RECOGNITION OF IMAGES DEGRADED BY LINEAR MOTION BLUR WITHOUT RESTORATION

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Abstract

The paper is devoted to the feature-based description of images degraded by linear motion blur. The proposed features are invariant with respect to motion velocity, are based on image moments and are calculated directly from the blurred image. In that way, we are able to describe the original image without the PSF identification and image restoration. In many applications (such as in image recognition against a database) our approach is much more effective than the traditional "blind-restoration" one.

The derivation of the motion blur invariants is a major theoretical result of the paper.

Numerical experiments are presented to illustrate the utilization of the invariants for blurred image description. Stability of the invariants with respect to additive random noise is also discussed and is shown to be sufficiently high. Finally, another set of features which are invariant not only to motion velocity but also to motion direction is introduced.

Index Terms: Blurred image, linear imaging system, motion blur, image moments, blur invariants

1 Introduction

The images of a moving scene or the images of a static scene taken by moving sensor represent the original objects mostly in an unsatisfactory manner. Due to the relative motion of the sensor and the scene and finite exposure time, such images are degraded by well-known degradation factor called *motion blur*.

Widely accepted standard linear model [1] describes the imaging process by convolution of an unknown original (or ideal) image $f(x,y)$ with space-invariant point spread function (psf) $h(x,y)$:

$$g(x,y) = (f * h)(x,y) \tag{1}$$

where $g(x,y)$ represents the observed image. The PSF $h(x,y)$ describes the imaging system and it is supposed to be unknown.

In case of linear horizontal motion, $h(x,y)$ has the following form:

$$h(x,y) = \begin{cases} \frac{1}{vt}\delta(y) & \iff 0 \le x \le vt \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

(v is the motion velocity, t is the exposure and δ is Dirac function).

Classical "blind-restoration" approach to dealing with motion blurred images consists of two following steps:

- Estimation of the PSF $h(x,y)$, i.e. estimation of the constant $c = vt$;
- Estimation of the ideal image $f(x,y)$ via restoration of the blurred image $g(x,y)$.

Both these steps have been dealt with extensively in literature during last two decades.

One group of methods for PSF identification is based on the investigation of zero patterns in the frequency domain [2]-[4] or spike patterns in the cepstral domain [5]. It is well-known [11] that there is an infinite number

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of paralell lines of zeros in Fourier spectrum of a motion blurred image, which are perpendicular to the motion direction. Knowing the distance between those lines, we can estimate the motion velocity.

Another group of methods is based on modeling of the image by a stochastic process. The original image is modeled as an AR process and the blur as a MA process. The blurred image is then modeled as an ARMA process and the MA process identified by this model is considered as a description of the PSF. In this way the problem of PSF estimation is transformed onto the problem of determining the parameters of an ARMA model [6]-[10].

After the PSF has been identified, the original image can be estimated via restoration of the blurred image by inverse filter, Wiener filter or by any other similar technique (see [1] and [11] for a survey).

Generally speaking, the above mentioned approach to image restoration is very complicated and time-consuming. In many cases, we do not need to know the original image itself, but we have only to know some representation of it (a typical example is a recognition of a blurred image against a database). However, such a representation should be independent of the motion velocity and direction and should describe really the original image, not the degraded one. In this paper, we introduce a set of features for image description which are invariant to motion blur (that means the feature values of $g(x,y)$ do not depend on the motion characteristics and they are the same as the feature values of $f(x,y)$). Image recognition may be then accomplished via classification in the feature space. In this way, we get rid of the necessity of the PSF identification and image restoration.

Motion blur invariant features introduced in this paper are based on image moments. In Section 2 we deal with ordinary and central moments of a blurred image and we express them as functions of moments of an ideal image and the PSF. Sections 3 and 4 perform the major part of the paper. An original algorithm for invariants derivation is presented and the invariants up to the 7th order are shown in explicit form. To demonstrate the performance of the invariants, numerical experiment is presented in Section 5. Section 6 is devoted to the invariant features in case of non-horizontal motion. Basic properties of the invariants (stability, uniqueness and efficiency) are briefly discussed in Section 7.

2 Blurred Image Moments

The two-dimensional $(p+q)$th order *ordinary moment* $m_{pq}^{(f)}$ of image $f(x,y)$ is defined by the integral

$$m_{pq}^{(f)} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^p y^q f(x,y)\,dx\,dy. \tag{3}$$

Point $(x_t^{(f)}, y_t^{(f)})$ given by the relations

$$x_t^{(f)} = \frac{m_{10}^{(f)}}{m_{00}^{(f)}},$$

$$y_t^{(f)} = \frac{m_{01}^{(f)}}{m_{00}^{(f)}}$$

is called *center of gravity* or *centroid* of image $f(x,y)$. The $(p+q)$th order *central moment* $\mu_{pq}^{(f)}$ is then defined as

$$\mu_{pq}^{(f)} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x - x_t^{(f)})^p (y - y_t^{(f)})^q f(x,y)\,dx\,dy. \tag{4}$$

The following two theorems describe how to express moments of the blurred image in terms of moments of the original image and the PSF.

Theorem 1: Let $f(x,y)$ and $h(x,y)$ be piecewise continuous functions which are non-zero only on bounded supports. Let $g(x,y)$ be given by the convolution

$$g(x,y) = (f * h)(x,y).$$

Then the relation

$$m_{pq}^{(g)} = \sum_{k=0}^{p}\sum_{j=0}^{q} \binom{p}{k}\binom{q}{j} m_{kj}^{(f)} m_{p-k,q-j}^{(h)}$$

holds for every p and q.

Proof:

$$m_{pq}^{(g)} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^p y^q g(x,y)\,dx\,dy = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^p y^q (f*h)(x,y)\,dx\,dy =$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^p y^q (\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(a,b) f(x-a, y-b) da db) dx dy =$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(a,b) (\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^p y^q f(x-a, y-b) dx dy) da db =$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(a,b) (\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x+a)^p (y+b)^q f(x,y) dx dy) da db =$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(a,b) (\sum_{k=0}^{p}\sum_{j=0}^{q} \binom{p}{k}\binom{q}{j} a^{p-k} b^{q-j} m_{kj}^{(f)}) da db = \sum_{k=0}^{p}\sum_{j=0}^{q} \binom{p}{k}\binom{q}{j} m_{kj}^{(f)} m_{p-k,q-j}^{(h)} \qquad \square$$

Lemma 1: Let $f(x,y)$, $g(x,y)$ and $h(x,y)$ be the functions from Theorem 1. Then the centroid of $g(x,y)$ is given by the sum of centroids of $f(x,y)$ and $h(x,y)$:

$$(x_t^{(g)}, y_t^{(g)}) = (x_t^{(f)} + x_t^{(h)}, y_t^{(f)} + y_t^{(h)}).$$

Proof:

$$x_t^{(g)} = \frac{m_{10}^{(g)}}{m_{00}^{(g)}} = \frac{m_{10}^{(f)} m_{00}^{(h)} + m_{00}^{(f)} m_{10}^{(h)}}{m_{00}^{(f)} m_{00}^{(h)}} = \frac{m_{10}^{(f)}}{m_{00}^{(f)}} + \frac{m_{10}^{(h)}}{m_{00}^{(h)}} = x_t^{(f)} + x_t^{(h)} \qquad \square$$

The proof for $y_t^{(g)}$ is similar.

Theorem 2: Let all assumptions of Theorem 1 be valid. Then the relation

$$\mu_{pq}^{(g)} = \sum_{k=0}^{p}\sum_{j=0}^{q} \binom{p}{k}\binom{q}{j} \mu_{kj}^{(f)} \mu_{p-k,q-j}^{(h)}$$

holds for every p and q.

Proof:

$$\mu_{pq}^{(g)} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x - x_t^{(g)})^p (y - y_t^{(g)})^q g(x,y) dx dy =$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x - x_t^{(f)} - x_t^{(h)})^p (y - y_t^{(f)} - y_t^{(h)})^q (f * h)(x,y) dx dy =$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x - x_t^{(f)} - x_t^{(h)})^p (y - y_t^{(f)} - y_t^{(h)})^q (\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(a,b) f(x-a, y-b) da db) dx dy =$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(a,b) (\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x - x_t^{(f)} - x_t^{(h)})^p (y - y_t^{(f)} - y_t^{(h)})^q f(x-a, y-b) dx dy) da db =$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(a,b) (\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x - x_t^{(f)} + a - x_t^{(h)})^p (y - y_t^{(f)} + b - y_t^{(h)})^q f(x,y) dx dy) da db =$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(a,b) (\sum_{k=0}^{p}\sum_{j=0}^{q} \binom{p}{k}\binom{q}{j} (a - x_t^{(h)})^{p-k} (b - y_t^{(h)})^{q-j} \mu_{kj}^{(f)}) da db = \sum_{k=0}^{p}\sum_{j=0}^{q} \binom{p}{k}\binom{q}{j} \mu_{kj}^{(f)} \mu_{p-k,q-j}^{(h)} \qquad \square$$

Lemma 2: Let $h(x,y)$ be given by relation (2). Then

- $\mu_{pq}^{(h)} = 0$ for every $q \neq 0$;
- if p is odd, then $\mu_{p0}^{(h)} = 0$.

The proof of Lemma 2 is straightforward.

Using Lemma 2 and Theorem 2, we get the following.

Lemma 3: Let

$$g(x,y) = (f * h)(x,y)$$

and let $h(x,y)$ be given by relation (2). Then

$$\mu_{pq}^{(g)} = \sum_{k=0}^{[p/2]} \binom{p}{2k} \mu_{p-2k,q}^{(f)} \mu_{2k,0}^{(h)}.$$

Lemma 4: Let $h(x,y)$ be given by relation (2). Then

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(x,y)dxdy = \mu_{00}^{(h)} = 1.$$

That means motion blur preserves global energy of the image.

3 Derivation of the Motion Blur Invariants

Motion blur invariants are supposed to be functions of image central moments, i.e. they are supposed to have a form $M = M(\mu_{00}, \mu_{20}, \ldots, \mu_{pq})$. By the order r of invariant M we understand the order of the highest moment μ_{pq}, i.e. $r = p + q$.

Derivation of the invariants up to the 3rd order is almost trivial. It is quite easy to prove by means of Lemma 3 that $\mu_{00}, \mu_{11}, \mu_{02}, \mu_{12}, \mu_{21}, \mu_{03}$ and μ_{30} are invariant with respect to linear motion blur.

We propose the following algorithm for the construction of motion blur invariants of order $r > 3$.

1. Let $r > 3$ be the order of desired invariant M. Let μ_{pq} be any central moment of order r (except μ_{r0} if r is even). Then we start with settings

$$K = [p/2] - 1$$

(symbol $[x]$ denotes integer part of x) and

$$I_0 = \mu_{pq}.$$

2. **for** $n = 0$ **to** K
Define D_n as

$$D_n = I_n^{(g)} - I_n^{(f)}.$$

D_n has the form

$$D_n = F_n(\mu^{(f)})\mu_{a_n0}^{(h)} + R_n(\mu^{(f)}, \mu^{(h)})$$

where F_n is a function of central moments of image $f(x,y)$ only and $\mu_{a_n0}^{(h)}$ is a central moment of the highest order of $h(x,y)$. No moment of $h(x,y)$ of the same order is contained in $R_n(\mu^{(f)}, \mu^{(h)})$. Moments of $g(x,y)$ were evaluated by means of Lemma 3. It holds

$$a_n = 2(K - n + 1).$$

Due to the properties of $h(x,y)$, a_n is even for every n. Define I_{n+1} as

$$I_{n+1} = I_n - \frac{1}{\mu_{00}} F_n(\mu)\mu_{a_n0}.$$

endfor

3. Set

$$M = I_{K+1}.$$

Note that this is an algorithm for *construction* of invariants, not for their evaluation. The invariants are evaluated by means of their explicit forms given in the next Section.

However, it is not clear if every function M produced by this algorithm is really motion blur invariant. The answer is given by the following Lemma.

Lemma 5: Let us consider the situation in Step 2 for the last n (i.e. $n = K$). Then

- D_K has the form

$$D_K = F_K(\mu^{(f)})\mu_{20}^{(h)}.$$

- If the function $F_K(\mu)$ is a motion blur invariant, then the function M produced as a result of the above described algorithm is also motion blur invariant.

Proof:

- Proof of the first assertion is straightforward.
-

$$M^{(g)} = I_{K+1}^{(g)} = I_K^{(g)} - \frac{F_K(\mu^{(g)})\mu_{20}^{(g)}}{\mu_{00}^{(g)}} =$$

$$I_K^{(f)} + F_K(\mu^{(f)})\mu_{20}^{(h)} - \frac{F_K(\mu^{(g)})(\mu_{00}^{(f)}\mu_{20}^{(h)} + \mu_{20}^{(f)})}{\mu_{00}^{(g)}} =$$

$$= I_K^{(f)} - \frac{F_K(\mu^{(f)})\mu_{20}^{(f)}}{\mu_{00}^{(f)}} = I_{K+1}^{(f)} = M^{(f)} \qquad \square$$

As you can see from the algorithm description, the number n_r of independent invariants of order r ($r > 1$) is

$$n_r = r + 1$$

if r is odd (each moment μ_{pq} of order r generates one independent invariant) and

$$n_r = r$$

if r is even (moment μ_{r0} does not generate any invariant).

Since the invariants should serve as features for image similarity or disimilarity assessment, it is sometimes inconvenient to use directly the invariants described above.

The invariants should be normalized in two ways: to be independent of average gray-level value of the image and to have the same "weight" in Euclidean metric space. To achieve this, we use *normalized motion blur invariants*

$$M' = \frac{M}{\mu_{00} \cdot (N/2)^r}$$

where N is the size of the image and r is the order of M.

The set $\mathcal{M}_r$ of invariants M' up to the rth order has

$$N_r = \sum_{k=2}^{r} n_k$$

elements. The N_r-dimensional metric space $(\mathcal{M}_r, \varrho)$ where ϱ is Euclidean metric is a suitable feature space for evaluating image-to-image distances.

4 Motion Blur Invariants in Explicit Form

Applying the above described algorithm, we can construct motion blur invariants of any order and express them in explicit form. A set of invariants up to the 7th order is listed below:

- Zero-order:

$$M_0 = \mu_{00}.$$

- 2nd order:

$$M_1 = \mu_{11},$$

$$M_2 = \mu_{02}.$$

- 3rd order:

$$M_3 = \mu_{12},$$

$$M_4 = \mu_{21},$$

$$M_5 = \mu_{03},$$

$$M_6 = \mu_{30}.$$

- 4th order:

$$M_7 = \mu_{04},$$
$$M_8 = \mu_{13},$$
$$M_9 = \mu_{22} - \frac{\mu_{20}\mu_{02}}{\mu_{00}},$$
$$M_{10} = \mu_{31} - \frac{3\mu_{20}\mu_{11}}{\mu_{00}}.$$

- 5th order:

$$M_{11} = \mu_{14},$$
$$M_{12} = \mu_{05},$$
$$M_{13} = \mu_{32} - \frac{3\mu_{20}\mu_{12}}{\mu_{00}},$$
$$M_{14} = \mu_{23} - \frac{\mu_{20}\mu_{03}}{\mu_{00}},$$
$$M_{15} = \mu_{41} - \frac{6\mu_{20}\mu_{21}}{\mu_{00}},$$
$$M_{16} = \mu_{50} - \frac{10\mu_{20}\mu_{30}}{\mu_{00}}.$$

- 6th order:

$$M_{17} = \mu_{06},$$
$$M_{18} = \mu_{15},$$
$$M_{19} = \mu_{24} - \frac{\mu_{20}\mu_{04}}{\mu_{00}},$$
$$M_{20} = \mu_{33} - \frac{3\mu_{20}\mu_{13}}{\mu_{00}},$$
$$M_{21} = \mu_{42} - \frac{\mu_{40}\mu_{02} + 6\mu_{20}M_9}{\mu_{00}},$$
$$M_{22} = \mu_{51} - \frac{5(\mu_{40}\mu_{11} + 2\mu_{20}M_{10})}{\mu_{00}}.$$

- 7th order:

$$M_{23} = \mu_{07},$$
$$M_{24} = \mu_{16},$$
$$M_{25} = \mu_{25} - \frac{\mu_{20}\mu_{05}}{\mu_{00}},$$
$$M_{26} = \mu_{34} - \frac{3\mu_{20}\mu_{14}}{\mu_{00}},$$
$$M_{27} = \mu_{43} - \frac{\mu_{40}\mu_{03} + 6\mu_{20}M_{14}}{\mu_{00}},$$
$$M_{28} = \mu_{52} - \frac{5(\mu_{40}\mu_{12} + 2\mu_{20}M_{13})}{\mu_{00}},$$
$$M_{29} = \mu_{61} - \frac{15(\mu_{40}\mu_{21} + \mu_{20}M_{15})}{\mu_{00}},$$
$$M_{30} = \mu_{70} - \frac{35\mu_{40}\mu_{30} + 21\mu_{20}M_{16}}{\mu_{00}}.$$

5 Numerical Experiment

In order to demonstrate the performance of the proposed blurred image features, a number of experiments were carried out. Major goal of the experiment described below was to prove the invariance of the features with respect to motion blur and to test their stability under additive random noise.

The test images used for the experiment are shown in Fig. 1. From top to bottom and from left to right you can see original Lena image of the size 256×256 pixels, Lena image blurred by slow horizontal motion ($v \cdot t = 30$ pixels), the blurred image corrupted by additive zero-mean Gaussian noise with standard deviation 10 and 30, respectively and Lena image blurred by fast horizontal motion ($v \cdot t = 80$ pixels). Last image – portrait of Lisa – was incorporated into the test set to show the discriminability of the invariants.

The invariants $M_0, M_1, \ldots, M_{22}$ were calculated for each image. However, in case of digital image f_{ij} of the size $N \times N$ we have to use a discrete version of eq. (4) for moment evaluation:

$$\mu_{pq} = \sum_{i=1}^{N} \sum_{j=1}^{N} (i - x_t)^p (j - y_t)^q f_{ij}. \tag{5}$$

The values of invariants are summarized in Table 1. It is clearly visible from Table 1 that $M_0, M_1, \ldots, M_{22}$ are invariant to motion velocity, sufficiently stable under additive noise and object discriminatory.

Figure 1: Top row: (a) Lena original, (b) horizontal motion blur ($v \cdot t = 30$ pixels), (c) image b corrupted by additive Gaussian random noise (STD = 10); bottom row: (d) image b corrupted by additive Gaussian random noise (STD = 30), (e) horizontal motion blur ($v \cdot t = 80$ pixels), (f) Lisa original

	Lena orig.	Motion 30 pix.	Motion 30 STD = 10	Motion 30 STD = 30	Motion 80 pix.	Lisa
$M_0[10^6]$	6.4586	6.4589	6.4607	6.4674	6.4590	5.0860
$M_1[10^{10}]$	0.2560	0.2559	0.2553	0.2612	0.2560	-0.6036
$M_2[10^{10}]$	3.4740	3.4742	3.4750	3.4816	3.4743	2.3215
$M_3[10^{11}]$	-1.1022	-1.1024	-1.0968	-1.0616	-1.1017	1.4777
$M_4[10^{11}]$	1.0456	1.0462	1.0570	1.0639	1.0450	-2.9602
$M_5[10^{11}]$	2.2436	2.2434	2.2313	2.2541	2.2444	0.4081
$M_6[10^{11}]$	-5.3930	-5.3925	-5.3901	-5.4370	-5.3934	6.8344
$M_7[10^{14}]$	3.3918	3.3920	3.3918	3.4093	3.3921	2.0830
$M_8[10^{14}]$	0.1776	0.1775	0.1772	0.1811	0.1776	-0.5208
$M_9[10^{14}]$	-0.0513	-0.0513	-0.0507	-0.0544	-0.0513	0.0426
$M_{10}[10^{14}]$	-0.2293	-0.2292	-0.2293	-0.2277	-0.2292	0.5672
$M_{11}[10^{15}]$	-1.5087	-1.5088	-1.4996	-1.4809	-1.5081	2.7966
$M_{12}[10^{15}]$	4.7607	4.7606	4.7382	4.7451	4.7622	0.7743
$M_{13}[10^{15}]$	-3.1958	-3.1954	-3.1843	-3.2365	-3.1957	0.6806
$M_{14}[10^{15}]$	0.0734	0.0739	0.0985	0.0330	0.0732	-2.8659
$M_{15}[10^{16}]$	-0.2281	-0.2283	-0.2280	-0.2314	-0.2282	0.9275
$M_{16}[10^{16}]$	1.7525	1.7524	1.7540	1.7645	1.7528	-3.5346
$M_{17}[10^{18}]$	3.9858	3.9860	3.9817	4.0315	3.9863	2.3215
$M_{18}[10^{18}]$	0.1484	0.1484	0.1483	0.1505	0.1485	-0.5753
$M_{19}[10^{18}]$	-0.0520	-0.0520	-0.0503	-0.0552	-0.0519	0.0603
$M_{20}[10^{18}]$	-0.2045	-0.2044	-0.2058	-0.2041	-0.2044	0.5220
$M_{21}[10^{18}]$	0.1186	0.1186	0.1191	0.1229	0.1185	-0.1292
$M_{22}[10^{18}]$	0.7187	0.7186	0.7195	0.7164	0.7187	-2.3919

Table 1 : The values of invariants of the images in Fig. 1

6 Motion Blur Invariants of Rotated and Scaled Images

The invariants introduced in Section 3 are invariant to motion velocity and to image translation but their values depend on image orientation, scale and motion direction. This fact may cause problems in some applications.

To obtain invariants to image scaling

$$\begin{aligned} x' &= s \cdot x, \\ y' &= s \cdot y \end{aligned}$$

it is sufficient only to replace central moments μ_{pq} by *normalized central moments* ν_{pq} in definitions of invariants M_i. Normalized central moments are defined as

$$\nu_{pq} = \frac{\mu_{pq}}{\mu_{00}^w}, \qquad w = \frac{p+q}{2} + 1.$$

To obtain invariants to image rotation

$$\begin{aligned} x' &= x \cdot \cos\alpha + y \cdot \sin\alpha, \\ y' &= -x \cdot \sin\alpha + y \cdot \cos\alpha \end{aligned}$$

we can utilize well-known rotation invariants introduced by Hu [12]. It can be proved that the following four of them are invariant also to motion blur:

$$\begin{aligned} \Phi_1 &= (\nu_{30} - 3\nu_{12})^2 + (3\nu_{21} - \nu_{03})^2, \\ \Phi_2 &= (\nu_{30} + \nu_{12})^2 + (\nu_{21} + \nu_{03})^2, \\ \Phi_3 &= (\nu_{30} - 3\nu_{12})(\nu_{30} + \nu_{12})((\nu_{30} + \nu_{12})^2 - 3(\nu_{21} + \nu_{03})^2) + \\ &\quad (3\nu_{21} - \nu_{03})(\nu_{21} + \nu_{03})(3(\nu_{30} + \nu_{12})^2 - (\nu_{21} + \nu_{03})^2), \\ \Phi_4 &= (3\nu_{21} - \nu_{03})(\nu_{30} + \nu_{12})((\nu_{30} + \nu_{12})^2 - 3(\nu_{21} + \nu_{03})^2) - \\ &\quad (\nu_{30} - 3\nu_{12})(\nu_{21} + \nu_{03})(3(\nu_{30} + \nu_{12})^2 - (\nu_{21} + \nu_{03})^2). \end{aligned}$$

Since linear motion of arbitrary direction can be decomposed into image rotation, horizontal motion and inverse rotation, the features Φ_1, Φ_2, Φ_3 and Φ_4 are invariant also to the motion direction.

The following experiment deals with image blur caused by non-horizontal motion. According to the analysis given above, rotation invariants Φ_1, Φ_2, Φ_3 and Φ_4 were used as the features for image description. Three test images are displayed in Fig. 2: original image, image blurred by horizontal motion ($v \cdot t = 50$ pixels) and image blurred by motion along a diagonal ($v \cdot t = 30$ pixels). Table 2 illustrates the property of invariance of these features.

Figure 2: (a) original image, (b) horizontal motion blur ($v \cdot t = 50$ pixels), (c) diagonal motion blur ($v \cdot t = 30$ pixels)

	(a) .	(b) .	(c) .
$\Phi_1[10^{-11}]$	5.4915	5.4130	5.4699
$\Phi_2[10^{-12}]$	5.7614	5.7471	5.8700
$\Phi_3[10^{-23}]$	3.8498	3.7067	3.7377
$\Phi_4[10^{-23}]$	9.4973	9.4345	9.8318

Table 2 : The values of invariants of the images in Fig. 2

7 Discussion

Every features which we want to use for image description should have the following basic properties:

- *Invariance:* Two images belonging to the same class should have the same values of the features.
- *Uniqueness:* Two images which do not belong to the same class should have different values of the features. In other words, the features should be object discriminatory.
- *Stability:* If two images are similar in some manner (in ℓ_2 norm for instance), then their feature values should be similar too, and vice versa.
- *Efficiency:* The features should be efficient to compute and store.

In this Section, we will discuss how the motion blur invariants M_i satisfy the above criteria.

Invariance was theoretically and experimentally proved in Sections 3 and 5.

By uniqueness of the feature we understand the ability to distinguish (in the feature space) among "similar" images. The results of the experiment described in Section 5 have shown the sufficiently high discriminability of the portrait photographs. However, from theoretical point of view, the set of invariants $\mathcal{M}_r$ is an incomplete feature system for every r. That means there may exist two different images $f_1(x,y)$ and $f_2(x,y)$ such that

$$M_j^{(f_1)} = M_j^{(f_2)} \qquad j = 0, 1, 2, \ldots, N_r.$$

There are two different kinds of stability to be investigated in case of blur invariants: stability under additive random noise and stability with respect to boundary effect.

So far we have considered noise-free model (1) only. Now let us consider imaging model with additive zero-mean random noise $n(x,y)$:

$$g(x,y) = (f * h)(x,y) + n(x,y). \tag{6}$$

Since the image $g(x,y)$ is then a random field, all its moments and all invariants can be viewed as random variables. It holds

$$E(\mu_{pq}^{(n)}) = E(\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^p y^q n(x,y)dxdy) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^p y^q E(n(x,y))dxdy = 0$$

and

$$E(\mu_{pq}^{(g)}) = E(\mu_{pq}^{(f*h)}) + E(\mu_{pq}^{(n)}) = \mu_{pq}^{(f*h)},$$

where $E(X)$ denotes the mean value of random variable X.

In practice, however, only single image $g(x,y)$ (i.e. only one realization of a random field) is available in most cases. We obtain $\mu_{pq}^{(g)}$ but we are not able to estimate mean values $E(\mu_{pq}^{(g)})$. Since the moments are computed by a summation over the whole image, they are supposed to be affected by additive noise very little. That means $\mu_{pq}^{(g)}$ are supposed to be close to $E(\mu_{pq}^{(g)})$ and we can use directly $\mu_{pq}^{(g)}$ for the computation of the invariants. It was confirmed by the experiments in Section 5 that the invariants evaluated by this technique are sufficiently stable.

Serious problems can be caused by boundary effects. Provided that the size of the original image is $N \times N$ pixels and the the size of PSF support is $1 \times L$ pixels, the correct size of the acquired image should be $N \times (N + L - 1)$ pixels. However, in practice the value of L is unknown and original and acquired images are considered to have the same size. Due to this fact, we lose some valuable information on image boundary. If $L \ll N$, the errors of invariant calculation caused by boundary effect are negligible. If L is relatively large (this comes in case of very fast motion), boundary effect should be taken into consideration. Stability of the invariants with respect to boundary effect should be studied in detail in the near future.

Computing complexity of the motion blur invariants is determined by computing complexity of central moments μ_{pq}. Direct evaluation of μ_{pq} by means of relation (5) requieres $O(N^2)$ operations. Although several methods for fast moment computing have been published recently (see [13] for a survey), they are not applicable in case of gray-level images.

8 Summary and Conclusion

The paper was devoted to the construction of image features invariant with respect to motion blur. The images degraded by relative motion of the object and the sensor can be described as an output of a linear shift-invariant imaging system, whose point spread function has the form (2).

A set of motion blur invariants based on image moments was introduced in this paper. An original algorithm for invariants construction was presented and some invariants were shown in explicit form. The derivation of the invariants is a major theoretical result of the paper.

Invariance of the features as well as their ability to distinguish among different images were demonstrated experimentally. Stability of the invariants with respect to additive random noise was also discussed and was shown to be sufficiently high.

Although the presented method for blurred image description produces very good results in some experiments, it has however several limitations. The most significant one arises from the fact that the whole image must be degraded uniformly. This assumption holds good for instance in case of a flat static scene and moving sensor. On the other side, if we scan a small moving object in front of a static background, our method is inapplicable.

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