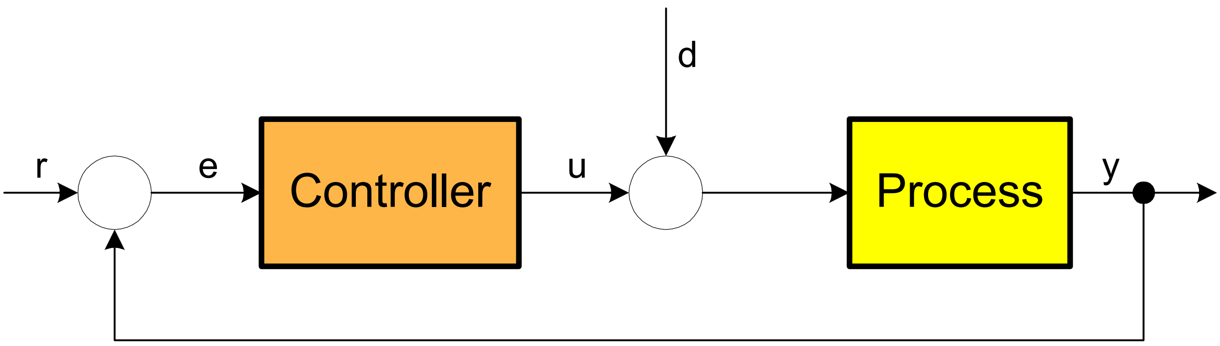

Fig. 14 Fig. 2 shows the process in a closed-loop configuration with the controller, where signals *r*, *u*, *d*, *y* and *e* represent a reference, controller output, input disturbance, process output and control error,

respectively. One possible controller design objective is to maintain the closed-loop magnitude (amplitude) as flat and as close to unity over as wide frequency range as possible [10].

If we assume, that there is no input disturbance (*d = 0*), the transfer function between the reference and the process output is:

$$G_{CL}(s) = \frac{Y(s)}{R(s)} = \frac{G_P(s)G_C(s)}{1+G_P(s)G_C(s)}. \tag{3}$$

The controller is determined in such a way that

$$G_{CL}(0) = 1 , \tag{4}$$

$$\lim_{\omega\to\infty}\left[\frac{d^{2k}\left|G_{CL}(j\omega)\right|}{d\omega^{2k}}\right] = 0 ;\ k = 1,2,\ldots,k_{\max} , \tag{5}$$

for as many k as possible [9,10,15]. This technique is variously called magnitude optimum (MO) [21], modulus optimum [1], or Betragsoptimum [9], and results in a fast and non-oscillatory closed-loop time response for a large class of process models [11,12,20].

Eq. (4) is simply fulfilled by using a controller structure containing the integral term[1], because the steady-state control error is zero. The number of conditions in Eq. (5) that can be satisfied depends on controller order. For a PI controller:

$$G_C(s) = K_P + \frac{K_i}{s} = K_P\left(1+\frac{1}{sT_i}\right), \tag{6}$$

where K_P, K_i, and T_i are respectively the proportional gain, the integral gain and the integral time constant, the MO method results in the following expressions for controller parameters [15]:

$$K_P = \frac{A_3}{2(A_1A_2 - A_0A_3)}, \tag{7}$$

$$K_i = \frac{A_2}{2\left(A_1A_2 - A_0A_3\right)}. \tag{8}$$

[1] Under the condition that the closed-loop response is stable

If the process is described by the following transfer function:

$$G_P = K_{PR}\frac{1+b_1 s+b_2 s^2+...+b_m s^m}{1+a_1 s+a_2 s^2+...+a_n s^n}, \tag{9}$$

where K_{PR} is the static gain of the process, then the so called »characteristic areas« A_0-A_3 of the process can be defined as [12, 13, 15]:

$$\begin{aligned}
A_0 &= K_{PR} \\
A_1 &= K_{PR}(a_1 - b_1 + T_{del}) \\
A_2 &= K_{PR}\left[b_2 - a_2 - T_{del}b_1 + \frac{T_{del}^2}{2!}\right] + A_1 a_1 \\
&. \\
&. \\
&. \\
A_k &= K_{PR}\left[(-1)^{k+1}(a_k - b_k) + \sum_{i=1}^{k}(-1)^{k+i}\frac{T_{del}^i b_{k-i}}{i!}\right] + \sum_{i=1}^{k-1}(-1)^{k+i-1}A_i a_{k-1}
\end{aligned} \tag{10}$$

Note that A_0 equals the steady-state gain of the process. The name "characteristic areas" is associated with the fact that they can be calculated from nonparametric process model in time domain by changing the steady state of the process and performing multiple integrations on the process input (u) and output (y) signals [7, 12]. This procedure is relatively easy to perform in practice and does not require explicit identification of the process transfer function parameters.

However, by using the original MO method, disturbance rejection is degraded when dealing with lower-order processes, since slow process poles might become almost entirely cancelled by controller zeros. This phenomenon is expected, since the MO method aims at achieving good reference tracking, so it optimizes the transfer function between the reference and the process output (r=1, d=0) $G_{CL}(s)=Y(s)/R(s)$ instead of the transfer function between the input disturbance (r=0, d=1) and the process output $G_{CLD}(s)=Y(s)/D(s)$. Optimizing the latter would prove itself a fruitless attempt, since this transfer function is not compatible with MO criterion (4) as $G_{CLD}(0)=0$.

The modification of MO criteria, hereafter denoted as the disturbance rejection magnitude optimum (DRMO), which has been previously proposed in [15], optimizes a modified transfer function between the input disturbance (*r*=0, *d*=1) and the process output:

$$G_{CLD1}(s) = \frac{K_i}{s}\frac{G_P(s)}{1 + G_P(s)G_C(s)}. \tag{11}$$

This transfer function is then applied to equations (4) and (5) instead of the transfer function (3). For the PI controller structure the following expressions have been obtained [15]:

$$K_P = \frac{\xi_2 - \mathrm{sgn}(\xi_2)A_1\sqrt{A_2^2 - A_1A_3}}{\xi_1}, \tag{12}$$

$$K_i = \frac{(1 + K_P A_0)^2}{2A_1}, \tag{13}$$

where ξ_1 and ξ_2 are

$$\begin{aligned} \xi_1 &= A_0^2 A_3 - 2A_0A_1A_2 + A_1^3 \\ \xi_2 &= A_1A_2 - A_0A_3 \end{aligned}. \tag{14}$$

The proposed DRMO method is quite efficient in improving disturbance rejection performance, especially for lower-order processes. The sufficient stability conditions are given in [15,20].

3. Study of decay ratios for DRMO tuning method

As previously mentioned, one possible criterion in time domain that determines the closed-loop systems time response, is the decay ratio. The aim of this chapter is to calculate the decay ratios for a wide batch of process models in a closed-loop configuration with a PI controller tuned by using the DRMO method. Anticipation is that the method in question gives decay ratios that are, in most cases, relatively similar for many different process models. The following batch of process models, covering processes of lower and higher orders, processes with delay, non-minimum phase processes and processes with zeros in left half-plane, has been selected:

$$G_{P1} = \frac{e^{-sT_{del}}}{1+sT}\,;\; T_{del} = \{12,11,10,6,2,1\};\; T = 12 - T_{del}\,, \tag{15}$$

$$G_{P2} = \frac{e^{-sT_{del}}}{(1+sT)^2}\,;\; T_{del} = \{11,10,8,6,4,1\};\; T = \frac{12 - T_{del}}{2}\,, \tag{16}$$

$$G_{P3} = \frac{1}{(1+sT_1)(1+sT_2)}\,;\; T_1 = \{11,10,9,8,7,6\};\; T_2 = 12 - T_1\,, \tag{17}$$

$$G_{P4} = \frac{1}{(1+sT_1)^2(1+sT_2)^2}\,;\; T_1 = \{5.5,5,4.5,4,3.5,3\};\; T_2 = \frac{12 - 2T_1}{2}\,, \tag{18}$$

$$G_{P5} = \frac{1}{(1+sT)^n}\,;\; n = \{3,4,5,6,7,8\};\; T = \frac{12}{n}\,, \tag{19}$$

$$G_{P6} = \frac{1}{(1+sT)(1+skT)(1+sk^2T)(1+sk^3T)}\,;\; k = \{0.9,0.7,0.5,0.4,0.3,0.2\};\; T = \frac{11}{k+k^2+k^3}\,, \tag{20}$$

$$G_{P7} = \frac{1-sT_z}{(1+sT_p)^3}\,;\; T_z = \{1,2,3,4,5,6\};\; T_p = \frac{12 - T_z}{3}\,, \tag{21}$$

$$G_{P8} = \frac{1+sT_z}{(1+sT_p)^3}\,;\; T_z = \{0.5,1,2.5,4.5,5.5,7\};\; T_p = \frac{12 + T_z}{3}\,, \tag{22}$$

$$G_{P9} = \frac{1}{(1+4s)(1+4s(1-i\alpha))(1+4s(1+i\alpha))}\,;\; \alpha = \{0.2,0.3,0.4,0.5,0.7,1\}. \tag{23}$$

These process models were used in the closed-loop configuration (Fig. 2) with input step disturbance of magnitude 1 (d=1(t) in Fig.2). The calculated decay ratios (1) for all the process models are depicted in Fig. 3.

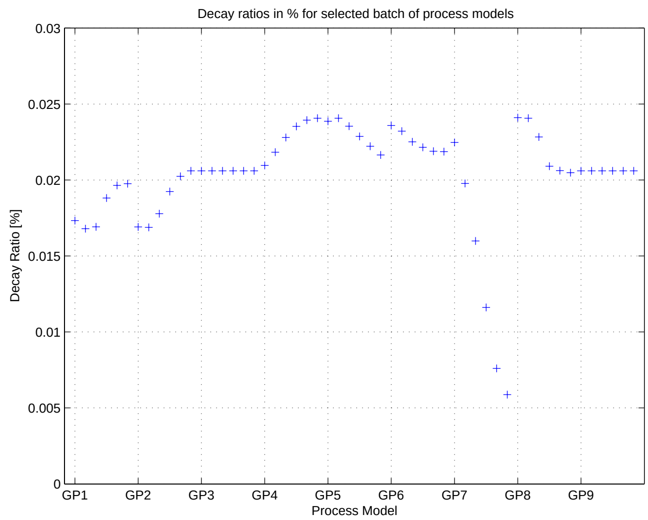

Fig. 3. Decay ratios for closed-loop systems with PI controller and processes G_{P1} to G_{P9}

Similar decay ratios (values between 0.017 and 0.024) can be observed for all the process models except the non-minimum phase process models. Some of the non-minimal phase processes (G_{P7}) give noticeably lower decay ratios. The histogram of decay ratios is shown in Fig. 4.

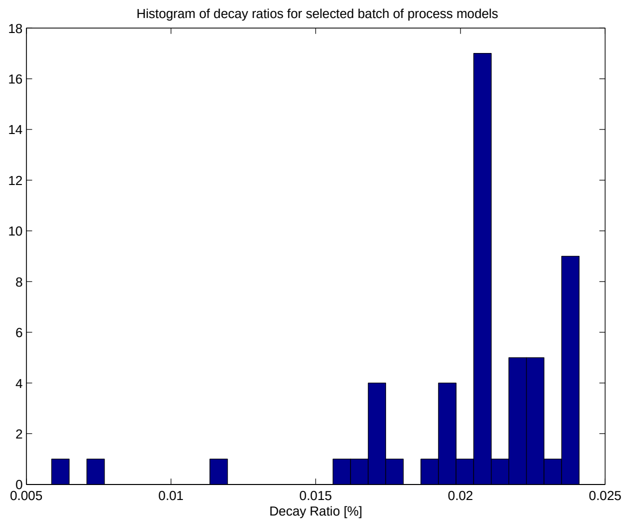

Fig. 4. A histogram of the decay ratios for closed-loop systems with PI controller tuned with DRMO method

4. Comparison with decay ratios of some other methods

The DRMO tuning method was compared with two frequency domain tuning methods. Two of the more popular such tuning methods are the Ästrom and Hagglund (Kappa-Tau or KT) tuning method [1] and the non-convex tuning in frequency domain [3,17]. A short description of the mentioned tuning methods follows in the next two sub-sections.

4.1 Kappa-Tau tuning method

This method [1] basically leans on the original Ziegler-Nichols rules. However, the process is characterized by three (instead of two) parameters. Desired maximum sensitivity (2) is used as a tuning parameter. If the process is stable, its dynamics is characterized by three parameters: the static gain K_P, the apparent lag T, and apparent dead time L (Fig. 5), which can all be obtained from a simple open-loop experiment.

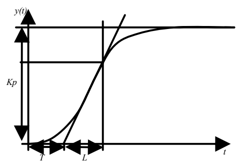

Fig. 5. Determination of process static gain, lag time and apparent dead time

The procedure of calculating the parameters is given in [1].

4.2 Non-convex based optimization tuning method

This method [3] is based on non-convex optimization in frequency domain. The controller parameters are being adjusted until a certain value of sensitivity M_s (2) is achieved is achieved at maximum integral gain (K_i). This method requires either the process model transfer function or the frequency characteristics of the process.

4.3 Results

Previously described sets of tuning rules for PI control [1,3,15] have been applied to the following process models:

$$G_{P1} = \frac{1}{(s+1)^3}, \tag{24}$$

$$G_{P2} = \frac{1}{(s+1)(1+0.2s)(1+0.04s)(1+0.008s)}, \tag{25}$$

$$G_{P3} = \frac{e^{-15s}}{(s+1)^3}, \tag{26}$$

$$G_{P4} = \frac{1-2s}{(s+1)^3}, \tag{27}$$

$$G_{P5} = e^{-s}. \tag{28}$$

The PI controller parameters for all three tuning methods are given in Table I.

Table I Controller parameters for processes (24) to (28).

Tuning method	*Parameters*	G_{P1}	G_{P2}	G_{P3}	G_{P4}	G_{P5}
DRMO	K_P	0,651	2,176	0,276	0,328	0,268
	K_i	0,455	4,041	0,045	0,176	0,804
KT M_s=1.4	K_P	0,535	1,32	0,077	0,141	0,005
	K_i	0,334	2,289	0,018	0,11	0,020
KT M_s=2	K_P	0,633	1,93	0,164	0,179	0,158
	K_i	0,325	2,591	0,027	0,101	0,472
NC M_s=1.4	K_P	1,145	3,036	0,280	0,340	0,023
	K_i	0,715	5,266	0,064	0,266	0,097
NC M_s=2	K_P	1,22	4,13	0,266	0,294	0,255
	K_i	0,685	6,988	0,048	0,184	0,854

Fig. 6 to 10 show the closed-loop responses on input disturbance for processes (24) to (28). In Fig. 11 the decay ratios are depicted for all three tuning methods.

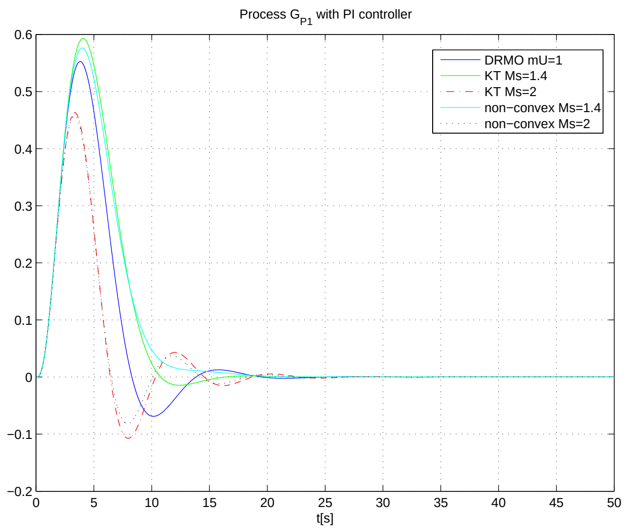

Fig. 6. Response on input disturbance (r=0, d=1) of process (24)

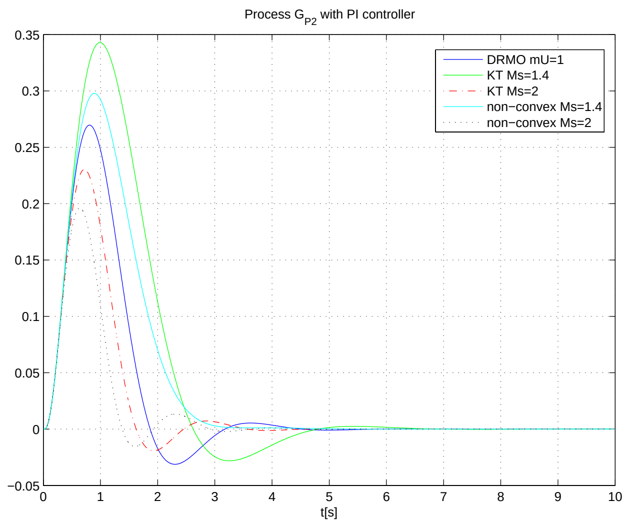

Fig. 7. Response on input disturbance (r=0, d=1) of process (25)

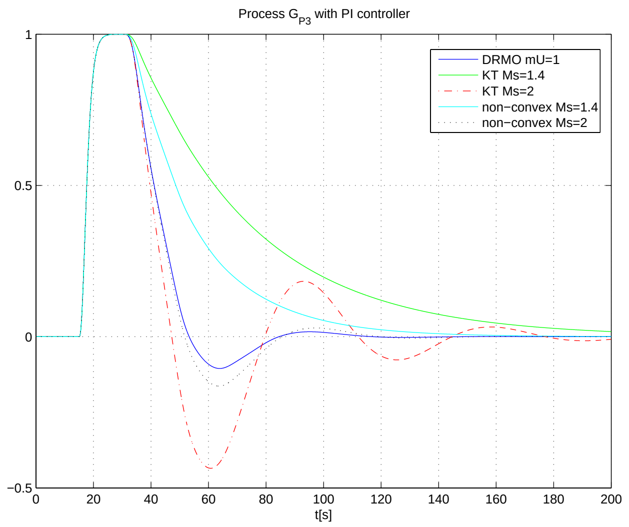

Fig. 8. Response on input disturbance (r=0, d=1) of process (26)

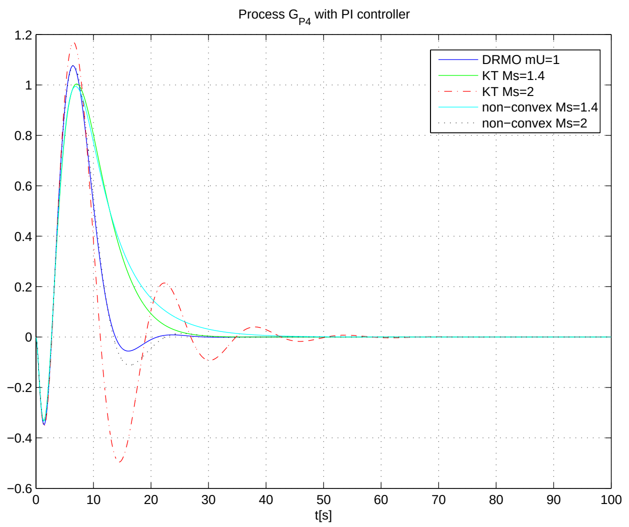

Fig. 9. Response on input disturbance (r=0, d=1) of process (27)

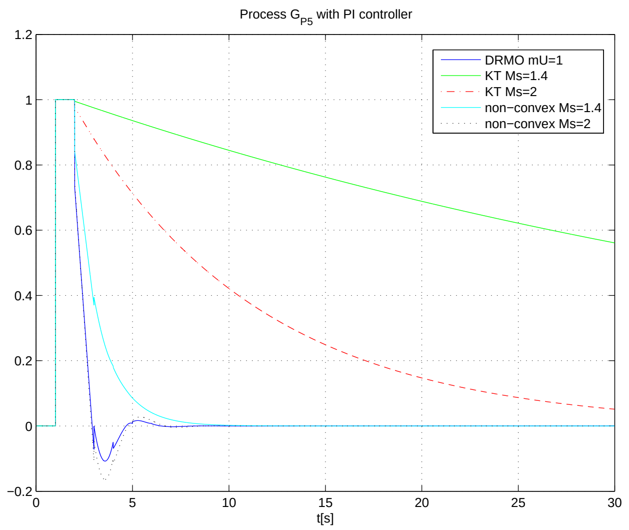

Fig. 10. Response on input disturbance (r=0, d=1) of process (28)

The decay ratios of the DRMO tuning method span between 0.009 and 0.022, while the other two methods have significantly wider intervals of decay ratios (0.017 to 0.09 for NC method and 0 to 0.185 for KT method). The results show that the responses on input disturbance, when the PI controller parameters are tuned with DRMO method, are more uniform in terms of decay ratios than responses from other two methods.

Fig. 11 depicts the maximum sensitivity function (2) for all the process models and tuning methods.

From Fig. 12 it is clear that the NC method guarantees constant maximum sensitivity over the tested batch of process models, since it sets M_s to a fixed value (by using optimization). On the other hand, the measured maximum sensitivity of the KT method varies significantly, even more than DRMO method.

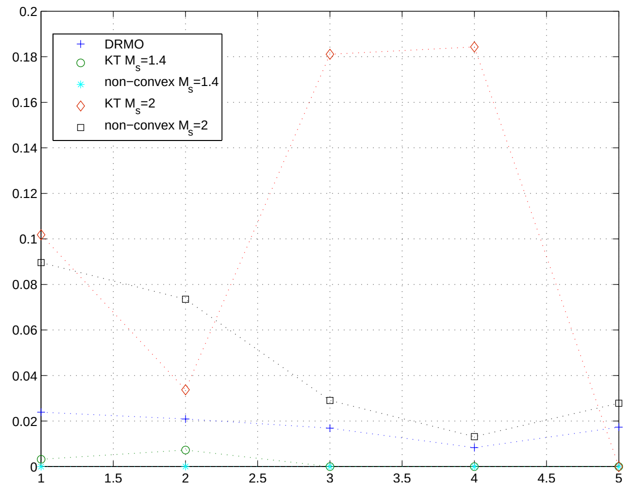

Fig. 11. Decay ratios of processes (24) to (28) for DRMO tuning method (+), KT tuning method with M_s=1.4 (○), KT tuning method with M_s=2 (◊), non-convex tuning method with M_s=1.4 (*) and non-convex tuning method with M_s=2 (□).

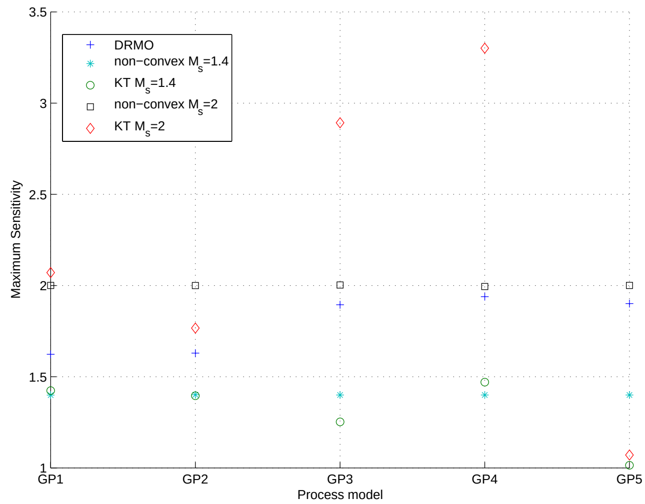

Fig. 12. Maximum sensitivity function for processes (24) to (28) for DRMO tuning method (+), KT tuning method with M_s=1.4 (○), KT tuning method with M_s=2 (◊), non-convex tuning method with M_s=1.4 (*) and non-convex tuning method with M_s=2 (□).

5. Conclusions

Disturbance rejection magnitude optimum (DRMO) tuning method applied to a PI controller results in efficient input disturbance rejection for many process models frequently encountered in process and chemical industries. The aim of this paper was to determine whether the closed-loop time responses are uniform in terms of the decay ratios. The experiment on a wide batch of process models established that decay ratios, except for non-minimal process- models, are between 0.017 and 0.024. Decay ratios of the non-minimum phase process models are significantly smaller.

The DRMO method, when compared with two other tuning methods, gives more uniform closed-loop responses in terms of decay ratios.

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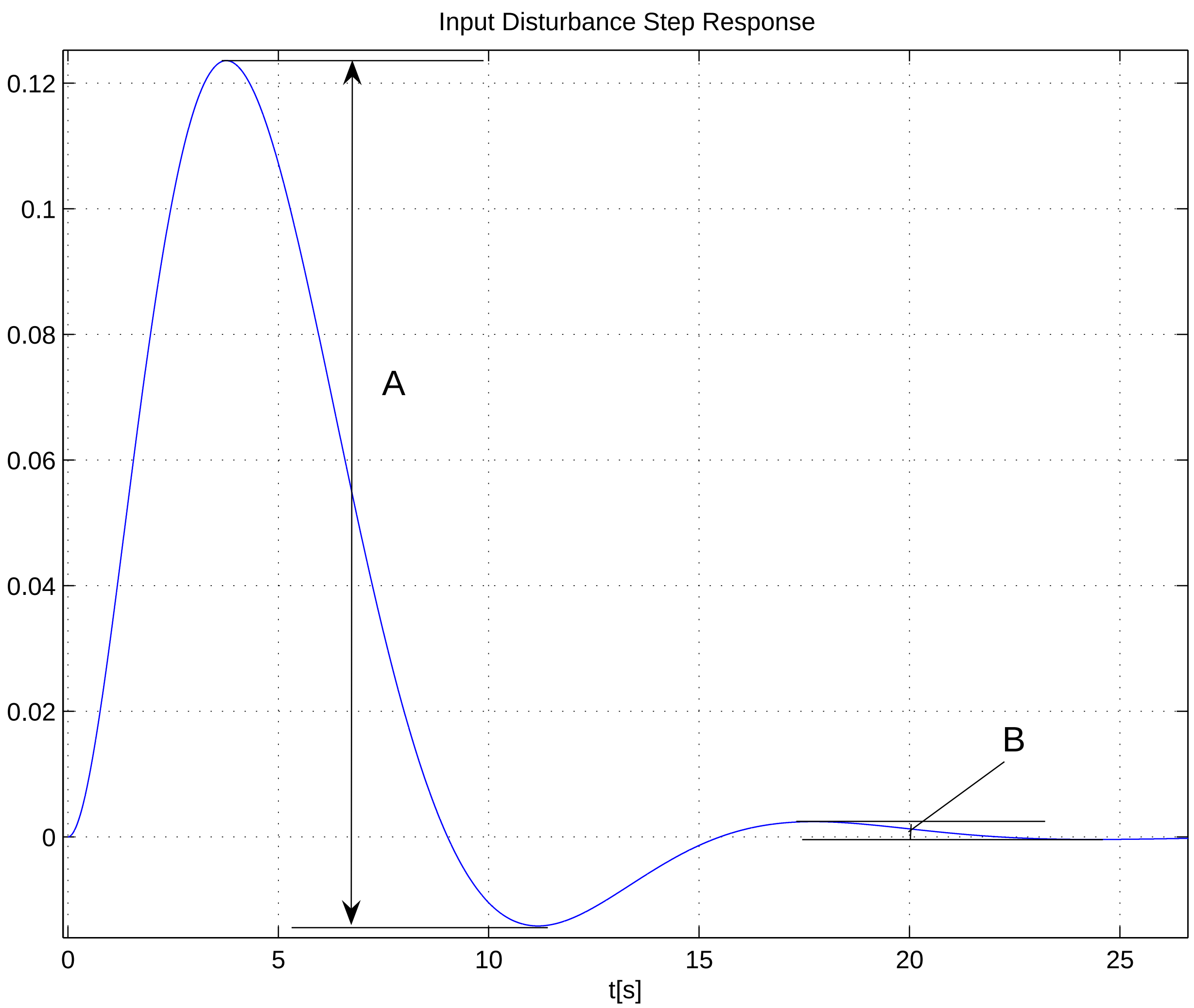

Fig. 13. Definition of decay ratio

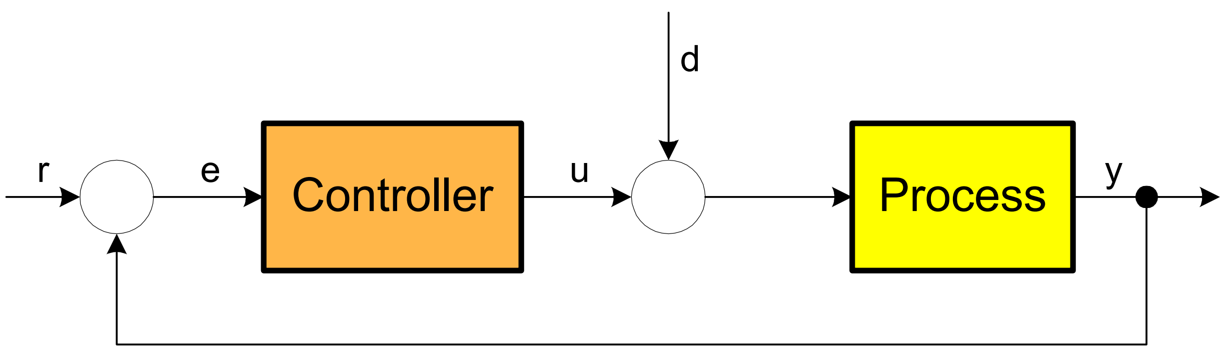

Fig. 14. Typical closed-loop configuration using a 1DOF controller.

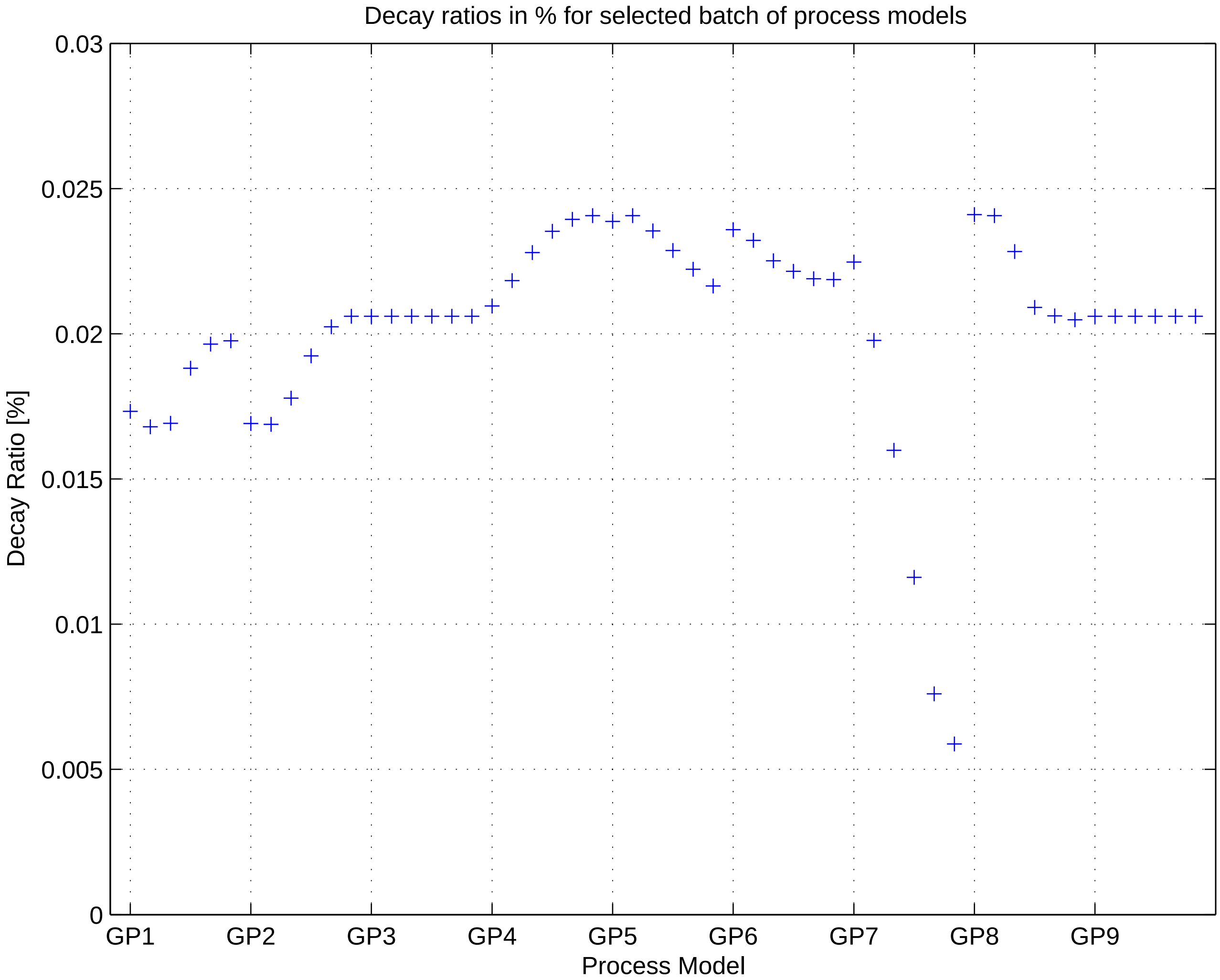

Fig. 15. Decay ratios for closed-loop systems with PI controller and processes G_{P1} to G_{P9}

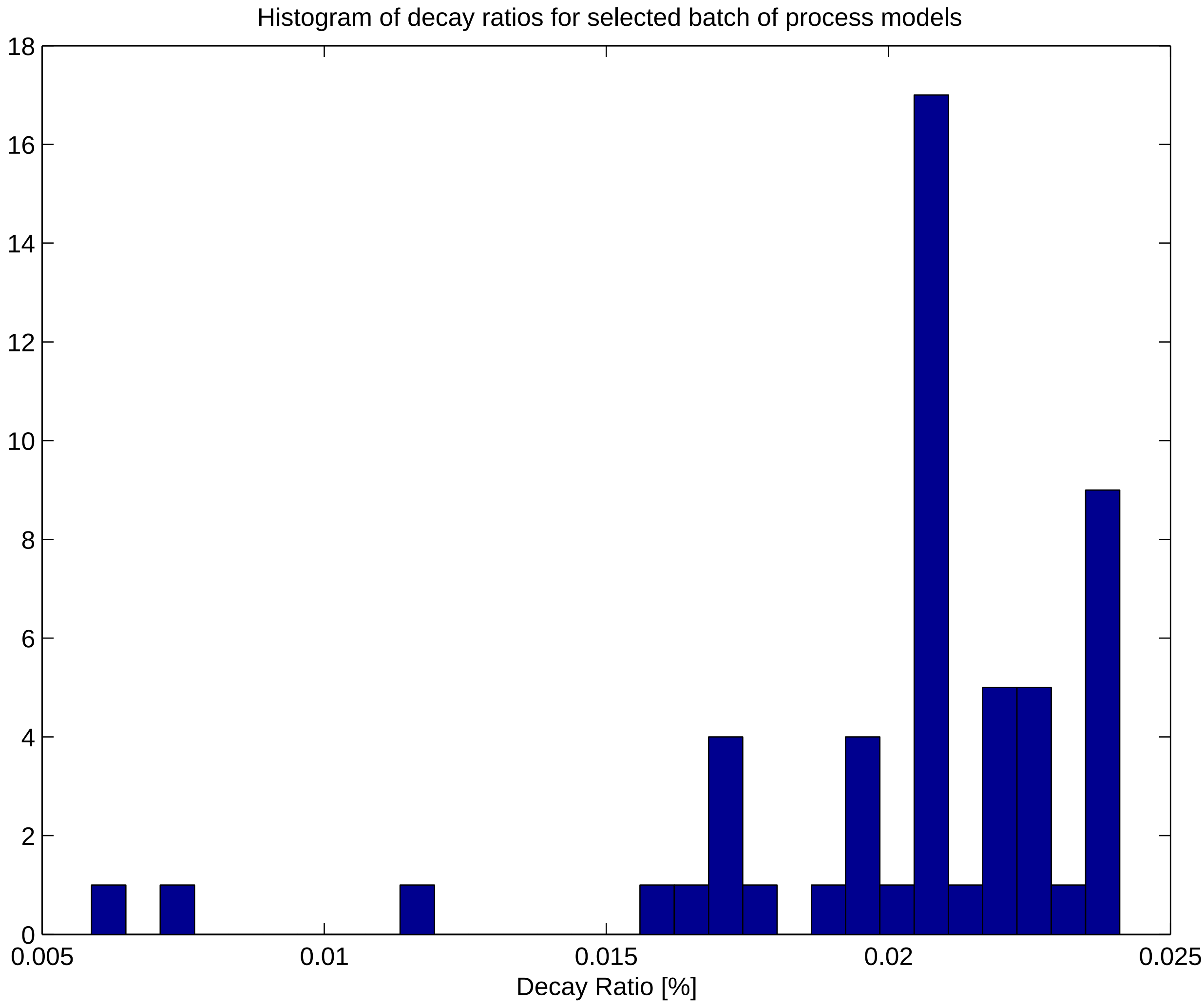

Fig. 16. A histogram of the decay ratios for closed-loop systems with PI controller and processes G_{P1} to G_{P9}

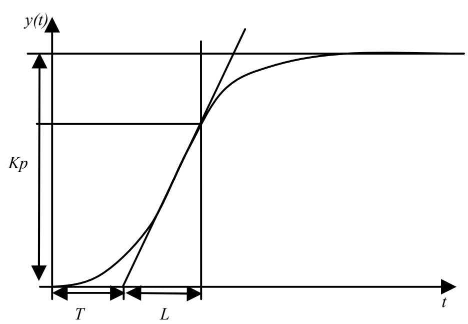

Fig. 17. Determination of process static gain, lag time and apparent dead time

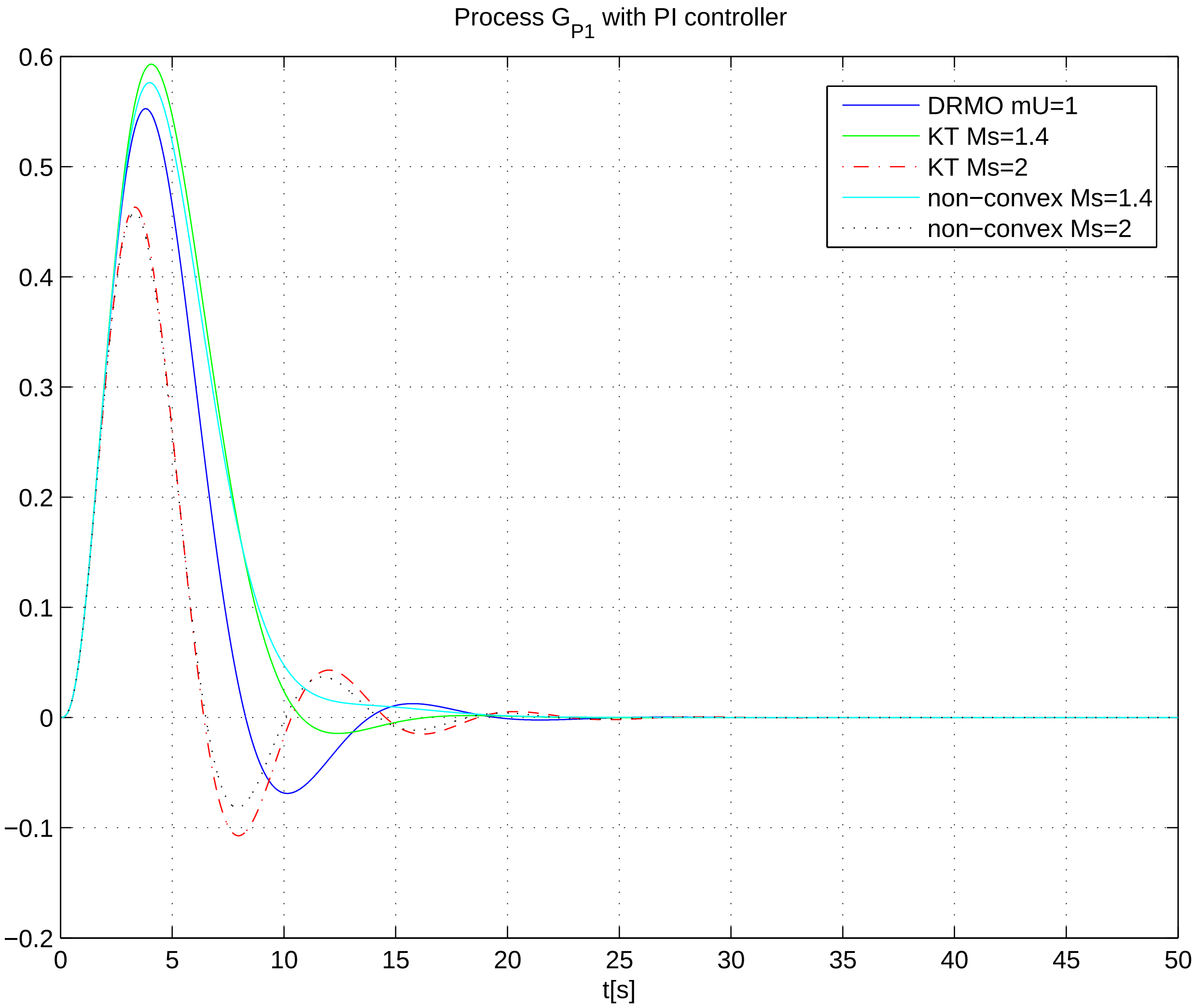

Fig. 18. Response on input disturbance (r=0, d=1) of process (24)

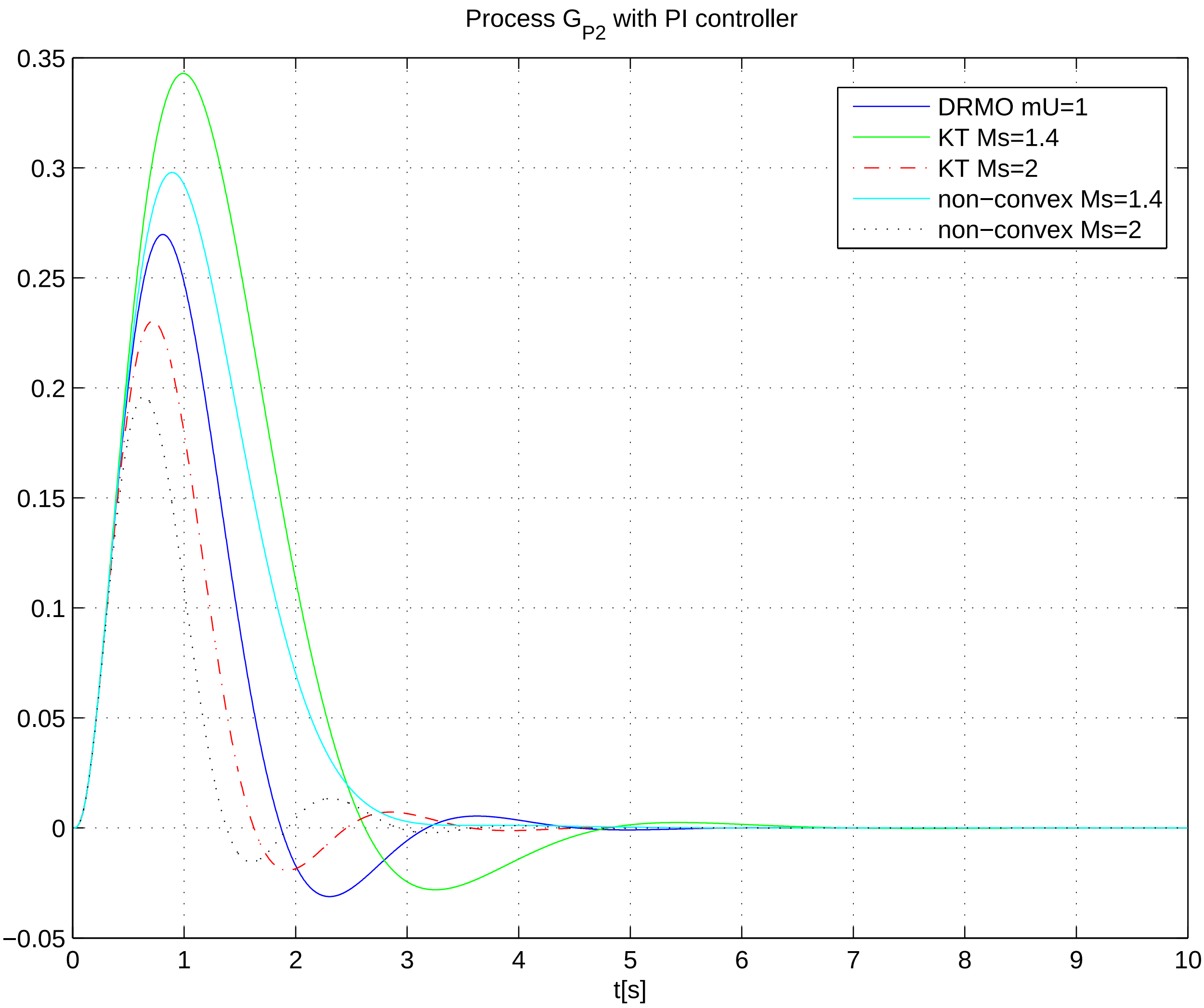

Fig. 19. Response on input disturbance (r=0, d=1) of process (25)

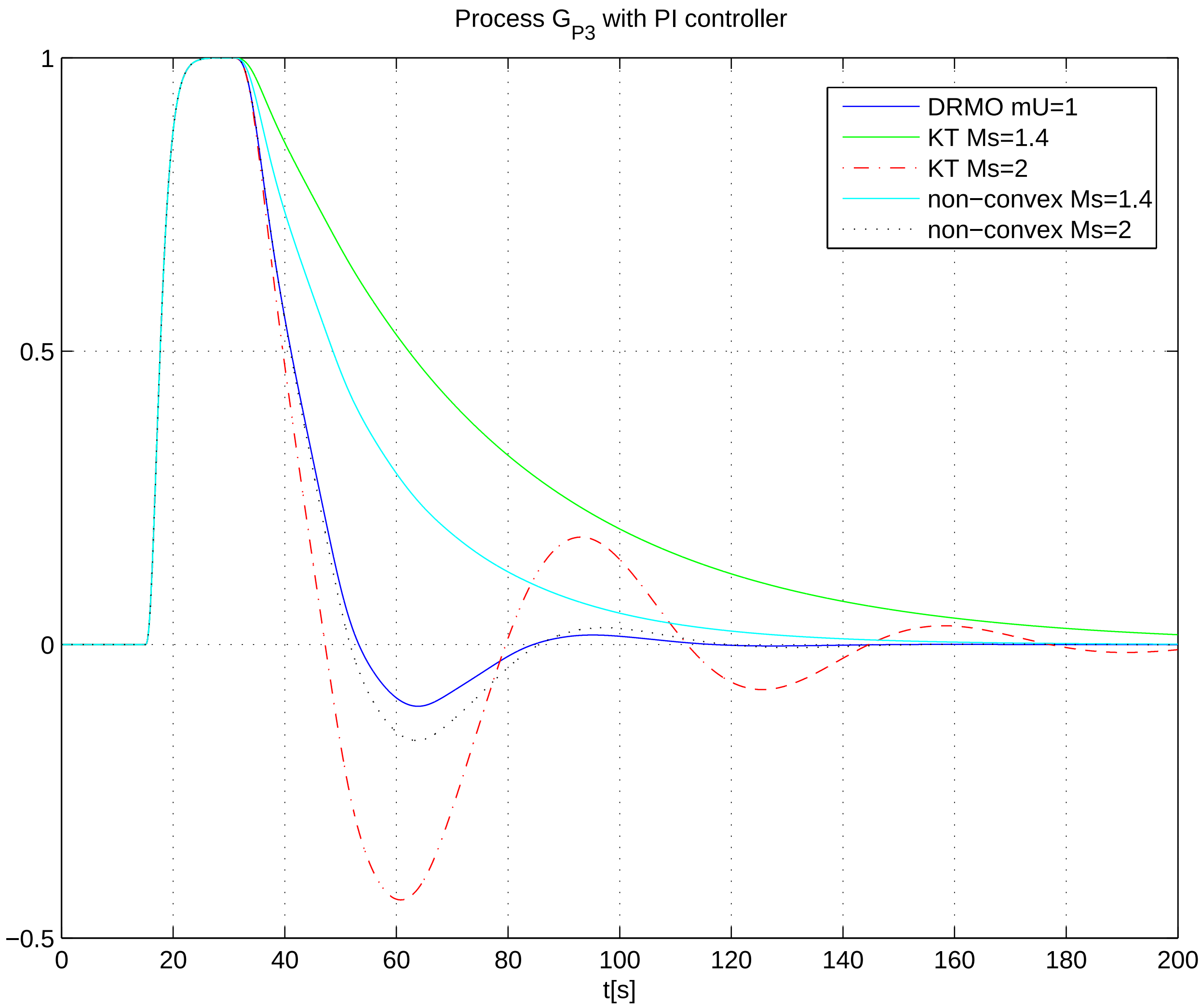

Fig. 20. Response on input disturbance (r=0, d=1) of process (26)

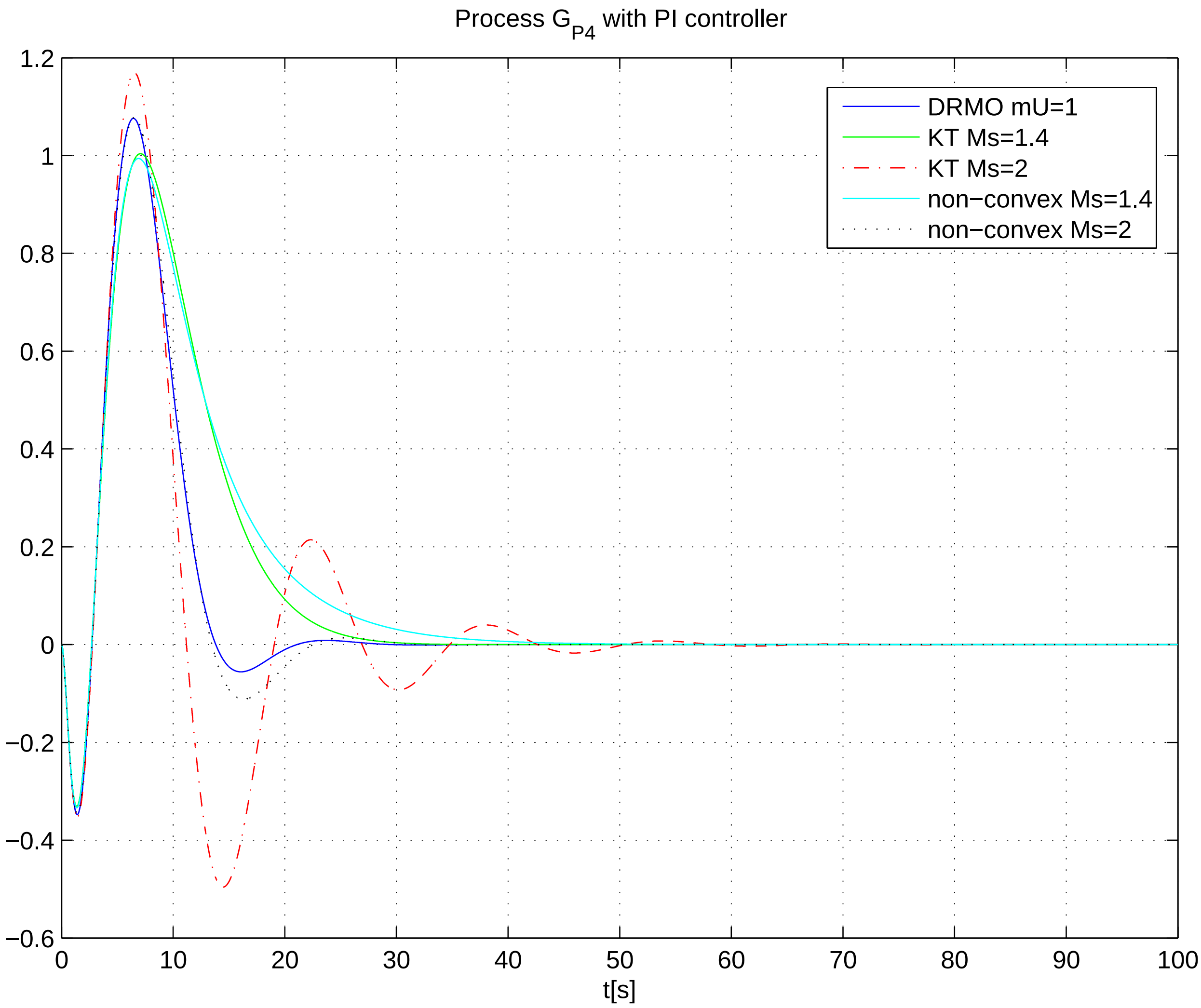

Fig. 21. Response on input disturbance (r=0, d=1) of process (27)

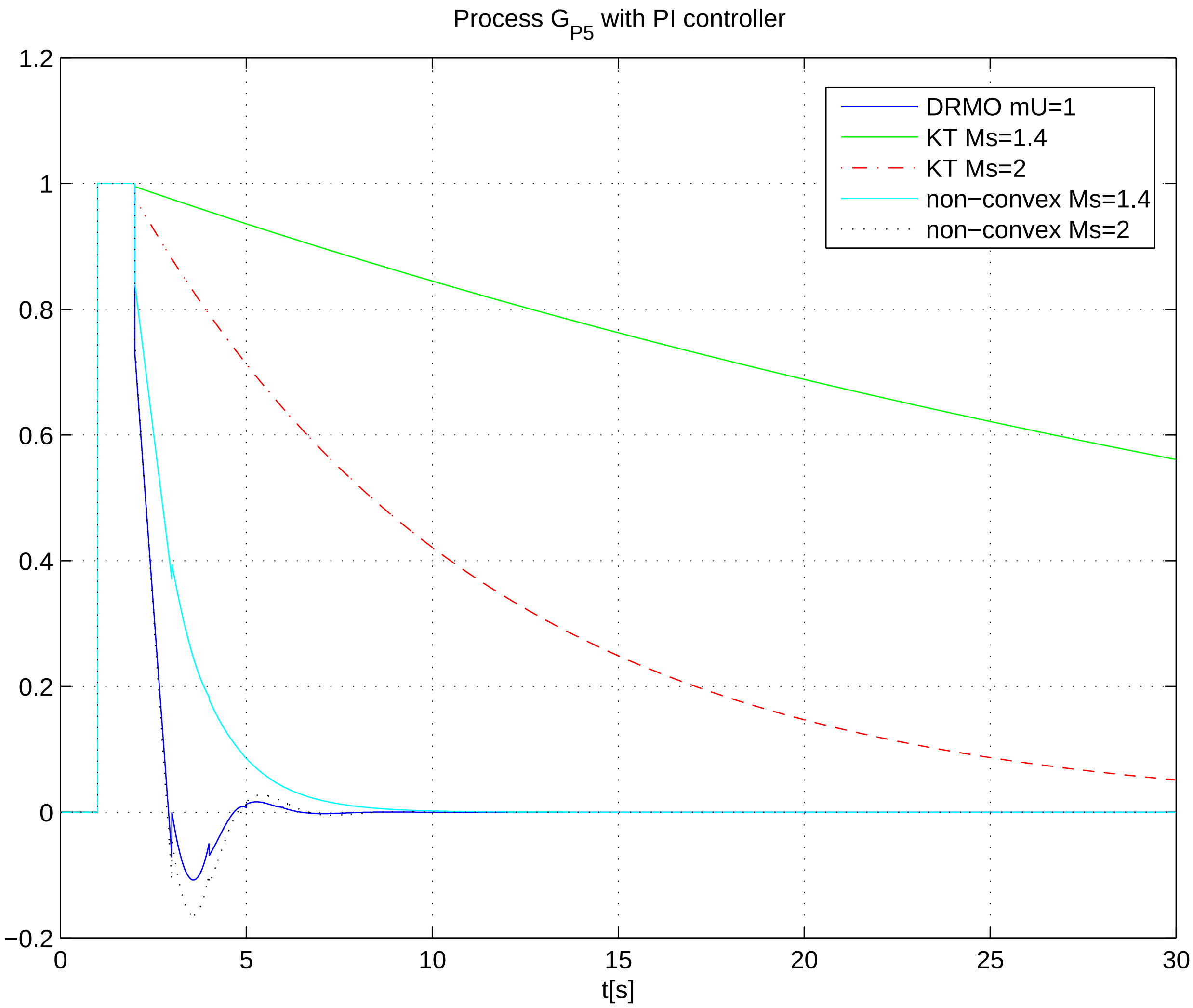

Fig. 22. Response on input disturbance (r=0, d=1) of process (28)

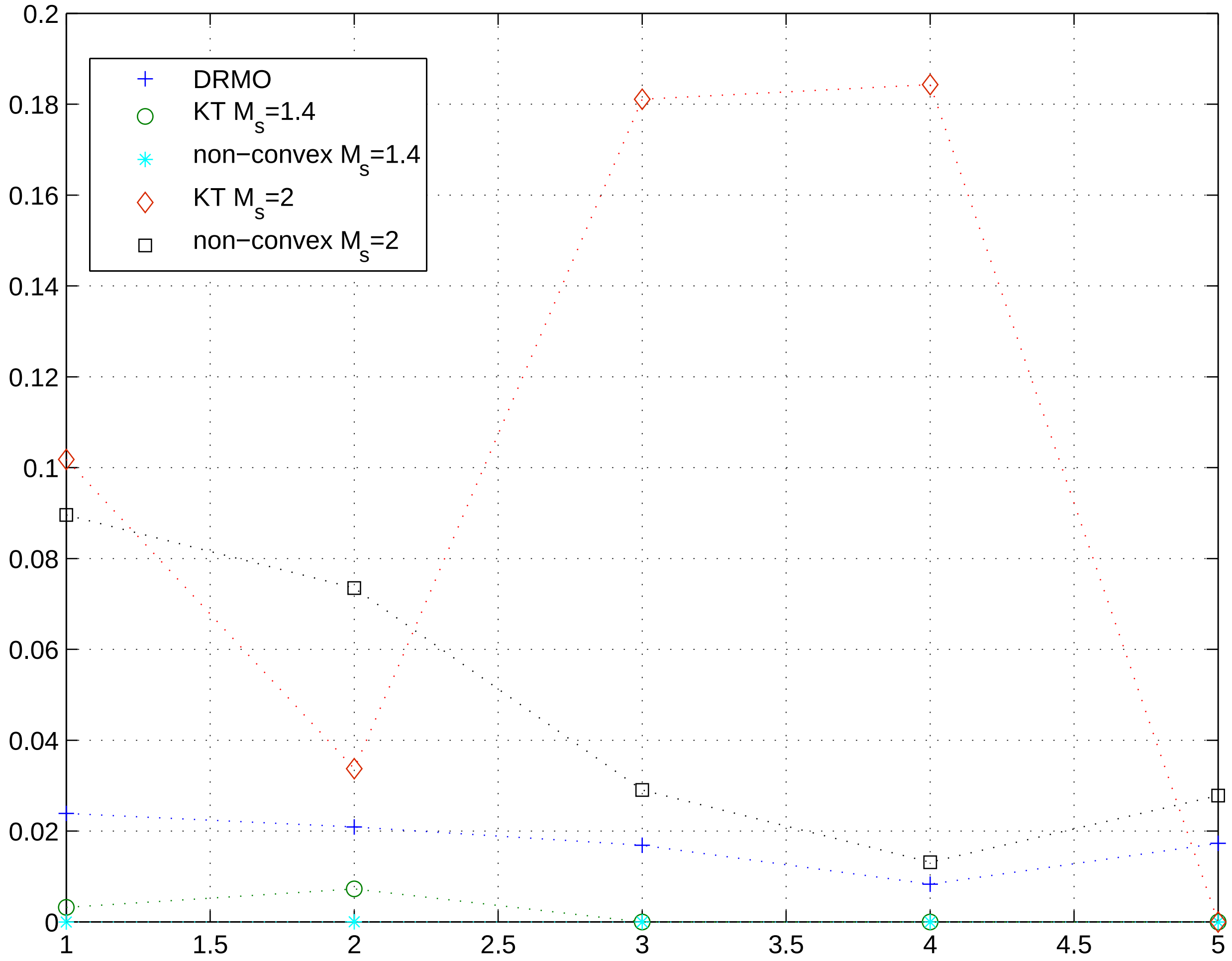

Fig. 23. Decay ratios of processes (24) to (28) for DRMO tuning method (+), KT tuning method with M_s=1.4 (○), KT tuning method with M_s=2 (◊), non-convex tuning method with M_s=1.4 (*) and non-convex tuning method with M_s=2 (□).

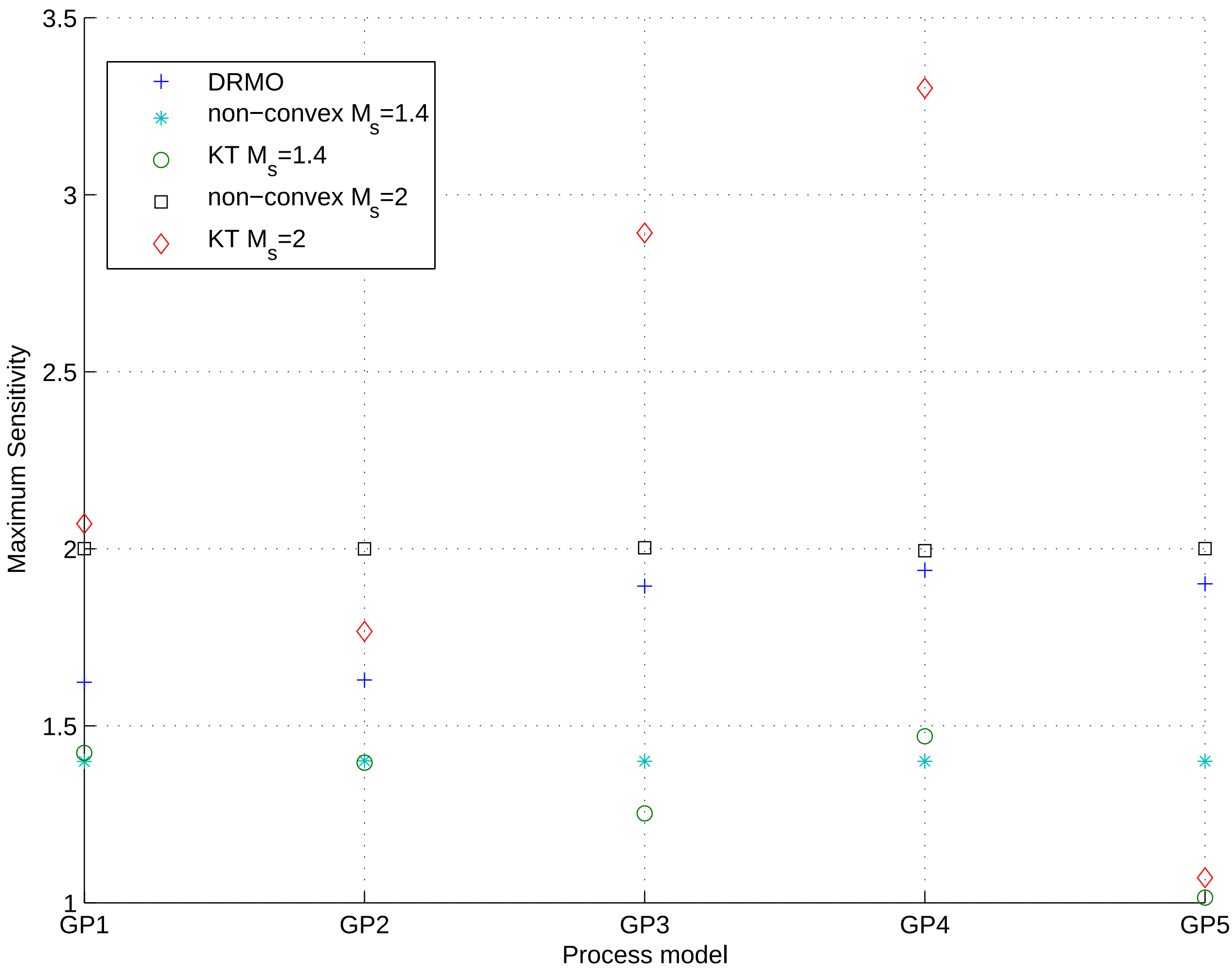

Fig. 24. Maximum sensitivity function for processes (24) to (28) for DRMO tuning method (+), KT tuning method with M_s=1.4 (○), KT tuning method with M_s=2 (◊), non-convex tuning method with M_s=1.4 (*) and non-convex tuning method with M_s=2 (□).

LIST OF FIGURES LEGENDS

Fig.6, Fig.7, Fig.8, Fig.9 and Fig.10 – (solid blue line) DRMO method; (solid green line) Astrom&Hagglund method with Ms=1.4; (red dashed line) Astrom&Hagglund method with Ms=2; (cyan solid line) Panagopoulos method with Ms=1.4; (black dashed line) Panagopoulos method with Ms=2

Fig.11 – (+) DRMO tuning method, (○) KT tuning method with M_s=1.4, (◊) KT tuning method with M_s=2, (*) non-convex tuning method with M_s=1.4, (□) non-convex tuning method with M_s=2.

Fig.12 – (+) DRMO tuning method, (○) KT tuning method with M_s=1.4, (◊) KT tuning method with M_s=2, (*) non-convex tuning method with M_s=1.4, (□) non-convex tuning method with M_s=2.