Blur and Affine Moment Invariants

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Abstract

The paper is devoted to the recognition of objects and patterns deformed by imaging geometry as well as by unknown blurring. We introduce a new class of features invariant simultaneously to blurring by a centrosymmetric PSF and to affine transformation. As we prove in the paper, they can be constructed by combining affine moment invariants and blur invariants derived earlier. Combined invariants allow to recognize objects in the degraded scene without any restoration and geometric normalization.

1. Introduction

Recognition of objects and patterns that are deformed in various ways has been a goal of much recent research. The degradations (geometric as well as radiometric) are introduced during the image acquisition process by such factors as imaging geometry, illumination changes, wrong focus, lens aberration, systematic and random sensor errors, object occlusion, etc. Finding a set of invariant descriptors is a key step to recognizing degraded objects regardless of the particular deformations. Many papers have been devoted to the invariants to transform of spatial coordinates, like rigid-body, affine, and projective transform (see [1], [2] for a survey and other references). Much less attention has been paid to invariants with respect to changes of the image intensity function (we call them radiometric invariants) and to combined radiometric-geometric invariants.

An important class of radiometric degradations we are faced with often in practice is image blurring. Blurring can be caused by camera defocus, atmospheric turbulence, vibrations, and by sensor or/and scene motion, to name a few. If the scene is flat, blurring can be usually described by a convolution $g(x, y) = (f * h)(x, y)$, where f is an original (ideal) image, g is an acquired image and h is a point spread function (PSF) of the imaging system. Since in most practical tasks the PSF is unknown, having the invariants to convolution is of prime importance when recognizing objects in a blurred scene. A pioneer work on this field was done by Flusser and Suk [3] who derived invariants to convolution with an arbitrary centrosymmetric PSF. From the geometric point of view, their descriptors were invariant to translation only. Despite of this, the invariants have found several successful applications, e.g. in face recognition on defocused photographs [4], in blurred digit recognition [5], and in focus/defocus quantitative measurement [6]. A significant improvement motivated by a problem of registration of blurred images was made by Flusser and Zitová. They introduced so-called combined blur-rotation invariants [7] and reported their successful usage in satellite image registration [8].

However, in a real world, the imaging geometry is projective rather than rigid-body. If the scene is flat and the camera far from the object in comparison to its size, the projective transform can be well approximated by an affine transform. Thus, having combined affine-blur invariants is in great demand.

2. Recalling the Theory of the Invariants

In this Section we introduce some basic terms and briefly recall the theorems on the invariants which will be used later in this contribution.

2.1. Basic Terms

Definition 1: By *image function* (or *image*) we understand any real function $f(x, y) \in L_1$ having a bounded support and a nonzero integral.

Definition 2: *Central moment* $\mu_{pq}^{(f)}$ of order $(p + q)$ of the

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image $f(x,y)$ is defined as

$$\mu_{pq}^{(f)} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x-x_c)^p (y-y_c)^q f(x,y)dxdy, \qquad (1)$$

where the coordinates (x_c, y_c) denote the centroid of $f(x,y)$.

Definition 3: *Affine transform* is a transform of spatial coordinates (x,y) into (u,v) defined by equations

$$\begin{aligned} u &= a_0 + a_1 x + a_2 y, \\ v &= b_0 + b_1 x + b_2 y. \end{aligned} \qquad (2)$$

Definition 4: Every PSF mentioned in this contribution is assumed to be *centrosymmetric* and *energy-preserving,* i.e.

$$h(x,y) = h(-x,-y),$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(x,y)dxdy = 1.$$

The assumption of centrosymmetry is not a significant limitation of practical utilization of the method. Most real sensors and imaging systems, both optical and non-optical ones, have the PSF with certain degree of symmetry. In many cases they have even higher symmetry than central one, such as axial or radial symmetry. Thus, the central symmetry is general enough to handle almost all practical situations.

2.2. Blur Invariants

In our earlier paper [3], the following theorem of blur invariants was proven.

Theorem 1: Let $f(x,y)$ be an image function. Let us define the following function $C^{(f)} : \mathcal{N}_0 \times \mathcal{N}_0 \rightarrow \mathcal{R}$.

If $(p+q)$ is even then

$$C(p,q)^{(f)} = 0.$$

If $(p+q)$ is odd then

$$C(p,q)^{(f)} = \mu_{pq}^{(f)} - \frac{1}{\mu_{00}^{(f)}} \sum_{n=0}^{p} \sum_{\substack{m=0 \\ 0<n+m<p+q}}^{q} \binom{p}{n}\binom{q}{m} C(p-n,q-m)^{(f)} \cdot \mu_{nm}^{(f)}. \qquad (3)$$

Then $C(p,q)$ is invariant to convolution with any centrosymmetric function $h(x,y)$, i.e.

$$C(p,q)^{(f)} = C(p,q)^{(f*h)}$$

for any p and q.

2.3. Affine Moment Invariants

Affine moment invariants (AMI's) were proposed by Reiss [9] and Flusser and Suk [10]. They originate from the classical theory of algebraic invariants The fundamental theorem, which was used for the derivation of the explicit forms of the invariants, can be formulated as follows.

Theorem 2: If the binary form of order p has an algebraic invariant of weight w and order k

$$I(a'_{p,0}, \cdots, a'_{0,p}) = \triangle^w I(a_{p,0}, \cdots, a_{0,p})$$

($\triangle$ denotes the determinant of the respective affine transform) then the moments of order p have the same invariant but with the additional factor $|J|^k$:

$$I(\mu'_{p0}, \cdots, \mu'_{0p}) = \triangle^w |J|^k I(\mu_{p0}, \cdots, \mu_{0p}),$$

where $|J|$ is the absolute value of the Jacobian.

We refer to [10], [9] for the proof of Theorem 2 and for a deeper explanation of the theory of the AMI's.

3. Combined Blur and Affine Invariants

By combined blur-affine invariants (CBAI's) we understand any functional defined on the set of image functions whose value does not change if the image function is convolved with a centrosymmetric PSF and transformed by an affine transform. The following theorem not only guarantees the existence of the CBAI's but also provides an explicit algorithm how to construct them.

Theorem 3: Let $I(\mu_{00}, \cdots, \mu_{PQ})$ be an affine moment invariant. Then $I(C(0,0), \cdots, C(P,Q))$, where $C(0,0) = \mu_{00}$ and all other blur invariants $C(p,q)$ are defined by Theorem 1, is a combined blur-affine invariant.

Proof: Since $I(C(0,0), \cdots, C(P,Q))$ is a function of blur invariants $C(p,q)$ only, it is also a blur invariant. To prove its invariance to affine transform, it is sufficient to prove that partial derivatives with respect to each of the parameters $a \in \{a_0, a_1, a_2, b_0, b_1, b_2\}$ of the affine transform equal zero. Since it holds for $I(\mu_{00}, \cdots, \mu_{PQ})$, it is sufficient to prove that this property is preserved when substituting $C(p,q)$ for μ_{pq}. Instead of doing that directly for the general affine transform, we can decompose (2) into six one-parametric transformations (horizontal translation, vertical translation, uniform scaling, stretching, horizontal skewing and vertical skewing) and prove the assertion separately for each of them, which is much easier.

To express the simplest CBAI's in explicit forms, we take five affine moment invariants of the 3rd and 5th orders:

$$\begin{aligned} I_1 = {} & (\mu_{30}^2\mu_{03}^2 - 6\mu_{30}\mu_{21}\mu_{12}\mu_{03} + 4\mu_{30}\mu_{12}^3 + 4\mu_{21}^3\mu_{03} \\ & -3\mu_{21}^2\mu_{12}^2)/\mu_{00}^{10}, \end{aligned}$$

$$
\begin{aligned}
I_2 \quad = \quad & (\mu_{50}^2\mu_{05}^2 - 10\mu_{50}\mu_{41}\mu_{14}\mu_{05} + 4\mu_{50}\mu_{32}\mu_{23}\mu_{05} \\
& +16\mu_{50}\mu_{32}\mu_{14}^2 - 12\mu_{50}\mu_{23}^2\mu_{14} + 16\mu_{41}^2\mu_{23}\mu_{05} \\
& +9\mu_{41}^2\mu_{14}^2 - 12\mu_{41}\mu_{32}^2\mu_{05} - 76\mu_{41}\mu_{32}\mu_{23}\mu_{14} \\
& +48\mu_{41}\mu_{23}^3 + 48\mu_{32}^3\mu_{14} - 32\mu_{32}^2\mu_{23}^2)/\mu_{00}^{14},
\end{aligned}
$$

$$
\begin{aligned}
I_3 \quad = \quad & (\mu_{30}^2\mu_{12}\mu_{05} - \mu_{30}^2\mu_{03}\mu_{14} - \mu_{30}\mu_{21}^2\mu_{05} \\
& -2\mu_{30}\mu_{21}\mu_{12}\mu_{14} + 4\mu_{30}\mu_{21}\mu_{03}\mu_{23} \\
& +2\mu_{30}\mu_{12}^2\mu_{23} - 4\mu_{30}\mu_{12}\mu_{03}\mu_{32} + \mu_{30}\mu_{03}^2\mu_{41} \\
& +3\mu_{21}^3\mu_{14} - 6\mu_{21}^2\mu_{12}\mu_{23} - 2\mu_{21}^2\mu_{03}\mu_{32} \\
& +6\mu_{21}\mu_{12}^2\mu_{32} + 2\mu_{21}\mu_{12}\mu_{03}\mu_{41} - \mu_{21}\mu_{03}^2\mu_{50} \\
& -3\mu_{12}^3\mu_{41} + \mu_{12}^2\mu_{03}\mu_{50})/\mu_{00}^{11},
\end{aligned}
$$

$$
\begin{aligned}
I_4 \quad = \quad & (2\mu_{30}\mu_{12}\mu_{41}\mu_{05} - 8\mu_{30}\mu_{12}\mu_{32}\mu_{14} + 6\mu_{30}\mu_{12}\mu_{23}^2 \\
& -\mu_{30}\mu_{03}\mu_{50}\mu_{05} + 3\mu_{30}\mu_{03}\mu_{41}\mu_{14} \\
& -2\mu_{30}\mu_{03}\mu_{32}\mu_{23} - 2\mu_{21}^2\mu_{41}\mu_{05} + 8\mu_{21}^2\mu_{32}\mu_{14} \\
& -6\mu_{21}^2\mu_{23}^2 + \mu_{21}\mu_{12}\mu_{50}\mu_{05} - 3\mu_{21}\mu_{12}\mu_{41}\mu_{14} \\
& +2\mu_{21}\mu_{12}\mu_{32}\mu_{23} + 2\mu_{21}\mu_{03}\mu_{50}\mu_{14} \\
& -8\mu_{21}\mu_{03}\mu_{41}\mu_{23} + 6\mu_{21}\mu_{03}\mu_{32}^2 - 2\mu_{12}^2\mu_{50}\mu_{14} \\
& +8\mu_{12}^2\mu_{41}\mu_{23} - 6\mu_{12}^2\mu_{32}^2)/\mu_{00}^{12},
\end{aligned}
$$

$$
\begin{aligned}
I_5 \quad = \quad & (\mu_{30}\mu_{41}\mu_{23}\mu_{05} - \mu_{30}\mu_{41}\mu_{14}^2 - \mu_{30}\mu_{32}^2\mu_{05} \\
& +2\mu_{30}\mu_{32}\mu_{23}\mu_{14} - \mu_{30}\mu_{23}^3 - \mu_{21}\mu_{50}\mu_{23}\mu_{05} \\
& +\mu_{21}\mu_{50}\mu_{14}^2 + \mu_{21}\mu_{41}\mu_{32}\mu_{05} - \mu_{21}\mu_{41}\mu_{23}\mu_{14} \\
& -\mu_{21}\mu_{32}^2\mu_{14} + \mu_{21}\mu_{32}\mu_{23}^2 + \mu_{12}\mu_{50}\mu_{32}\mu_{05} \\
& -\mu_{12}\mu_{50}\mu_{23}\mu_{14} - \mu_{12}\mu_{41}^2\mu_{05} + \mu_{12}\mu_{41}\mu_{32}\mu_{14} \\
& +\mu_{12}\mu_{41}\mu_{23}^2 - \mu_{12}\mu_{32}^2\mu_{23} - \mu_{03}\mu_{50}\mu_{32}\mu_{14} \\
& +\mu_{03}\mu_{50}\mu_{23}^2 + \mu_{03}\mu_{41}^2\mu_{14} - 2\mu_{03}\mu_{41}\mu_{32}\mu_{23} \\
& +\mu_{03}\mu_{32}^3)/\mu_{00}^{13}.
\end{aligned}
$$

and substitute $C(p,q)$ for μ_{pq}. Since the even-order $C(p,q)$'s equal zero by definition, only those AMI's composed from odd-order moments are meaningful for this purpose. The explicit forms of $C(p,q)$'s can be found in [3] or derived directly from Theorem 1. Those needed for substitution into the above AMI's are listed below.

$$C(3,0) = \mu_{30}, C(2,1) = \mu_{21}, C(1,2) = \mu_{12}, C(0,3) = \mu_{03},$$

$$C(5,0) = \mu_{50} - \frac{10\mu_{30}\mu_{20}}{\mu_{00}},$$

$$C(4,1) = \mu_{41} - \frac{2}{\mu_{00}}(3\mu_{21}\mu_{20} + 2\mu_{30}\mu_{11}),$$

$$C(3,2) = \mu_{32} - \frac{1}{\mu_{00}}(3\mu_{12}\mu_{20} + \mu_{30}\mu_{02} + 6\mu_{21}\mu_{11}),$$

$$C(2,3) = \mu_{23} - \frac{1}{\mu_{00}}(3\mu_{21}\mu_{02} + \mu_{03}\mu_{20} + 6\mu_{12}\mu_{11}),$$

$$C(1,4) = \mu_{14} - \frac{2}{\mu_{00}}(3\mu_{12}\mu_{02} + 2\mu_{03}\mu_{11}),$$

$$C(0,5) = \mu_{05} - \frac{10\mu_{03}\mu_{02}}{\mu_{00}}.$$

4. Numerical Experiment

We have done numerous experiments demonstrated the numerical properties, invariance and discrimination power of the combined invariants. We have used artificial data (where the image blurring and the deformation were computer-generated) as well as real data, where the blurring was caused by wrong focus of the camera and the deformation came from perspective geometry. Here, we present one of the real experiments.

The picture of a comb was captured by a digital still-frame camera in three positions: orthogonal view, weak perspective view, and strong projective view. Although the obtained image deformation is projective (i.e. nonlinear), it can be approximated by affine transform (2). In each case, the distance of the camera from the object was 50 cm. From each position we took three images gradually focused on 50 cm (in-focus image), 10 cm (weak defocusing) and 0.2 cm (strong defocusing), see Fig. 1. The values of five combined invariants from the previous section were computed for all combinations of blur and projection. To compare the values with that of another object, a photograph of the Earth was captured, see Fig. 2. The images were normalized to the same contrast and the background was segmented and zeroed. The values of the invariants are shown in Table 1.

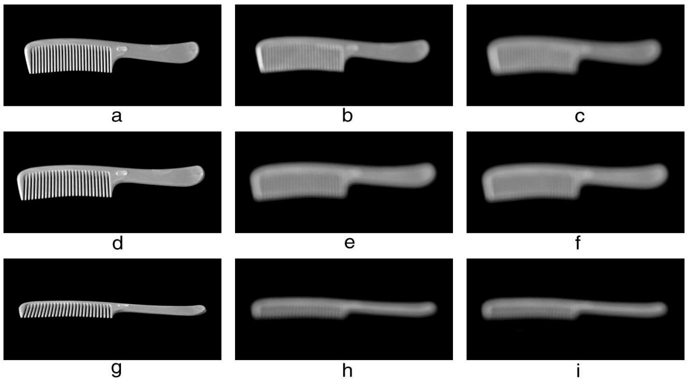

Figure 1. The object "Comb", (a) in focus, (b) weak defocus, (c) strong defocus, (d) in focus and weak perspective deformation, (e) weak defocus and weak perspective deformation, (f) strong defocus and weak perspective deformation, (g) in focus and strong perspective deformation, (h) weak defocus and strong perspective deformation and (i) strong defocus and strong perspective deformation.

In an ideal case, the invariants should be constant for all comb images. However, we can observe that they vary. The changes occurring due to defocus are slight, while the

Object	Deformation	Focus [cm]	$I_1[10^{-6}]$	$I_2[10^{-7}]$	$I_3[10^{-8}]$	$I_4[10^{-7}]$	$I_5[10^{-9}]$	$I_6[10^{-7}]$
Comb	-	50	-1.391	-1.488	2.084	4.775	-1.266	-2.939
Comb	-	10	-1.291	-1.531	2.301	4.533	-1.748	-4.983
Comb	-	0.2	-1.221	-1.514	2.228	4.377	-1.747	-5.475
Comb	weak	50	-1.175	-1.057	2.426	3.762	-1.586	-1.098
Comb	weak	10	-1.085	-1.174	2.565	3.715	-2.141	-3.094
Comb	weak	0.2	-1.067	-1.138	2.554	3.632	-2.154	-2.928
Comb	strong	50	-0.8072	-0.2247	3.426	1.925	-1.493	3.070
Comb	strong	10	-0.8642	-0.2231	4.681	2.225	-1.516	9.309
Comb	strong	0.2	-0.8068	-0.03673	6.182	2.001	-4.480	9.578
Earth	-	50	0.01390	0.1895	1.979	-0.1768	-0.6951	2.341

Table 1. The values of the combined invariants

Figure 2. The object "Earth".

changes due to the perspective projection are a bit stronger. This is caused namely by nonlinear term in the perspective transform to which the invariants are sensitive. Fortunately the difference between the values of different objects are much greater than the intra-class variations.

It is worth noting that in computer generated experiments the invariant values stay really constant regardless of the degree of deformation and blurring.

5. Conclusion

The paper is devoted to the image features which are invariant simultaneously to blurring by a filter with centrally symmetric PSF and to the affine transformation. The major theoretical result of this work is Theorem 3, showing that the combined invariants can be constructed by substituting blur invariants into the affine moment invariants.

Practical applications of the new invariants may be found in object recognition in blurred and noisy environment, in template matching, image registration, etc.

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