ON ENTRY-WISE ORGANIZED FILTERING

E.A. Suzdaleva *

* *Department of Adaptive Systems*
Institute of Information Theory and Automation of the Academy of Sciences of the Czech Republic
Pod vodárenskou věží 4, 18208 Prague 8 Czech Republic
fax : +420266052068 and e-mail : suzdalev@utia.cas.cz

Abstract: The paper deals with state estimation in a factorized form. By the concept of the factorized filtering we mean such data preprocessing, as a result of which the n-dimensional state-space model can be decomposed into n one-dimensional models (factors). The key idea is the factors are expected to open a way to describe jointly continuous and discrete probability distributions. A special transformation of the state-space model is considered. The general solution of the factorized state estimation is discussed.

Keywords: State-space model, Bayesian filtering, factors.

1. INTRODUCTION

The filtering problem is the subject of research in the different areas, the applications are hardly countable. Navigational and guidance systems, radar tracking, sonar ranging are only several examples just to name, see Cipra (1993). A filter tries to obtain an optimal estimate of desired quantities from data provided by a noisy environment. Any measurement will be corrupted to some degree by noise and device inaccuracies, so the filter removes the measurement noise and extracts valuable information from signal. A lot of signal proccesing techniques is available in the field of the filtering problems, see Oppenheim and Wilsky (1983). The Kalman filter is a special case, addressed to the problems in which the system can be described through a linear model and the measurement noise is white and Gaussian, according to Welch and Bishop (1995). The appearance of the Kalman filter stimulated the subsequent research on its extending and applying, as Welch and Bishop (2005) say. This moves us to explaining the aim of the work.

The idea of the paper is to use the factorized filtering and find the ways to allow its systematic implementation. By the concept of the factorized filtering we mean such data preprocessing, as a result of which the n-dimensional state-space model can be decomposed into n one-dimensional models, namely, the factors. The factorized data preprocessing seems to be a solution of a difficulty caused by different probability distributions for separate factors. In Gaussian and linear case it leads to the special Kalman filtering combined with parameter estimation. The factor here is equivalent to a special state-space model. In general, the i-th state factor can be described by conditional probability distribution

$$f(x_{i;t+1}|x_{i+1;t+1}, \ldots, x_{\mathring{x};t+1}, x_t, u_t) \sim \quad (1)$$

$$\sim \mathcal{N}(Ax_{i;t} + Bu_{i;t}, R_{i;t})$$

where x_t is a state of system, u_t is the input.

The key idea of the use of factorized filtering is the factors are expected to open a way to describe jointly continuous and discrete probability distributions, Kárný et al. (2005). Let's try to explain the idea. Generally, the joint probability distribution can be presented in the factorized form using the chain rule.

$$f(d(t)|d(t-1),\Theta) = \tag{2}$$

$$= \prod_{i=1}^{\mathring{d}} f(d_{i;t}|d_{i+1;t},\ldots,d_{\mathring{d};t},d(t-1),\theta_{i;t},\ldots,\theta_{\mathring{\theta};t})$$

when $d_{\mathring{d}+1;t},\ldots,d_{\mathring{d};t}$ denotes the empty set.

Let $\mathring{d} = 3$ and respectively $d(t) = [d_{1;t}, d_{2;t}, d_{3;t}]'$. Three factors of the data vector are in essence the random variables with their probability distributions. Obviously, the case when *all* three random variables have continuous *or* discrete probability distributions may not even need to use the factorizing, since the random values can be described as a vector.

Assume that $d_{1;t}$ and $d_{2;t}$ have continuous probability distribution, while $d_{3;t}$ has the discrete one. In that case the data factors $d_{1;t}$ and $d_{2;t}$ may need Gaussian model and can be united in the vector, while $d_{3;t}$ needs some another model. But the most unsuccessful case from that point of view is the presence of discrete probability distribution for $d_{2;t}$, when both $d_{1;t}$ and $d_{3;t}$ might be described by Gaussian model. We have no choice but to observe each factor individually. It should be noted furthermore, that the factors open a way to use the different models for different entries of data.

Now we have told about data vector in general, but what if the situation could have been projected to the state-space model? The existence of both continuous and discrete state factors in any order will seriously complicate the problem. It remains only to try to process the state factors individually.

Thus we have shown why the factorized filtering problem requires a solution and described the aim of the paper.

To say briefly about the contents of the paper, the preliminaries with short description of conceptual Bayesian solution of the state estimation problem will be available. Further the paper deals with the general factorized filtering. A separate section will be devoted to the transformation of state-space model.

2. PRELIMINARIES

The section gives the general Bayesian solution of the filtering problem. One of the basic objectives of a decision maker in this paper is to estimate the state and predict the output of the observed system. The state-space model defines the conditional probability distributions

$$f(x_{t+1}|x_t,u_t),\ f(y_t|u_t,x_t) \tag{3}$$

It is considered that the state x_t and input u_t are known, and neither the output y_t nor the next state x_{t+1} depend on the past history of the system, hence

$$f(y_t|u_t,d(t-1),x_t) = f(y_t|u_t,x_t) \tag{4}$$

$$f(x_{t+1}|x_t,d(t)) = f(x_{t+1}|x_t,u_t) \tag{5}$$

The state-space model practically combines (4) and (5) in a single more complex model with all parameters known. As the one-step-ahead Bayesian predictor for this case we have the following recursion, according to Peterka (1981)

$$f(y_t|u_t,d(t-1)) = \tag{6}$$

$$= \int f(y_t|u_t,x_t) f(x_t|d(t-1)) dx_t$$

$$f(x_t|d(t)) = \frac{f(y_t|u_t,x_t) f(x_t|d(t-1))}{f(y_t|u_t,d(t-1))} \tag{7}$$

$$f(x_{t+1}|d(t)) = \tag{8}$$

$$= \int f(x_{t+1}|x_t,u_t) f(x_t|d(t)) dx_t$$

We will try to explain briefly what the use of the recursion is based on. We shall assume that the second integrand in (6), i.e. the probability distribution $f(x_t|d(t-1))$, is known for some $t = t_0$ and can be calculated for $t = t_0 + 1$, in other words, $f(x_1|d(0))$ is known. Knowing the probability distribution $f(x_t|d(t-1))$ we can calculate the probability distribution of the next output y_t for any u_t. As a result of this operation the new data pair $d(t) = \{u_t, y_t\}$ can be predicted. This new pair will be used for the next step of recursion.

In order to obtain the second part (7) of the recursion the basic Bayesian operations have been applied. We have

$$f(y_t,x_t|u_t,d(t-1)) =$$
$$= f(y_t|u_t,d(t-1),x_t) f(x_t|u_t,d(t-1)) =$$
$$= f(x_t|d(t)) f(y_t|u_t,d(t-1))$$

It should be noted that the second equality in this relation occured due to the new data pair $d(t) = \{u_t, y_t\}$ prediction. From this relation and from (4) the second step (7) of the recursion follows. The probability distribution for the state x_t is updating with respect to the new data pair $d(t) = \{u_t, y_t\}$.

The recursive relation (8) is understood as update of the state x_t in time. This part has been obtained with the help of applying the basic Bayesian operations

$$f(x_{t+1}|d(t)) = \int f(x_{t+1},x_t|d(t)) dx_t =$$

$$= \int f(x_{t+1}|x_t, d(t)) f(x_t|d(t)) dx_t \quad (9)$$

and due to (5).

The attempt to factorize this data processing recursive algorithm will be discussed in the next sections.

3. TRANSFORMATION OF THE STATE-SPACE MODEL

Assume the system is described by the state-space model

$$x_{t+1} = Ax_t + Bu_t + \omega_t \quad (10)$$

$$y_t = Cx_t + Du_t + e_t \quad (11)$$

where A, B, C, D are known matrices of appropriate dimensions. The noises ω_t and e_t are supposed to be white and Gaussian with zero mean values and known covariances.

Before we proceed to the immediate problem solving, we suggest to make some transformation of the state-space model (10)-(11) that leads to the triangular form of matrices of the state both from (11) and (10). The presence of triangular matrices of the state is strongly needed from the positions of our approach to the factorized filtering, because according to our state-space model it gives a possibility to observe dependencies of the individual factors. That would be quite understandable, if we imagined the structure of the state-space model after multiplying the matrices and vectors. Besides, it essentially simplifies the computational process.

Thus we propose to make the following transformation of (10)-(11) introducing the transformation matrix T

$$y_t = CT^{-1}Tx_t + Du_t + e_t \quad (12)$$

$$Tx_{t+1} = TAT^{-1}Tx_t + TBu_t + T\omega_t \quad (13)$$

Denoting the new state $\tilde{x}_t$=Tx_t we obtain

$$y_t = CT^{-1}\tilde{x}_t + Du_t + e_t \quad (14)$$

$$\tilde{x}_{t+1} = TAT^{-1}\tilde{x}_t + TBu_t + T\omega_t \quad (15)$$

Obviously our basic requirements is the transformation matrix T must satisfy the following conditions (for the sake of simplicity it is assumed that the output y_t and the state x_t have dimensions $\mathring{y} = 2$ and $\mathring{x} = 3$ correspondingly).

$$CT^{-1} = \tilde{C} = \begin{bmatrix} \tilde{c}_{11} & \tilde{c}_{12} & \tilde{c}_{13} \\ 0 & \tilde{c}_{22} & \tilde{c}_{23} \end{bmatrix} \quad (16)$$

$$TAT^{-1} = \tilde{A} = \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \tilde{a}_{13} \\ 0 & \tilde{a}_{22} & \tilde{a}_{23} \\ 0 & 0 & \tilde{a}_{33} \end{bmatrix} \quad (17)$$

It should be noted that during transformation we have obtained $T\omega_t$ at (15). In order not to lose a triangular structure of noise and its uncorrelatedness, let's put some orthogonal matrix $\mathcal{F}$ such that the product of T and $\mathcal{F}$ would give us the triangular matrix. It means

$$T\mathcal{F}\mathcal{F}'\omega_t = T\mathcal{F}\tilde{\omega}_t \quad (18)$$

where $\tilde{\omega}_t = \mathcal{F}'\omega_t$ and

$$E[\tilde{\omega}_t\tilde{\omega}_t'] = E[\mathcal{F}'\omega_t\omega_t'\mathcal{F}] = \mathcal{F}'I\mathcal{F} = I \quad (19)$$

Solution of this task (search of the orthogonal matrix) is out of scope of this paper now, but we can return to it later.

There is a possibility to make one more transformation of equation (11).

$$Hy_t = HCT^{-1}\tilde{x}_t + HDu_t + He_t \quad (20)$$

which leads to

$$\tilde{y}_t = HCT^{-1}\tilde{x}_t + HDu_t + He_t \quad (21)$$

where H is a square matrix of dimension $(\mathring{y} \times \mathring{y})$.

This kind of transformation must be fulfilled before making the square root matrix from covariance matrix of noise. The matrix H should transform CT^{-1} into triangular one. As regards He_t, we will handle it in the similar way as we treated with $T\omega_t$, i.e. with the help of the orthogonal matrix $\mathcal{G}$.

$$H\mathcal{G}\mathcal{G}'e_t = H\mathcal{G}\tilde{e}_t \quad (22)$$

where $H\mathcal{G}$ is a triangular matrix.

Then with the help of multiplying (21) by the inverse matrix $\mathcal{G}^{-1}$ we will obtain necessary triangular structure of the output equation.

The new transformation will enable to fulfill the requirements (16)-(17). The matrix H can be found if the matrices A, B, C and D are known.

Now we can speak about transformation of the whole state-space model as $\{x_t, y_t\} \rightarrow \{\tilde{x}_t, \tilde{y}_t\}$ that can be produced separately before fulfilling the filter algorithm steps.

4. GENERAL FACTORIZED FILTERING

It is assumed that both matrices of the state at (11) and (10) are triangular either with the help of transformation of the state-space model (Section 3) or according to initial conditions. This section offers only the calculation of the factorized filter.

4.1 *Data update factorizing*

At this step we try to predict the new data pair. For the sake of simplicity it is assumed that the output y_t and the state x_t have dimensions $\mathring{y} = 2$ and $\mathring{x} = 3$ correspondingly. According to the chain rule the factorized form of the recursive relation (6) is

$$f(y_{1;t}|y_{2;t}, u_t, d(t-1))\times \quad (23)$$

$$\times f(y_{2;t}|u_t, d(t-1)) =$$
$$= \int\int\int f(y_{1;t}|y_{2;t}, u_t, x_{1;t}, x_{2;t}, x_{3;t})\times$$
$$\times f(y_{2;t}|u_t, x_{2;t}, x_{3;t})\times$$
$$\times f(x_{1;t}|x_{2;t}, x_{3;t}, d(t-1))f(x_{2;t}|x_{3;t}, d(t-1))\times$$
$$\times f(x_{3;t}|d(t-1))dx_{1;t}dx_{2;t}dx_{3;t}$$

It should be noted that the conditional probability distribution $f(y_{2;t}|u_t, x_{2;t}, x_{3;t})$ demonstrates the triangular structure of the output equation (11). Due to the triangular structure of the model the factors $f(y_{2;t}|u_t, x_{2;t}, x_{3;t})$ and $f(y_{2;t}|u_t, d(t-1))$ do not depend on $x_{1;t}$ and the right-hand side of relation (23) can be expressed as a recursive calculation shceme. Let's take (23) and try to calculate the probability distribution while integrating over the factor $x_{1;t}$.

$$f(y_{1;t}|y_{2;t}, u_t, d(t-1))\times \quad (24)$$

$$\times f(y_{2;t}|u_t, d(t-1)) =$$
$$= \int\int\int f(y_{1;t}|y_{2;t}, u_t, x_{1;t}, x_{2;t}, x_{3;t})\times$$
$$\times f(x_{1;t}|x_{2;t}, x_{3;t}, d(t-1))f(y_{2;t}|u_t, x_{2;t}, x_{3;t})\times$$
$$\times f(x_{2;t}|x_{3;t}, d(t-1))f(x_{3;t}|d(t-1))dx_{1;t}dx_{2;t}dx_{3;t} =$$
$$= \int\int \alpha_1 f(y_{2;t}|u_t, x_{2;t}, x_{3;t})f(x_{2;t}|x_{3;t}, d(t-1))\times$$
$$\times f(x_{3;t}|d(t-1))dx_{2;t}dx_{3;t}$$

where

$$\alpha_1 = \int f(y_{1;t}|y_{2;t}, u_t, x_{1;t}, x_{2;t}, x_{3;t})\times \quad (25)$$

$$\times f(x_{1;t}|x_{2;t}, x_{3;t}, d(t-1))dx_{1;t}$$

Obviously, after that it remains to calculate recursively

$$\alpha_2 = \int\int \alpha_1 f(y_{2;t}|u_t, x_{2;t}, x_{3;t})\times \quad (26)$$

$$\times f(x_{2;t}|x_{3;t}, d(t-1))f(x_{3;t}|d(t-1))dx_{2;t}dx_{3;t}$$

Thus at this step the new data have been predicted. They will be used to update the probability distribution for the state x_t. Let's express relation (7) as a product of the conditional probability distributions for the factors

$$f(x_{1;t}|x_{2;t}, x_{3;t}, d(t))f(x_{2;t}|x_{3;t}, d(t))\times \quad (27)$$

$$\times f(x_{3;t}|d(t)) \propto f(y_{1;t}|y_{2;t}, x_{1;t}, x_{2;t}, x_{3;t}, u_t)\times$$
$$\times f(y_{2;t}|x_{2;t}, x_{3;t}, u_t)f(x_{1;t}|x_{2;t}, x_{3;t}, d(t-1))\times$$
$$\times f(x_{2;t}|x_{3;t}, d(t-1))f(x_{3;t}|d(t-1))$$

Let's take all the probability distributions, which depend on $x_{1;t}$ and aggregate them.

$$f(x_{1;t}|x_{2;t}, x_{3;t}, d(t)) \propto \quad (28)$$

$$\propto f(y_{1;t}|y_{2;t}, x_{1;t}, x_{2;t}, x_{3;t}, u_t)\times$$
$$\times f(x_{1;t}|x_{2;t}, x_{3;t}, d(t-1))$$

We have got rid of $x_{1;t}$ at (27). Now the following joint probability distribution remains

$$f(x_{2;t}|x_{3;t}, d(t))f(x_{3;t}|d(t)) \propto \quad (29)$$

$$\propto f(y_{2;t}|x_{2;t}, x_{3;t}, u_t)f(x_{2;t}|x_{3;t}, d(t-1))\times$$
$$\times f(x_{3;t}|d(t-1))$$

We know that knowledge about $y_{2;t}$ incorporates into both $x_{2;t}$ and $x_{3;t}$ according to our state-space model and due to the triangular structure. Because of the unequal dimensions of the output y_t and state x_t we have some obvious difficulties here. The matrix C of state at (11) is of dimension $(\mathring{y} \times \mathring{x})$ and in consequence of that $\mathring{y} \neq \mathring{x}$ we can observe the dependencies mentioned above. At this step it is enough to calculate the *joint* conditional probability distribution $f(x_{2;t}|x_{3;t}, d(t))f(x_{3;t}|d(t))$. We will need the factors $f(x_{2;t}|x_{3;t}, d(t))$ and $f(x_{3;t}|d(t))$ for time update, but knowing the joint probability distribution and applying the basic Bayesian operations, we can always calculate the conditional ones. It can be seen that

$$f(x_{2;t}|x_{3;t}, d(t)) = \frac{f(x_{2;t}, x_{3;t}, |d(t))}{f(x_{3;t}|d(t))} = \quad (30)$$

$$= \frac{f(x_{2;t}, x_{3;t}, |d(t))}{\int f(x_{2;t}, x_{3;t}|d(t))dx_{2;t}}$$

So we fulfilled the data update and could move to the next recursion step, namely, to re-calculation of the state at moment $(t+1)$.

4.2 *Time update factorizing*

At this step the operation of time update is fulfilled. It should be reminded that an assumption about triangular matrix A at (10) has been made. Let's rewrite (8) at the factorized form, taking into account that the factors $x_{1;t}$ and $x_{2;t}$ gradually disappear in the conditional probability distributions due to the triangular structure.

$$f(x_{1;t+1}|x_{2;t+1},x_{3;t+1},d(t))\times \tag{31}$$

$$\times f(x_{2;t+1}|x_{3;t+1},d(t))f(x_{3;t+1}|d(t)) =$$

$$= \int\int\int f(x_{1;t+1}|x_{2;t+1},x_{3;t+1},x_{1;t},x_{2;t},x_{3;t},u_t)\times$$

$$\times f(x_{2;t+1}|x_{3;t+1},x_{2;t},x_{3;t},u_t)f(x_{3;t+1}|x_{3;t},u_t)\times$$

$$\times f(x_{1;t}|x_{2;t},x_{3;t},d(t))f(x_{2;t}|x_{3;t},d(t))\times$$

$$\times f(x_{3;t}|d(t))dx_{1;t}dx_{2;t}dx_{3;t}$$

Due to triangular structure the product of the conditional probability distributions for the factors $x_{1;t+1}$, $x_{2;t+1}$ and $x_{3;t+1}$ can be converted into the recursive scheme. Much as we operated calculating the output prediction, we will start to integrate over $x_{1;t}$. It should be only noted that at the state equation (10) the matrix A is of dimension $(\mathring{x}\times\mathring{x})$ and after necessary transformation we obtain a triangular matrix without complexities we had while updating data. So let's integrate the right-hand side of (31) recursively over $x_{1;t}$, $x_{2;t}$ and $x_{3;t}$.

$$\int\int \beta_1 f(x_{2;t+1}|x_{3;t+1},x_{2;t},x_{3;t},u_t)\times \tag{32}$$

$$\times f(x_{3;t+1}|x_{3;t},u_t)f(x_{2;t}|x_{3;t},d(t))\times$$

$$\times f(x_{3;t}|d(t))dx_{2;t}dx_{3;t} =$$

$$= \int \beta_1\beta_2 f(x_{3;t+1}|x_{3;t},u_t)f(x_{3;t}|d(t))dx_{3;t} = \beta_3$$

where

$$\beta_1 = \tag{33}$$

$$= \int f(x_{1;t+1}|x_{2;t+1},x_{3;t+1},x_{1;t},x_{2;t},x_{3;t},u_t)\times$$

$$\times f(x_{1;t}|x_{2;t},x_{3;t},d(t))dx_{1;t}$$

and

$$\beta_2 = \int \beta_1 f(x_{2;t+1}|x_{3;t+1},x_{2;t},x_{3;t},u_t)\times \tag{34}$$

$$\times f(x_{2;t}|x_{3;t},d(t))dx_{2;t}$$

In that way this step completes the recursion defined by relations (6)-(8). It should be noted that the prior probability distribution $f(x_{t=1}|d(0))$ must be available for applying the recursive algorithm solving the problem of filtering. It only remains to rewrite the recursion as step-by-step algorithm what the next section is devoted to.

4.3 *Algorithm*

At this section we will only resume the calculating described above and present it as the recursive algorithm. Previously we have considered only a case, directed at matrices (2×3), but hopefully we could extend it to the universal one, when $\mathring{y} = m$, $\mathring{x} = n$. Thus, taking into account the appropriate matrix dimensions, the factorized filtering algorithm includes the following steps.

$$\alpha_1 = \tag{35}$$

$$= \int f(y_{1;t}|y_{2;t},\ldots,y_{\mathring{y};t},u_t,x_{1;t},\ldots,x_{\mathring{x};t})\times$$
$$\times f(x_{1;t}|x_{2;t},\ldots,x_{\mathring{x};t},d(t-1))dx_{1;t}$$

If $\mathring{y} < \mathring{x}$, then for α_j, $1 < j < \mathring{y}$ it is valid

$$\alpha_j = \int \alpha_1\ldots\alpha_{j-1}\times \tag{36}$$

$$\times f(y_{j;t}|y_{j+1;t},\ldots,y_{\mathring{y};t},u_t,x_{j;t},\ldots,x_{\mathring{x};t})\times$$
$$\times f(x_{j;t}|x_{j+1;t},\ldots,x_{\mathring{x};t},d(t-1))dx_{j;t}$$

$$\alpha_{\mathring{y}} = \int\int \alpha_1\ldots\alpha_{\mathring{y}-1}\times \tag{37}$$

$$\times f(y_{\mathring{y};t}|u_t,x_{\mathring{x}-1;t},x_{\mathring{x};t})f(x_{\mathring{x}-1;t}|x_{\mathring{x};t},d(t-1))\times$$
$$\times f(x_{\mathring{x};t}|d(t-1))dx_{\mathring{x}-1;t}dx_{\mathring{x};t}$$

If $\mathring{y} = \mathring{x}$, what is the most successful case, then α_j, $1 \le j < \mathring{y}$, is calculated in the same way as for $\mathring{y} < \mathring{x}$. As regards $\alpha_{\mathring{y}}$

$$\alpha_{\mathring{y}} = \int \alpha_1\ldots\alpha_{\mathring{y}-1}\times \tag{38}$$

$$\times f(y_{\mathring{y};t}|u_t,x_{\mathring{x};t})f(x_{\mathring{x};t}|d(t-1))dx_{\mathring{x};t}$$

We suggest that $\max\mathring{y} = \mathring{x}$.

The next step is calculated equally for both cases $\mathring{y} < \mathring{x}$ and $\mathring{y} = \mathring{x}$.

$$f(x_{1;t}|x_{2;t},\ldots,x_{\mathring{x};t},d(t)) \propto \tag{39}$$

$$\propto f(y_{1;t}|y_{2;t},\ldots,y_{\mathring{y};t},x_{1;t},\ldots,x_{\mathring{x};t},u_t)\times$$
$$\times f(x_{1;t}|x_{2;t},\ldots,x_{\mathring{x};t},d(t-1))$$

$$f(x_{j;t}|x_{j+1;t},\ldots,x_{\mathring{x};t},d(t)) \propto \tag{40}$$

$$\propto f(y_{j;t}|y_{j+1;t},\ldots,y_{\mathring{y};t},x_{j;t},\ldots,x_{\mathring{x};t},u_t)\times$$
$$\times f(x_{j;t}|x_{j+1;t},\ldots,x_{\mathring{x};t},d(t-1))$$

where $1 < j < \mathring{y}$.

If $\mathring{y} < \mathring{x}$, then the conditional probability distribution for two last factors remains the joint one.

$$f(x_{\mathring{x}-1;t}|x_{\mathring{x};t},d(t))f(x_{\mathring{x};t}|d(t)) \propto \tag{41}$$

$$\propto f(y_{\mathring{y};t}|x_{\mathring{x}-1;t},x_{\mathring{x};t},u_t)f(x_{\mathring{x}-1;t}|x_{\mathring{x};t},d(t-1))$$
$$\times f(x_{\mathring{x};t}|d(t-1))$$

$$f(x_{\mathring{x}-1;t}|x_{\mathring{x};t}, d(t)) = \tag{42}$$

$$= \frac{f(x_{\mathring{x}-1;t}, x_{\mathring{x};t}, |d(t))}{f(x_{\mathring{x};t}|d(t))} =$$

$$= \frac{f(x_{\mathring{x}-1;t}, x_{\mathring{x};t}, |d(t))}{\int f(x_{\mathring{x}-1;t}, x_{\mathring{x};t}|d(t))dx_{\mathring{x}-1;t}}$$

If $\mathring{y} = \mathring{x}$, then the last factor should be calculated

$$f(x_{\mathring{x};t}|d(t)) \propto \tag{43}$$

$$\propto f(y_{\mathring{y};t}|x_{\mathring{x};t}, u_t)f(x_{\mathring{x};t}|d(t-1))$$

The time update steps includes

$$\beta_1 = \tag{44}$$

$$= \int f(x_{1;t+1}|x_{2;t+1}, \ldots, x_{\mathring{x};t+1}, x_{1;t}, \ldots, x_{\mathring{x};t}, u_t)\times$$

$$\times f(x_{1;t}|x_{2;t}, \ldots, x_{\mathring{x};t}, d(t))dx_{1;t}$$

$$\beta_j = \int \beta_1 \ldots \beta_{j-1}\times \tag{45}$$

$$\times f(x_{j;t+1}|x_{j+1;t+1}, \ldots, x_{\mathring{x};t+1}, x_{j;t}, \ldots, x_{\mathring{x};t}, u_t)\times$$

$$\times f(x_{j;t}|x_{j+1;t}, \ldots, x_{\mathring{x};t}, d(t))dx_{j;t}$$

where $1 < j < \mathring{x}$. And the last factor

$$\beta_{\mathring{x}} = \int \beta_1 \ldots \beta_{\mathring{x}-1} f(x_{\mathring{x};t+1}|x_{\mathring{x};t}, u_t)\times \tag{46}$$

$$\times f(x_{\mathring{x};t}|d(t))dx_{\mathring{x};t}$$

Thus we have aggregated all calculations into the stepwise algorithm.

5. CONCLUSION

The paper has described the general solution of the state estimation in factorized form. The particular example for the number of the output factors $\mathring{y} = 2$ and for the state factors $\mathring{x} = 3$ respectively and the general recursive algorithm have been presented. The next task expected to be solved is to specialize the factorized filtering and try to apply the approach to linear and Gaussian case, which will lead to special factorized Kalman filter.

ACKNOWLEDGMENTS

The work has been partially supported by project AV ČR S1075351 and by Ministry of Transportation of the Czech Republic, project 1F43A/003/120.

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