CASCADE CONTROL METHODS OF A SIMPLE NONLINEAR LIMB MODEL

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Abstract: The cascade control of an elbow-like nonlinear limb model is described in this paper. The control design method is based on linearization and backstepping using the special cascade structure of the system. Using the proposed control structure, a regulation problem is solved. The control can be extended to the trajectory following task.

Keywords: Biomedical Modelling, Musculoskeletal control, Non-linear Control, Backstepping.

1. INTRODUCTION

Motivation and aim: Even the simplest limb model exhibits strongly nonlinear dynamic behavior that calls for applying the results of nonlinear systems and control theory. The control of musculoskeletal structures has considerable importance in the field of human locomotion control, designing and controlling muscle prothesis and artificial limbs. Last but not least the techniques of FES (functional electrical stimulation) - of patients with some kind of paralysis - can be improved with appropriate control methods.

Application of nonlinear control for biomechanical systems has appeared in the literature several times in the past decades. A study proposed by Levine and Zajac [8] has shown, that in the case of the pedaling problem, the control to achieve maximal acceleration for a simple skeletal system is bang-bang. The article of Kaplan [7] provides an optimal control method for muscle activity in the case of the pedalling problem using non-derivative quasi-Newton methods and numerical gradient methods. A further research of acceptable controllers (incl. input-output linearization) for the cycling problem was proposed by Abbot in his degree thesis [1]. Except some articles, the most of the solutions are based on linearization of the model, or on optimization methods with significant need of computing power. In general the usage of nonlinear control engineering methods is not prevalent in literature in this field, and also the applied solutions can be refined with new methods (utilize the cascade properties of such systems). Second, the proof of stability in the case of nonlinear system and control theory is always a challenge for systems with complex state-space models. In this case the system described has a quite complex dynamics, but we can avoid some of these difficulties by applying feedback linearization ([5] or [6]), and by using the backstepping method.

2. THE SIMPLE NONLINEAR LIMB MODEL

2.1 Basic assumptions and system structure

A nonlinear input-affine state-space model has been developed [2] for a simple one-joint system with a flexor and an extensor muscle (see Fig. 1) which is suitable for nonlinear system analysis and control design.

During the modeling, the following basic assumptions have been made:

- The nonlinear properties of the force-length relation and the force-contraction velocity relation are taken into account.
- Exerted forces depend linearly on the activation state of muscles, following the principles in [12], [11] and [9].
- The tendon dynamics is not modeled separately as described in [3].

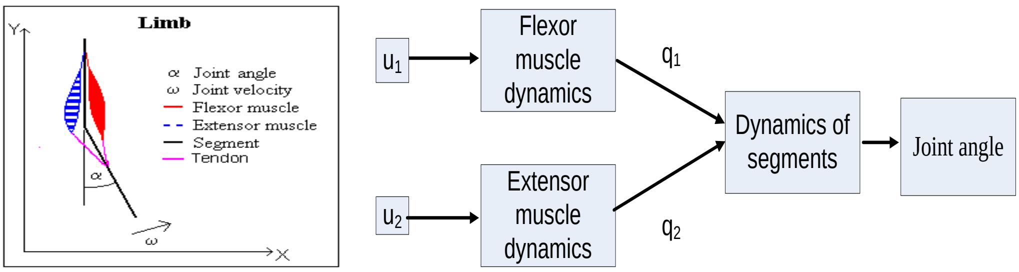

Fig. 1: The scheme and structure of the modeled system

The manipulable inputs of the model are the normalized activation signals of the muscles, the measurable output is the joint angle. The number of state variables is 4.

2.2 Model equations

Segmental dynamics The dynamics of the segments in the open-loop case can be described by the following equations:

$$\frac{d\alpha}{dt} = \omega \quad \frac{d\omega}{dt} = \frac{M_m(q_1, q_2, \alpha, \omega) + ml_C cos(\alpha - \pi/2)g}{\Theta + ml_C^2} \tag{1}$$

where α [rad] is the joint angle, $-\pi/2$ [rad] is the angle between the global coordinate-system's x axis, and the not-moving upper segment of the limb, ω [rad/s] is the anglular velocity, Θ $[kgm^2]$ is the moment of inertia defined to the mass-center point of the bone, m [kg] is the mass of the moving limb part, l_C [m] is the distance between the moving limb part's center of mass point and the joint axis, M_m [Nm] is the resulting joint torque of the muscles, and g $[m/s^2]$ is the gravitational acceleration. q_1 and q_2 denote the activation states of the muscles.

If we study the model around $\alpha = \pi/2$ joint angle, the forces of ligaments, bones and the passive force of the muscles can be neglected, so we can write

$$M_m = F^f_{max} f^f_{LM}(\alpha) f^f_{VM}(\alpha, \omega) d^f q_1 + F^e_{max} f^e_{LM}(\alpha) f^e_{VM}(\alpha, \omega) d^e q_2 \tag{2}$$

where f_{LM}^{f} and f_{LM}^{e} denote the function of the parabolic force-length characteristics in the case of the flexor and extensor muscle, f_{VM}^{f} and f_{VM}^{e} denote the function of the arch. tangent-like force-contraction velocity characteristics in the case of the flexor and extensor muscle, d denotes the moment arm, and F_{max} denotes the maximal force. These characteristics are fully described in [3] and they can be seen in Fig. 2.

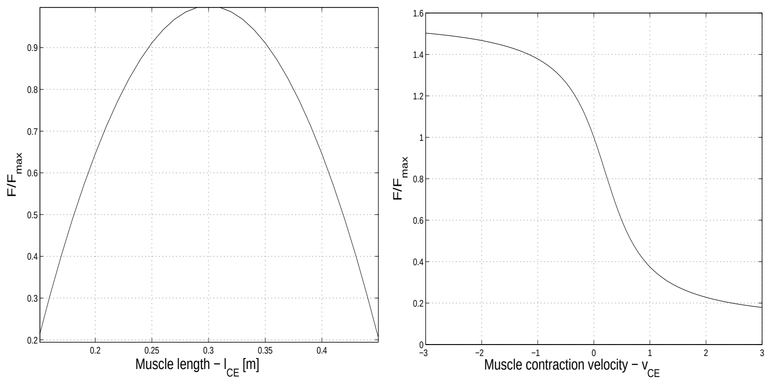

Fig. 2: F_{LM} and F_{VM}

Muscle dynamics The differential equation of the muscle defines the connection between $q(t)$, the activation state of the muscle and the activation signal $u(t)$. With $u(t) \in [0, 1]$ the equation taken from Zajac [12] is:

$$\frac{dq}{dt} = -\left(\frac{1}{\tau_{act}}(\beta + [1-\beta]u(t)\right) q + \frac{1}{\tau_{act}} u(t) \tag{3}$$

where τ_{act} [s] is the activation time, showing how quick the muscle reacts on the external activation signal coming from the nervous system, β is a constant, describing the correlation between the decrease of the activation state and the external activation signal. If β = 1 then the external activation signal does not affect the decrease of the activation state, if β = 0 then it strongly affects the decrease. $q_1(t)$ denotes the activation state of the flexor muscle, and $q_2(t)$ denotes the activation state of the extensor muscle.

State-space equations With the notation x_i for the state-space variables ($x_1 = q_1$, $x_2 = q_2$, $x_3 = \alpha$, $x_4 = \omega$), the equations are as follows:

$$\frac{dx_1}{dt} = -\left(\frac{1}{\tau_{act}}(\beta + [1-\beta]u_1(t))\right) x_1 + \frac{1}{\tau_{act}} u_1(t)$$
$$\frac{dx_2}{dt} = -\left(\frac{1}{\tau_{act}}(\beta + [1-\beta]u_2(t))\right) x_2 + \frac{1}{\tau_{act}} u_2(t)$$
$$\frac{dx_3}{dt} = x_4 \quad \frac{dx_4}{dt} = \frac{(M_m(x_1, x_2, x_3, x_4) + ml_C cos(x_3 - \pi/2)g)}{\Theta + ml_C^2} \tag{4}$$

where the first two equations describe the dynamics of the muscles, and the second two describe the dynamics of the limb. $u_1(t)$ denotes the activation signal of the flexor muscle, and $u_2(t)$ denotes that of the extensor muscle.

If we rearrange (4) we can get the following general form of input-affine systems:

$$\dot{x} = f(x) + \sum_{i=1}^{m} g_i(x) u_i(t) \quad y = h(x) \tag{5}$$

where

$$g_1(x) = \left[-\frac{1}{\tau_{act}}(1-\beta)x_1 + \frac{1}{\tau_{act}},\ 0,\ 0,\ 0\right]^T \quad g_2(x) = \left[0,\ -\frac{1}{\tau_{act}}(1-\beta)x_2 + \frac{1}{\tau_{act}},\ 0,\ 0\right]^T$$

$$f(x) = \left[-\frac{1}{\tau_{act}}\beta x_1,\ \ -\frac{1}{\tau_{act}}\beta x_2,\ x_4, \frac{M(x_1,x_2,x_3,x_4) + ml_C cos(x_3 - \pi/2)g}{\Theta + ml_{com}^2}\right]^T$$

We assume that we can measure all the state variables, i.e. $h(x) = x$.

2.3 Model structure

Fig. 1 shows that the dynamics of the muscle activation do not depend on the dynamics of the limb, therefore the model has a cascade structure. Thus, the complete dynamics of the limb can be divided into two parts:

- The activation dynamics of the muscles, that is described by two simple first order systems, with the activation signals as input and the activation states as output.
- The movement dynamics of the limb with the activation states as input and the joint angle as output. That corresponds to a second order dynamics.

3. CONTROL STRUCTURE DESIGN BY USING THE BACKSTEPPING METHOD

In this study we examine a steady-state point, which exhibits unstable properties in the open loop case.

3.1 Structure and backstepping

The backstepping technique, which is described below, can be used for nonlinear systems that can be written (possibly after a coordinates transformation) as a cascade of subsystems.

Backstepping is based on the idea, that if we can design a virtual feedback ξ_{ref} for the second subsystem of a cascaded structure which is influenced by the output of the first manipulable subsystem, we can determine a feedback for the input of the first subsystem which stabilizes the closed-loop cascade structure.

As detailed in [10] if we have the system structure

$$\dot{z} \;\; = \;\; f(z) + g(z)\xi \qquad \dot{\xi} = a(z,\xi) + b(z,\xi)u$$

and there exists a virtual feedback $\xi_{ref} = \alpha(z)$

such, that $z = 0$ is an asymptotically stable equilibrium of

$$\dot{z} = f(z) + g(z)\alpha(z) \tag{6}$$

with a Ljapunov function V that is positive definite at $z = 0$, and the form of $a(z,\xi)$ and $b(z,\xi)$ is known. Then the feedback law for the input can be expressed as follows [4]:

$$u = b^{-1}(\xi,z)(A_1(\alpha(z) - \xi) - a(\xi,z) + \frac{d\alpha}{dz}\dot{z}) \tag{7}$$

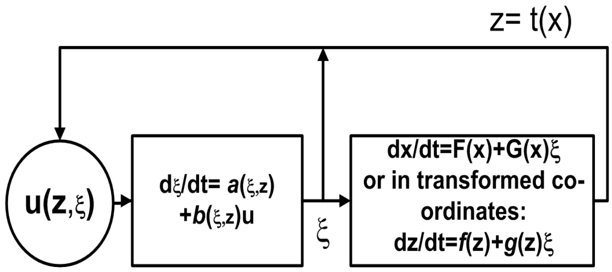

Fig. 3: General structure for the backstepping method

where A_1 is the feedback gain of the error $y_1 := \xi - \alpha(z)$. In this case we obtain the feedback transformed system

$$\dot{z} \;=\; [f(z) + g(z)\alpha(z)] + g(z)y_1 \qquad \dot{y}_1 = -A_1 y_1 \tag{8}$$

with storage function $S_1 = V(z) + \frac{1}{2}y_1^2$, satisfying

$$\frac{dS_1}{dt} \leq -\|y_1\|^2. \tag{9}$$

In our case $\dot{z} = f(z)+g(z)\xi$ corresponds to the dynamics of the limb, and $\dot{\xi} = a(z,\xi)+b(z,\xi)u$ denotes the muscle dynamics. Furthermore $a(\xi, z)$ and $b(\xi, z)$ are in the form of $a(\xi)$ and $b(\xi)$. From (1) and (2) it can be easily seen, that the limb dynamics can also be transformed to the form of (8), because q_1 which is considered as input to the second subsystem, appears in the state-space equation in a linear way.

Our aim is to control the output x_3 (joint angle). The virtual feedback $\alpha(z)$ is determined via a controller designed for the segmental dynamics, which uses linearization and pole placement in this case. The reference signal for the muscle activation state ($q_1^{ref} = \xi_{ref} = \alpha(z)$), and the manipulable input - the muscle activation signal - is determined via this virtual feedback.

Furthermore we have to note, that in this case, the transformed coordinates are identical to the original ones, so $z_1 = x_1 = \alpha$, $z_2 = x_2 = \omega$.

3.2 Sub-controllers

Control of segments (2nd subsystem) The dynamics of the limb has a relative degree of 2.

In the case of the segments, as an example we apply (exact) feedback linearization, and a simple pole-placement design. In this case only the flexor muscle is used as input to get a SISO structure. The control to the normalized variables is applied around a steady-state point at $\alpha = \pi/2$ joint angle. The coordinates of the steady-state point x^* are the following:

$$x_1^* = 0.1456,\; x_2^* = 0.1,\; x_3^* = \pi/2,\; x_4^* = 0 \tag{10}$$

Feedback linearization

As described in [5] for a nonlinear n-dimensional SISO system we need to apply the feedback

$$u = \frac{1}{L_g L_f^{n-1} h(x)}(-L_f^n h(x) + v(t)) \tag{11}$$

and a suitable nonlinear coordinate transformation to obtain a linear system of order n which is influenced by the input u - including the external input $v(t)$. Here $L_f h(x)$ denotes the Lie-derivative of $h(x)$ along f.

This means, that the state-space model of the feedback linearized closed loop system is a simple double-integrator in the new coordinates:

$$\dot{z}_1 = z_2 \quad \dot{z}_2 = v \quad y = z_1 \tag{12}$$

where z_1 and z_2 can be determined by using the coordinate-transformation $z_i = L_f^{i-1} h(x)$.

For the system in the new coordinates we can design a pole-placement controller:

$$v = -Kz \tag{13}$$

where K is a suitable state-feedback vector.

While tuning K, one had to keep in mind the time-domain behavior of the output response, and the fact that the virtual feedback has an input constraint ($q_1 \in [0, 1]$), too.

In fact, we can not act on the system at the point of muscle activation states, so we have to define the value of (11) as reference signal for the flexor muscle's activation state (virtual feedback).

Muscle control (1st subsystem) We have to use (7) to determine the muscle activation signal.

3.3 Simulation results

Simulations were performed using MATLAB by numerically solving the differential equations. The poles for the pole-placement controller were set to [-15 -20], and A_1 was set to -100. The starting values were the same as the coordinates of the steady state point, except for the joint angle ($\alpha = x_3$) which was 0.2. As we can see in Fig. 4 the error has stable linear dynamics.

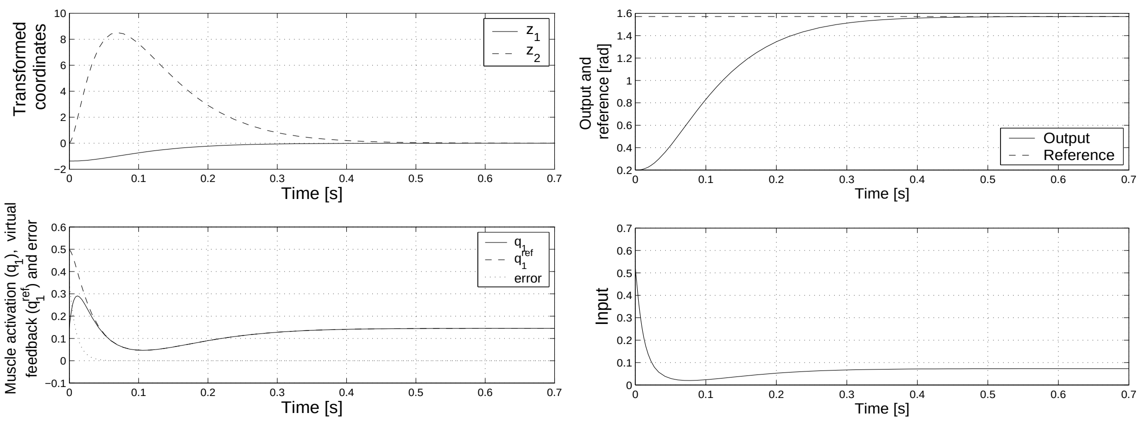

Fig. 4: Transformed coordinates, muscle activation states, virtual feedback (the reference signal for muscle activation) , the error, output, reference signal and input for regulation

Fig. 4 shows, that the input satisfy the input constraint ($u(t) \in [0, 1]$), even in this case, when the starting position is very far from the reference.

4. CONCLUSIONS AND FUTURE WORK

We described a stabilizing control of an elbow-like nonlinear limb model in this paper. The control design method is based on linearization and backstepping using the special cascade structure of the system. Several constants are available for controller tuning with which an effective compromise could be found in any examined case in matching the input constraints and optimizing the time-response of the system.

The possible tasks for the future could be to involve a gamma-loop model described in [3] in the mechanism of control as a load-estimator and corrector structure. Furthermore it is planned to utilize both of the muscles for a MIMO-fedback linearizing control, and compare the state trajectories of the closed-loop system with trajectories recorded in the case of real movement patterns.

5. ACKNOWLEDGEMENTS

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