CONTINUOUS-TIME MODELING OF URBAN TRAFFIC

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Abstract: The present work concerns continuous-time modeling of traffic flow. The model is proposed for one lane. The general idea is to use available measurements to compute global characteristic of the whole lane. The approach models traffic flow by describing the dynamics of an artificial object, so-called congested area. The considered approach is supposed to form a basis for further modeling of roads, cross-roads and city's microregions.

Keywords: traffic control, state estimation, state prediction, macroscopic models

1. INTRODUCTION

The main motivation for modeling of traffic systems is the growing quantity of vehicles at the roads. This phenomena leads to many traffic jams at highways and in cities. Majority of modern city roads is equipped by detectors providing the information about the state of the traffic network. And many cross-roads are controlled by signal lights.
This control is often noneffective and is not able to prevent the jams. The noneffectiveness is given by the local control because the signal schemes are designed for one cross-road and do not respect each another. Thus it calls for the global optimization of the signal lights. The present detector technology is able to provide the on-line data which can be used for the on-line control optimization over some area or microregion.
The good on-line optimization is conditioned by the proposing of the good model. This text take a part in the finding this model.

1.1 Traffic flow terms

Unit vehicle, [uv] represents the "normalized" vehicle with known properties: acceleration a, deceleration d, desired speed V, length of the vehicle and reaction time of the driver T_{reak}. We use the term "vehicle" instead the unit vehicle from here onward.

Density $\rho(t)$, [uv/m] characterize the number of vehicles moving at the given part of the lane normalized per meter.

Detectors are equipments providing information about the microregion. The detectors can measure only two magnitudes: intensity and occupation.

Occupancy $O(x, t)$, [%] characterizes ratio of the detector activating time given the detector sampling time T_s: $O = T_a/T_s$.

Intensity $I(x, t)$, [uv/s] characterizes number of vehicles passing through the point x per time unit.

Signal lights are providing four signals repeating in given order: red, orange, green and yellow. The yellow and orange signal are signed by yellow light but they are defined for the terminological differentiation the red-green and green-red transition. The time of one repetition is called cycle time T_C and the activity period of particular signals is given by signal plan.

Congested area is the part of the lane, where the vehicles influence each other. We will assume that congested area is only one and it is at the downstream end of the lane. The rest of the lane is called free-flow area. The x coordinate of the border between congested and free-flow area is denoted by l.

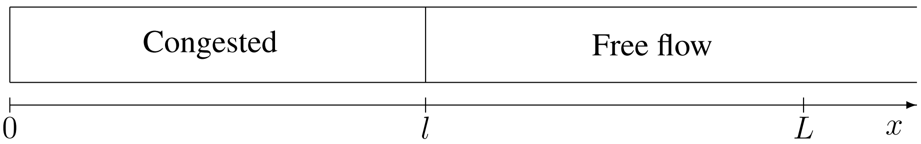

Fig. 1: Congested and free-flow areas

1.2 Assumptions

- The architecture properties of the lane are fixed and known. The position on the lane is given by coordinate $x \in [0, L]$, where L is length of the line. Zero position ($x = 0$) corresponds the stop-line (see Fig. 1).
- All vehicles in free-flow area are moving with the desired speed. And the desired speed V is assumed as equal to the maximal allowed speed.
- The signal plan of the traffic lights is known. The signal equipment has only two signs: red and green, i.e. Yellow and orange signs have zero length. And the state of signal light is characterized by function $G(t)$:

$$G(t) = \begin{cases} 1, & \text{the green signal in time } t, \\ 0, & \text{the red signal in time } t. \end{cases} \quad (1)$$

- The lane is equipped by three detectors (so-called "stop", "far-away" and "strategic") with known position and length - see Fig. 2. Note: The present model uses only the strategic detector, but the next version will use all detectors.
  For simplify the notation, the strategic detector is assumed to be at the end of the lane $x = L$.

1.3 Aim of the work

The measured variables do not characterize the situation. Thus, they cannot be useful for direct optimization. The main reason for this is that the measured variables have the space-discreet

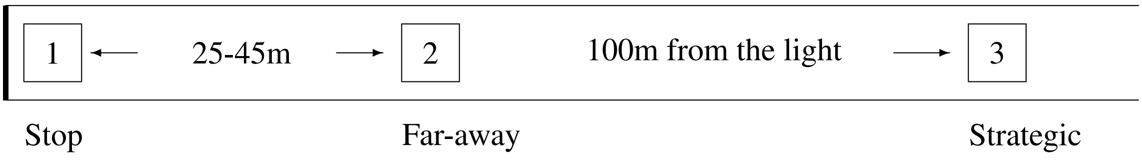

Fig. 2: Positions of detectors

character and we need the global lane characteristic.
One of several global characteristic is the length of congested area, which corresponds with the real length of queue. The proposed model is based on description of congested area and its dynamics.

2. DEVELOPMENT OF THE MODEL

This section concerns with the developing of the model. The models is based on the microscopic description of physical phenomena but it provides the transformation to the macroscopic variables.

2.1 Modeled phenomena

The congested area can be taken as an artificial object object which proposes the following phenomena.

Density, condensation and dissolution
The density ρ is the main magnitude, which influences the condensation of the congested area. From the microscopic point of view, the process of dissolution starts at the downstream end of the congested area, where the first vehicle leaves the queue - the average density is descending (dissolution) because the length of the congested area is not changing. The first car makes the hole and other vehicles are moving forward to fill it. So that the hole is propagating through the area - but in the opposite direction to the vehicles. When the hole reaches the end of the congested area, the length l is reducing - the condensation.
There are many holes because all vehicles, left the area, make their own holes. The holes propagate through the area with the constant speed c. It means that the first condensation begins after delay $\Delta T = l/c$ since the first vehicle left the lane.
We need to describe mentioned process by the macroscopic variables like density and length. Let us assume the congested area with N vehicles and length l. We obtain the average density as follows: $\rho = N/l$. And if the density is not maximal - there are holes to fill - then the length l will descend and the density will grow to the maximal possible density ρ_{Max} while the length l will descend to the minimum for the given number of vehicles $l_{min} = N/\rho_{Max}$, where ρ_{Max} is known constant.

The output speed
The output speed V_1 is related with the green signal. When the green signal switch on, the first vehicle starts to move with acceleration a. When the vehicle leaves its place, then the vehicle behind starts to move, but the second vehicle leaves the lane with a higher speed than the first one, because it has longer distance to the stop line.
From macroscopic point of view, the output speed is growing with the constant acceleration a. The drivers reaction-time take the part only by the first vehicle in the queue because the next

ones have lot of time to prepare for their starting.

End of queue
All vehicles start with acceleration a, so the last vehicle in the queue is also moving with this acceleration but starts with some delay. The speed of the last vehicle characterizes the speed of the congested end - the condensation speed - and relies at the dissolution and its propagating in the congested area.

2.2 Model equations

Despite continuous modeling used, the model variables will be calculated by computer and the final realization will be discrete. So that the following equations should be in the computer-friendly shape.

Table 1: Used notations

t	time
Δt	simulation step
x	coordinate
V	desired speed
ρ_M	maximal density
a	vehicles acceleration
d	vehicles deceleration
T_{reak}	reaction time
l	length of congested area
ρ	average density
V_3	condensation speed
I_3	upstream incoming intensity
V_1	outgoing speed
I_1	downstream outgoing intensity

Note: The indexing of variables is analogical to the detectors indexing, for example a_1 the acceleration of the speed V_1.

The outgoing speed
The outgoing speed V_1 is calculated as the speed of the accelerated moving with an acceleration $a_1(t)$. Respecting the influence of the signal lights, we obtained:

$$V_1(t + \Delta t) = G(t + \Delta t) \left(V_1(t) + \int_t^{t+\Delta t} a_1(\tau) G(\tau - T_{reak}) G(\tau) d\tau \right). \tag{2}$$

The terms have following sense: $G(t+\Delta t)$ makes the speed equal to zero, if the red sign lights. $G(\tau - T_{reak})$ introduces the reaction delay of the first driver, when the green signal switched on. This delay shifts all drivers by T_{reak}. $G(\tau)$ characterizes the end of the green signal, where the reaction time is zero.
The positive value of the speed V_1 starts the dissolution. It is useful to define the dissolution function:

$$\delta(t) = \begin{cases} 1, & V_1(t) > 0 \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

The condensation speed
The condensation speed V_3 can be expressed using the same idea as V_1 but with the green signal

function $G(t)$ replaced by dissolution function $\delta(t)$:

$$V_3(t+\Delta t) = V_3(t) + \int_t^{t+\Delta t} a_3(\tau)\delta(\tau - \frac{l(\tau)}{c})d\tau, \tag{4}$$

where the term $l(\tau)/c$ characterizes time delay needed to propagate of the holes from the downstream end of the congested area to the upstream end.

The incoming intensity
The intensity $I_3(t)$ is measured by the strategic detector, which is placed at the coordinate L. This intensity is only time-depend variable, which can be used via the following formula (Helbing *et al.*, 2005):

$$I(x,t) = I(x_0, t - \Delta\tau(x_0, x)) \tag{5}$$

where $\Delta\tau(x_0, x)$ is time needed to reach point x from x_0.
The vehicles are moving with the desired speed V until they reach the end of congested area - they slow down to the speed V_3 and join the congested area. We obtain:

$$I(x,t) = I(L, t - \Delta\tau(L,x)) = I_3(t - \Delta\tau(L,x)), \tag{6}$$

where the time-shift $\Delta\tau(L,x)$ can be calculated via formula for the moving with constant deceleration d:

$$\Delta\tau(L,x) = \begin{cases} \sqrt{\frac{2(L-x)}{d}} + \frac{L-x}{V} - \frac{(V-V_3)^2}{2dV}, & \text{if } \frac{V-V_3}{d} \leq \sqrt{\frac{2(L-x)}{d}} \\ \frac{2(L-x)}{V-V_3}, & \text{if } \frac{V-V_3}{d} > \sqrt{\frac{2(L-x)}{d}} \end{cases} \tag{7}$$

the first formula corresponds to, when the vehicle has enough distance for decelerating. The second case characterizes situation when the distance is not enough for decelerating and the deceleration is greater than d - this case is necessary in situation when the coordinate x is near to L.
Using the formula (6), (7), the number of vehicles N can be expressed:

$$N(t+\Delta t) = N(t) + \int_t^{t+\Delta t} I(l,\tau)d\tau - \int_t^{t+\Delta t} V_1(\tau)\rho(\tau)d\tau \tag{8}$$

where the second term expresses the incoming number and the third one the outgoing number of vehicles.

The length and density
The length of congested area l depends on the condensation speed $V3$ and incoming intensity I_3:

$$l(t+\Delta t) = l(t) + \int_t^{t+\Delta t} \frac{I(l,\tau)}{\rho(\tau)}d\tau - \int_t^{t+\Delta t} V_3(\tau)d\tau \tag{9}$$

where the second term characterizes the outgoing vehicles and the third one the incoming vehicles (by analogy with (8)).
The variable $\rho(\tau)$ is the average density at time τ and the term $I(l,\tau)/\rho(\tau)$ characterizes the queue enlargement induced by the incoming intensity. The condensation is characterized by the last integral.
The next time average density can be calculated via formula:

$$\rho(t+\Delta t) = \frac{N(t+\Delta t)}{l(t+\Delta t)} \tag{10}$$

2.3 Assumptions and conditions

There are several conditions, which must be fulfilled to easy model implementation:

Maximal speed is the easiest assumption - the speed V_1 cannot step over the maximal outgoing speed $V_{1,Max}$ which is given by architecture properties of the cross-road. This restriction influences the values of the acceleration $a_1(t)$ (see (2)) by making it zero, when the speed V_1 reaches the maximal value. By analogy with that, the speed V_3 is restricted by the desired speed V and influences the acceleration $a_3(t)$ in equation (4).

Red signal makes the speed V_1 equal to zero. This exception is included in the first term of the equation (2).

Length of congested area l cannot leave the interval $[0, L]$. So that the outgoing and incoming intensity terms in the equations (8) and (9) must be changed to keep the value l in the mentioned interval.

Minimal density behavior is one of the disadvantages of the model. If the density decreases under the value ρ_{min}, then the model "accelerates" the vehicles in the congested area. The reason is the use of the average density which supposes uniform distribution of vehicles in the congested area.
Use follows to solve this: $\rho \leq \rho_{min} \quad \Rightarrow \quad \rho \leftarrow 0$

Output intensity cannot reach value higher than number of cars in the congested area:

$$I_1(t) \leq \rho(t)l(t) \tag{11}$$

This condition is related with the mentioned minimal length restriction.

3. PRELIMINARY TEST OF THE MODEL

This section deals with the the preliminary tests of the model. The comparative testing using the AIMSUN simulation program (*GETRAM: AIMSUN version 4.2 User manual, 2004)* was used.

3.1 Model implementation

The developed model was implemented as the MATLAB function. The simulation step was set to value $\Delta t = 1$ s. The following assumptions were adapted:

- The desired speed $V = 50$ km/hour was used as the input and maximal output speed.
- The accelerations $a_1(t)$ and $a_3(t)$ are defined by:

$$a_i(t) = \begin{cases} 3 \text{ m/s}^2 & V_i(t) < V \\ 0 \text{ m/s}^2 & V_i(t) = V \end{cases} \qquad \text{where } i \in \{1, 3\} \tag{12}$$

- The used value of deceleration is $d = 4$ m/s^2.
- Effective vehicle length equals to 5 meters and corresponding value of the maximal density $\rho_{Max} = 0.2$ uv/m.

- Drivers reaction time equals 1.83 s.
- The speed c defined by function: $c(t) = \frac{1}{T_{reak}\rho(t)}$. This dependency is calculated at every simulation step, because the propagation of the dissolution is function of the density.

The AIMSUN model has the following characteristics: one lane ($L = 100$ m) with three detectors, one signal equipment and one class of vehicles. This setup has only few variables can be set: signal lights and the input intensity.
The set of simulations each of length 3600 s is prepared to illustrate the physical behavior of the model:
Table 2: Set of simulations

NR.	Green time [s]	Cycle time [s]	Intensity [uv/hour]	Note
1	15	75	100	Under-saturated
2	15	75	200	Under-saturated
3	15	75	300	Saturated
4	15	75	400	Saturated
5	15	75	500	Over-saturated
6	15	75	600	Over-saturated
7	15	75	700	Over-saturated
8	20	75	300	Under-saturated
9	30	75	300	Under-saturated
10	40	75	300	Under-saturated

The saturation means the equilibrium between input and output, when the congested area is not empty at the end of the green signal and the length of congested area grows slow. The saturated (it means neither under-saturated nor over-saturated) situation is very unreal because the length of congested is either zero or the maximum and the change between this takes about 5 minutes and not whole hour.

3.2 Model variables

The length of congested area l is the most important output from the model. The simulation set number 2 is used as an example of the length modeling (see Figure 3). This case characterize the under-saturation on the lane when the length l does not reach the maximum - the length of lane. The disadvantage which can be seen from the Figure 3 is the time shift between model and AIMSUN, which is one or two seconds - the model has the faster dynamics than AIMSUN one.

Density and speeds: An example of the density and modeled speeds behavior is shown at Figure 4. There was used the model output from the over-saturated simulation set number 7.
The density ρ is descending (see $t = 600$ s) and the output speed V_1 starts to grow at the start of the green signal ant the dissolution also propagates. The propagation takes about 20 s and then the speed V_3 starts growing.
The behavior of these variables seems be good but the descending of the speed V_3 is to steep to real deceleration - this disadvantage should be removed in the further version of the model.

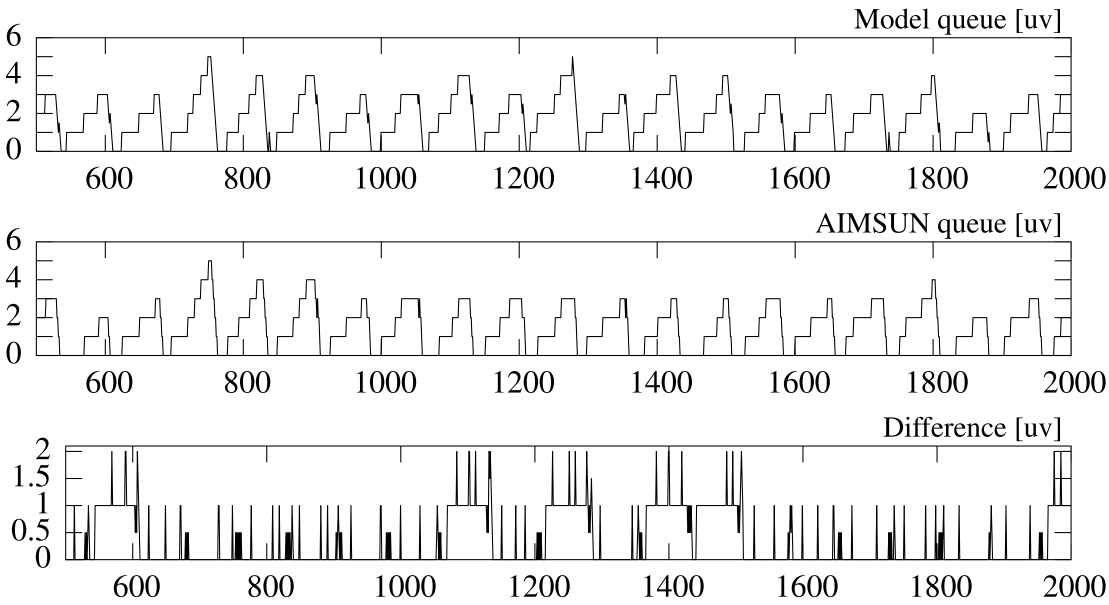

Fig. 3: Length of congested area

3.3 The results

The quality results compared with AIMSUN data is expressed by (results see Table 1):

$$Q = \frac{100\%}{\max_{t\in\{1\dots3600\}} l_{real}(t)} \frac{\sum_{t\in\{1\dots3600\}} |l(t) - l_{real}(t)|}{3600} \tag{13}$$

where l is the modeled length of congested area and the l_{real} are AIMSUN values.

Table 3: The results of simulations

NR.	Green time [s]	Cycle time [s]	Intensity [uv/hour]	Q[%]	Note
1	15	75	100	3.58	Under-saturated
2	15	75	200	6.91	Under-saturated
3	15	75	300	31.74	Saturated
4	15	75	400	29.56	Saturated
5	15	75	500	16.71	Over-saturated
6	15	75	600	16.90	Over-saturated
7	15	75	700	13.95	Over-saturated
8	20	75	300	7.59	Under-saturated
9	30	75	300	8.27	Under-saturated
10	40	75	300	9.86	Under-saturated

It is easy to see that the model provides good results in under-saturated situations, acceptable results in over-saturated situation and bad results in saturated situations.

4. CONCLUSION

The continuous-time modeling of the lane was proposed. The set of simulations and their results illustrate the model potential.
The results of the simulations are acceptable because model provides good results in the typical under- and over-saturated situations. This property qualifies the model for further use as the

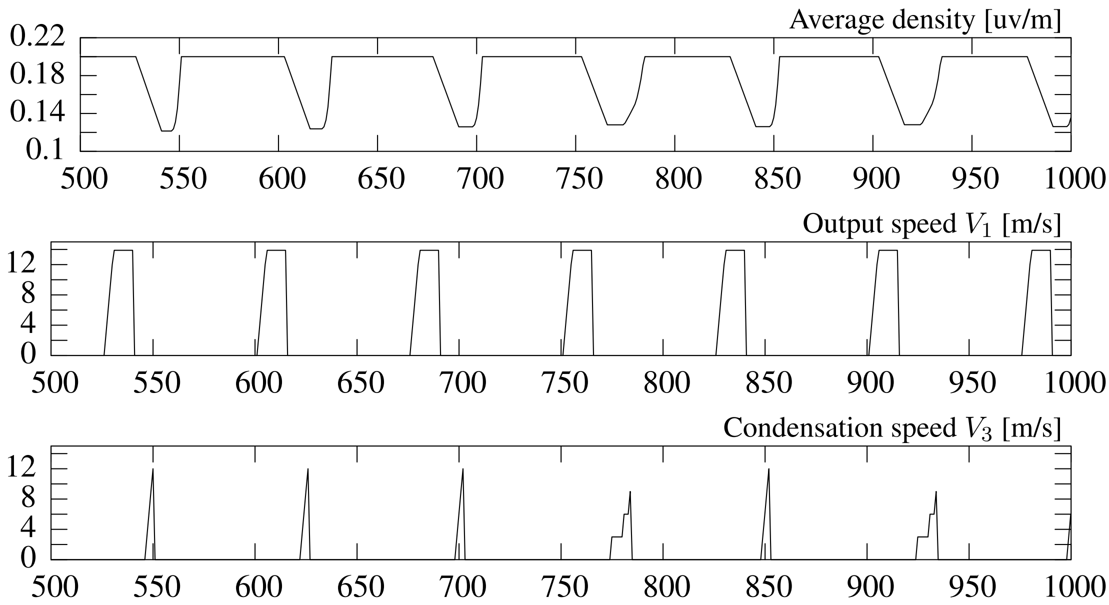

Fig. 4: Modeled variables

basic structure for designing the model of the microregion including multi-lane roads and cross-roads. New relationships must be included into the model for preparation to the connection in the microregion model - especially variables connected with the propagation of information from one lane to another one.
The used simulations were restricted by many assumptions. Here is the list of the assumption which should be generalized in the further research:

- The average density should not be used. The space-continuous density gives better results - despite the higher complexity of the solution.
- The yellow and orange signal should be used. This feature influence the driver reaction times.

To reach this features, the model should be enriched about the information from another detectors of neighboring cross-roads because proposed model uses only the strategic detector information.

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