Experimental determination of elastic coefficients of dry bovine bone

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Abstract

The elastic properties of single parts of a human skeleton are necessary to know for modelling bone tissue-implants interactions as well as for diagnostic purposes. This paper contributes to methodology of evaluation of elastic properties of bones by ultrasonic wave inversion. The method was developed on composite structures like as plates and cylindrical shells. The final results are then demonstrated on the bovine cortical bone specimen.

The properties are supposed to exhibit orthotropic or transversally isotropic symmetry. The quasi-longitudinal and transverse waves are generated from wave diffraction on liquid/specimen interface. The wave velocity fields obtained by the ultrasonic scanning technique are used as an input to inversion procedure for all elastic constants determination.

The experimental results are confronted with the numerical modelling of wave propagation and the stability of resulting data is evaluated by the statistical method based on Monte-Carlo simulation. The suggested approach has a potential for qualify of such measurements performed on fresh bones and also for improvement in-situ ultrasonic techniques.

Keywords: Matrix of elastic coefficients; Ultrasonic immersion technique; Inverse problem; Monte-Carlo simmulation.

1 Introduction

The determination of elastic coefficients bones is important mainly for micro-mechanical modelling that conduces to new findings concerning the micro-structure of a bone tissue. This knowledge may, for example, help to answer a bone tissue remodelling problem. Bone tissue is, from the mechanical point of view, an inhomogeneous, anisotropic, and visco-elastic material, in its principle composite material. Compared to other tissues of the human body, the strain of a bone tissue is comparatively small, hence it is possible to assume a linear dependence between the stress and the strain. The visco-elasticity of a cortical bone is, in the terms of time dependency on material constants, relatively small. Therefore, it is possible to contemplate a cortical bone as a linear elastic material, which is approximately homogeneous and anisotropic with an orthotropic material symmetry [18, 19, 21, 22, 30].

Determination of the elastic coefficients of a bone tissue is very important for the description of mechanical properties of bones. Static mechanical tests (e.g. compressive, bending, and torsional tests) are currently used for assessment of the elastic coefficients of bones. Elastic coefficients are possible to detect experimentally by means of dynamics tests (ultrasonic tests).

The purpose of this study is a measurement of elastic coefficients of a cortical bone as a orthotropic material [18, 19, 21, 22, 30] in its material symmetry via an immersion technique. This experimental method consists in specimen positioning between transmitting and receiving ultrasonic transducers, whereas entire measuring configuration is immersed into a liquid. The specimen is rotated in various directions and velocities of longitudinal and transversal waves in a broad range of directions are measured. Elastic coefficients of specimen can be obtained either analytically from velocity measurements in different directions [12, 14] or the problem is possible to solve as multi dimensional optimization approach [25, 28, 26, 29] (inverse problem).

The acoustic scanner for immersion measurement of longitudinally and transversally propagating waves through anisotropic specimen was used in this study. Test measurements were performed on an etalon composite specimen CFRP (Carbon Fiber Reinforced Plastic). CFRP is homogenous and anisotropic (transversely isotropic material symmetry) material with principal directions identical to the fiber direction. The specimens were plate-shaped with the thickness of approximately 2, 3.8 and 8 mm. Ultrasonic wave velocities along various directions in this specimens were known from previous PS/PR (PS point source, PR point receiver) measurement [28, 26, 27]. Similarly, the experiment can be arranged for measurements of a cylindrical specimen, where the scanning plane is set to be a axial plane of a tube. This approach has been verified on an isotropic plexiglass tube.

Latest measurements were realized on cortical bovine bone as an elastic orthotropic medium [18, 19, 21]. However, not all of nine independent elastic coefficients can be determined from propagation in the axial plane. Therefore, prospective measurements are planned to obtain the reminder coefficients.

2 Theory

2.1 Elastic waves in anisotropic solid

For a linear, elastic, homogenous, anisotropic material the generalised Hookes law

$$\sigma_{ij} \;=\; C_{ijkl}\varepsilon_{kl} \;, \tag{1}$$

is valid, where σ_{ij} and ε_{kl} are the are the stress and infinitesimal strain tensors and C_{ijkl} is often rewritten in shorten Voight's notation as a matrix of elastic coefficients c_{ij} [6, 10]. The equation of motion

$$\rho = \frac{\partial^2 u_i}{\partial t^2} = C_{ijkl}\frac{\partial^2 u_k}{\partial x_j \partial x_l} \;, \tag{2}$$

where ρ is the mass density, and $\mathbf{u}(\mathrm{x},t)$ is a planar elastic wave propagating in direction $\mathbf{n}$ throught observed material, can be obtained substituting Hooke's law into an equation of equlibrium [6, 24],

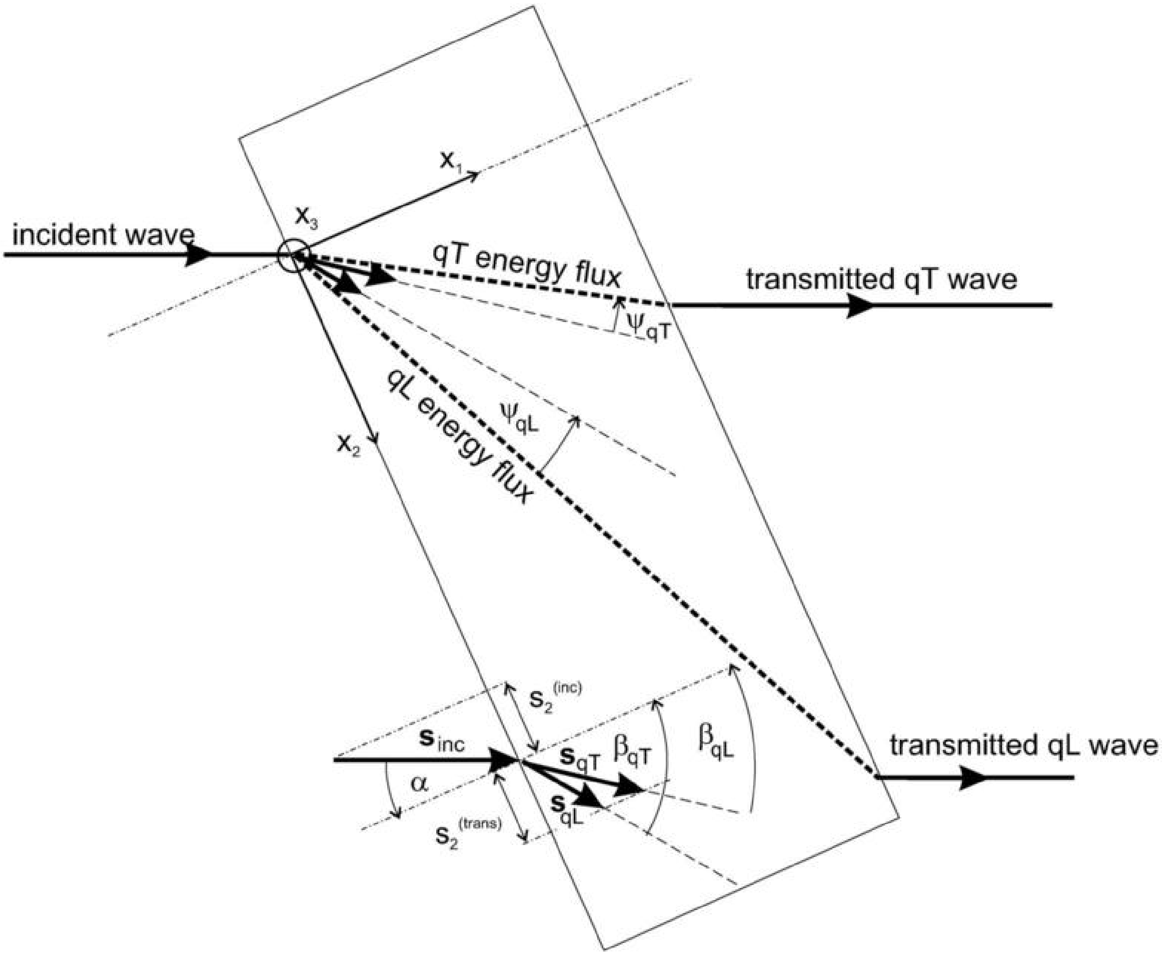

Figure 1: An interaction of a single planar wave with an anisotropic plate-like specimen.

considering the infinitesimal strain tensor

$$\varepsilon_{kl} = \frac{1}{2}\left(\frac{u_k}{x_l} + \frac{u_l}{x_k}\right) . \tag{3}$$

A planar elastic wave $\mathbf{u}(\mathrm{x},t)$ propagating in a material can be represented by

$$\mathbf{u} = \mathbf{U}e^{i(\mathbf{kx}-\omega t)} \tag{4}$$

where $\mathbf{U}$ is the wave amplitude, ω is the angular frequency, $\mathbf{x}$ is the position vector, and $\mathbf{k}$ is the wave vector. On substituting (4) into (2), it is obvious, that ω and U_k must satisfy the following system of a so-called Christoffel equation

$$\left(C_{ijkl}k_jk_l - \rho\omega^2\delta_{ik}\right)U_k = 0 , \tag{5}$$

where δ_{ik} is the Kronecker's symbol.

The phase velocity of planar wave (4) and wave vector $\mathbf{k}$ are defined as

$$v_\phi = \frac{\omega}{k} \tag{6}$$

$$\mathbf{k} = k \cdot \mathbf{n} , \tag{7}$$

where k is the wave number, and $\mathbf{n}$ is wave normal. Thus, the Christoffel equation can be rewritten in the terms of the phase velocity

$$\left(C_{ijkl}n_jn_l - \rho v_\varphi^2 \delta_{ik}\right) = 0 \,. \tag{8}$$

Introducing the Christoffel's tensor

$$\Gamma_{ik} = C_{ijkl}n_jn_l \,, \tag{9}$$

the Christoffel's equation can be treated as an eigenvalue problem of following relation

$$\left|\Gamma_{ik} - \rho v_\varphi^2 \delta_{ik}\right| = 0 \,. \tag{10}$$

The Christoffel's coefficients for general anisotropic material are well described in literature [14, 24, 3].

Transversely isotropic material is described by five independent elastic constants

$$c_{ij} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}\left(c_{11}-c_{12}\right) \end{pmatrix} \text{ and } c_{66} = \frac{1}{2}\left(c_{11}-c_{12}\right) \,. \tag{11}$$

The x_3 axis is directed along axis of symmetry (fiber direction in composite materials, direction of Haversian system in bones) and plane x_1x_2 plane is isotropic plane. Christoffel equation (5) for wave propagating in x_1x_3 plane is

$$\begin{bmatrix} c_{11}k_1^2 + c_{44}k_1^2 - \rho\omega^2 & \left(c_{23}+c_{44}\right)k_1k_3 \\ \left(c_{23}+c_{44}\right)k_1k_3 & c_{44}k_1^2 + c_{33}k_1^2 - \rho\omega^2 \end{bmatrix} \begin{bmatrix} U_1 \\ U_3 \end{bmatrix} \tag{12}$$

and for wave propagating in x_1x_3 is

$$\begin{bmatrix} c_{11}k_1^2 + c_{66}k_2^2 - \rho\omega^2 & \left(c_{12}+c_{66}\right)k_1k_2 \\ \left(c_{12}+c_{66}\right)k_1k_2 & c_{66}k_1^2 + c_{11}k_2^2 - \rho\omega^2 \end{bmatrix} \begin{bmatrix} U_1 \\ U_3 \end{bmatrix} \,. \tag{13}$$

It is obvious from equations (12) and (13) that two modes of wave propagation in x_1x_2 exists , i.e. the quasi-longitudinal (qL) and quasi-transverse (qT) modes. The unknown elastic constants for transversely isotropic elastic material can be obtained by qL and qT wave propagation measurements in only two planes.

The acoustic energy travels in anisotropic materials with group velocity v_G which differs in direction and magnitude from phase velocity. However, in our case of immersion measurements, we directly obtain the phase velocities [14, 11] instead of group velocities resulting from the PS/PR measurements [28, 26, 27].

2.2 Waves on a solid/fluid interface

Reflection and refraction of elastic waves at a boundary between two anisotropic media is described by the Snell-Descartes law [24]. Even in our simpler case, where one of the media is considered to be a non-viscous fluid, evaluation of paths of reflected energy fluxes is a significantly difficult issue.

An interaction of a single planar wave with an anisotropic plate-like specimen is outlined in Fig.1. The incident wave of a slowness vector $\mathbf{s}_{\text{inc}}$ is refracted into more separate waves in the specimen, each travelling at its own group velocity in the direction of its energy flux. In Fig.1, only two of these waves (qL and one qT) are shown for clarity. Besides them, one more qT mode

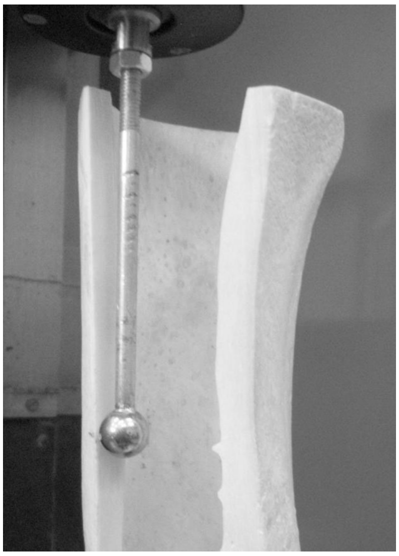

Figure 2: Bone specimen and its shape measurement.

can be generated, or the surface wave can arise instead of one of the bulk modes, depending on the angle of incidence [14, 24].

According to the Snell-Descartes law, the projection of the incident wave slowness into the boundary $s_2^{(\mathrm{fluid})}$ is conserved, following conditions for refraction angles $\beta_{\mathrm{qL}}$and β_{qT}

$$\frac{v_\varphi^{\mathrm{fluid}}}{v_\varphi^{\mathrm{qL}}} = \frac{\sin\alpha}{\sin\beta_{\mathrm{qL}}} \qquad \text{and} \qquad \frac{v_\varphi^{\mathrm{fluid}}}{v_\varphi^{\mathrm{qT}}} = \frac{\sin\alpha}{\sin\beta_{\mathrm{qT}}} \,. \tag{14}$$

Consequently, the angles $\beta_{\mathrm{qL}}$and β_{qT}determine the directions of refracted wave's slowness vectors, containing angles $\Psi_{\mathrm{qL}}$and Ψ_{qT} with corresponding energy fluxes. At the second interface, the Snell-Descartes law is implemented again, resulting in a set of separate parallel planar (transmitted) waves of the same direction as the incident wave.

When the opposite surfaces of the specimen are not perfectly parallel, i.e. the specimen is slightly wedge-shaped, the transmitted waves are planar again, but, their directions may vary from the direction of the incident wave. Furthermore, other deviations from a perfect rectangularity of a specimen may distort both the parallelism and the planarity of the transmitted waves.

3 Materials and methods

3.1 Etalon Samples

The composite anisotropic and isotropic materials of plate and tube shapes were used in this study. These materials were utilized as etalon specimens for the experimental device and methodology testing.

The plate specimens were made of unidirectional CFRP (Carbon Fibre Reinforced Plastic, manufacturer La-Composite Letov ATG, Ltd.) with orientation of fibres parallel to specimens surfaces. The material symmetry of specimens was presumed as transversally isotropic, where the rotational axis x_3 was given by direction of the fibres. The dimensions of specimens were 120 mm x 120 mm, the approximate thicknesses were 2, 3.8 and 8 mm. All five elastic constants (c11, c12, c13, c33 and c44) were known from previous ultrasonic measurements (PS/PR technique) [28, 26, 27].

The tube specimen was made from plexiglass (inner diameter = 24 mm , the outer diameter = 30 mm). The tube was provided with the slot in the axial direction, so the only one wall of

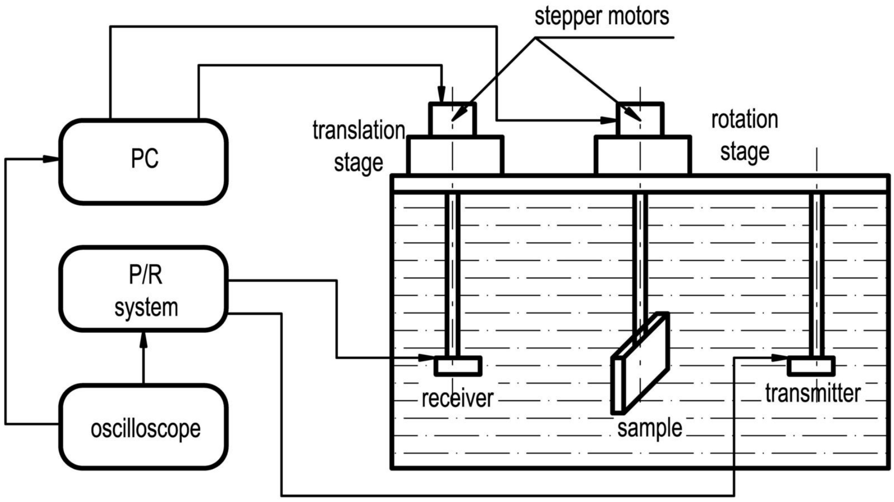

Figure 3: Experimental configuration

specimen was exposed to the wave propagation. The similar slot was cut into bone sample.

3.2 Bone sample

For the ultrasonic measurements, a dry bovine cortical femur diaphysis without marrow was used. Bovine bones were assumed to be orthotropic and x_3 axis was parallel to the bone fibers. The epiphyses and axial slot were cut to obtain cortical bone sample (fig. 2) and then marrow was removed. Afterwards, the bone was boiled, immersed into the lye and sterilized in the autoclave. The shape, thickness and curvature of specimens were measured by contact probe, which was located in CNC milling machine (fig 2).

We have decided to use a dry bone instead of a wet bone [18, 19, 21, 22, 30, 17], because we were interesting in precise determination of elastic properties separated from the natural visco-elastic behaviour of bones.

Because of the pulse character of our measurements, we require the examined material to be non-dispersive. In the other words, we need the measured phase velocities to be frequency-independent, containing, thus, only information about the elastic properties. However, the presence of any attenuation implies directly a dispersion [16]. That is why we have decided to minimize the bones attenuation by drying it. In future, the results for dried bones can be compared with these obtained on wet samples. Then the attenuation characteristics can be determined separately from the elastic ones, known from the dried bones.

Anyway, the elastic coefficients determined for the dried bones should be close to those obtained from conventional tensile tests, whereas the dynamic properties of bones result complexly from their visco-elasticity, material dispersion and attenuation.

3.3 Experimental setup

The ultrasonic immersion scanner (fig. 3) was designed to measure the time of flight (TOF) and the amplitude of received pulse after transmission through a sample. The specimen is rotated and immersed in water between two ultrasonic transducers. The transmitting transducer is fixed and receiving transducer is adjustable by translation stage. The scanner allows to measure longitudinal and quasi-transverse waves in a wide range of direction. The device can be modified for a pulse-echo measurement, using a tile as an acoustic mirror instead the receiving transducer.

The entire experimental process is controlled from a PC using two stepping motors for moving the sample and the receiver transducer. High frequency Pulse/Receiver system (JSR Ultrasonics

DPR 50+) is used for generating and receiving pulse, which is connected with two 1.25 MHz, 0.5 in diameter ultrasonic transducers made by Panametrics, Inc. The receiving signals from the sensors were recorded by a digital oscilloscope (DSO LeCroy 9304AM).

3.4 Inverse problem

Inverse problem consists in determination of elastic coefficients of the examined material from a set of phase velocities in various directions [4, 23]. Then the formulation of the corresponding inverse problem is quite simple. Let us recapitulate that the phase velocities for a given wave normal **n** are evaluated from eigenvalues of the Christoffel matrix (9), which is uniquely determined by this wave normal n and the elastic coefficients c_{ij}.

The superscript *exp* will be used to distinguish the experimentally obtained velocities (v_φ^{exp}) from those evaluated for known elastic coefficients via the above described theory (v_φ). Let a given wave normal **n** and a corresponding phase velocity $v_\varphi^{exp}(\mathrm{n})$ of at least one mode of propagation is available. Then the problem consists in determination of matrix $\Gamma(\mathbf{n})$ so that some of its eigenvalues are equal to the experimentally obtained velocity $v_\varphi^{exp}(\mathrm{n})$. For known class of symmetry, the structure of $\Gamma(\mathbf{n}))$ is known as well and the problem can be formulated by following nonlinear equation in c_{ij} :

$$\det[\Gamma(c_{ij}, \mathbf{n}) - \rho(v_\varphi^{exp}(\mathbf{n}))^2 \mathbb{I}] = 0 \,, \tag{15}$$

where $\mathbb{I}$ is the unit matrix.

However, the number of unknown elastic coefficients is usually too large to be uniquely determined from such single equation (15). Then the phase velocities is additional directions are taken and a system of nonlinear equations arises

$$\begin{array}{c} \det\left[\Gamma\left(c_{ij}, \mathbf{n}^{(1)}\right) - \rho\left(v_\varphi^{exp}\left(\mathbf{n}^{(1)}\right)\right)^2 \mathbb{I}\right] = 0 \\ \vdots \\ \det\left[\Gamma\left(c_{ij}, \mathbf{n}^{(N)}\right) - \rho\left(v_\varphi^{exp}\left(\mathbf{n}^{(N)}\right)\right)^2 \mathbb{I}\right] = 0 \end{array} \,, \tag{16}$$

for exactly correct values of $v_\varphi^{exp}\left(\mathbf{n}^{(1\cdots N)}\right) = v_\varphi\left(\mathbf{n}^{(1\ldots N)}\right)$ the equations (16) are not mutually independent and they can be satisfied all at once by correct values of c_{ij}. When the values $v_\varphi^{exp}\left(\mathbf{n}^{(1\cdots N)}\right)$ are experimentally distorted, the problem must be solved by an optimization procedure, which determines the coefficients c_{ij} so that the system (16) is optimally satisfied. As a suitable criterion for optimal satisfaction of (16) a least squares measure

$$\sum_{n=1}^{N} \left\{\det\left[\Gamma\left(c_{ij}, \mathbf{n}^{(n)}\right) - \rho\left(v_\varphi^{exp}\left(\mathbf{n}^{(1)}\right)\right)^2 \mathbb{I}\right]\right\}^2 \to \min_{c_{ij}} \,, \tag{17}$$

will be used.

When the solution of the direct problem can be obtained for every **n** and every c_{ij} in form $v_\varphi(c_{ij}, \mathbf{n})$, the whole problem can be reformulated into minimization of a quadratic sum Q

$$Q = \sum_{n=1}^{N} \left(v_\varphi\left(c_{ij},\ \mathbf{n}^{(n)}\right) - v_\varphi^{exp}\left(\mathbf{n}^{(n)}\right)\right)^2 \to \min_{c_{ij}} \,. \tag{18}$$

Let it be highlighted, that the wave normal **n** in (16), (17), (18) is known. This fact crucially simplifies both the formulation and the solution of the inverse problem for phase velocities in comparison with similar problem for group velocities [12, 25, 29, 2, 7, 8, 15].

For every wave normal **n** and for known elastic coefficients c_{ij} , all three eigenvalues of the Christoffel matrix (9) can be at least numerically evaluated. Then the resultant phase velocities

$v_\varphi^{(1,2,3)}$, corresponding to particular modes of propagation, can be compared with the experimental data $v_{\varphi\,1\ldots N}^{exp}$ in directions $\mathbf{n}_{1\ldots N}$. If $\tilde{c}_{ij}$ is a quess of elastic coefficients, the quadratic sum (18) takes form

$$Q = \sum_{n=1}^{N} \left(v_\varphi \left(\tilde{c_{ij}} \,, \mathbf{n}_n \right) - v_\varphi^{exp} \left(\mathbf{n}_n \right) \right)^2 \ . \tag{19}$$

The coefficients $(\mathrm{c_{ij}})$ that minimize the function (19) are sought.

For numerical multidimensional minimization, a preprogrammed Matlab routine fminsearch.m [1] is used, which employs the simplex search method. It is a direct search method that does not use numerical or analytic gradients. A simplex in n-dimensional space is characterized by the n+1 distinct vectors that are its vertices. In two-dimensional space, a simplex is a triangle; in three-dimensional space, it is a pyramid. At each step of the search, a new point in or near the current simplex is generated. The function value at the new point is compared with the function's values at the vertices of the simplex and, usually, one of the vertices is replaced by the new point, giving a new simplex. This step is repeated until the diameter of the simplex is less than the specified tolerance. Such method is especially proper when every evaluation of the minimized function is complicated and covers a considerably long time period, as it is in case of solution of the eigenvalue problem (10). The simplex method was proved suitable on many of similar optimizing procedures [12, 9, 5]

3.5 Error estimation

To estimate the accuracy of the optimization procedure's results, no appropriate analytical approach is available. The only possible solution is, thus, a Monte Carlo simulation, based on running the whole optimization process several times with randomly distorted input data. A Gaussian statistic made over the set of results is then expected to reveal the reliability of optimized coefficients [28, 20, 13]. In this work, we have treated the wave arrival times to be determined accurately. In the other words, the inaccuracy of the wave front arrival's detection was expected to be incomparably smaller than other possible sources of procedure's failure taken into account. These are the variability of the specimen's thickness and, even more important, the variability of the zero angle determination. Both of these inaccuracies were involved in our Monte Carlo simulations, considering the thickness and the specimen's orientation to be normally distributed about the correct values. Moreover, for the bone specimen, some variability of the mass density was admitted. Then the procedure was repeated 30 times to generate a representative set of output data.

Although this set cannot be expected to be governed by a normal distribution, its variability can be approximatively expressed by usual Gaussian statistic quantities, namely by the standard deviations

$$\mathrm{SD}_{ij} = \left(\frac{1}{n-1} \sum_{k=1}^{n} \left(c_{ij}^{(k)} - \overline{c_{ij}}^2 \right) \right)^{1/2} \ , \tag{20}$$

where by the overlining we denote the mean value of cij , averaged for all n passes of the inverse procedure. In our case, $n = 30$.

Then we present our results in a form

$$c_{ij} = c_{ij}^{(undistorted)} \pm \mathbf{SD}_{ij} \ , \tag{21}$$

where the original procedure's result $c_{ij}^{(undistorted)}$ is usually not exactly equal to the mean value c_{ij}.

Obviously, the presented standard deviations cannot be treated absolutely, but they bring a valuable insight in how sensitive and stable the optimization procedure is for an each particular coefficient.

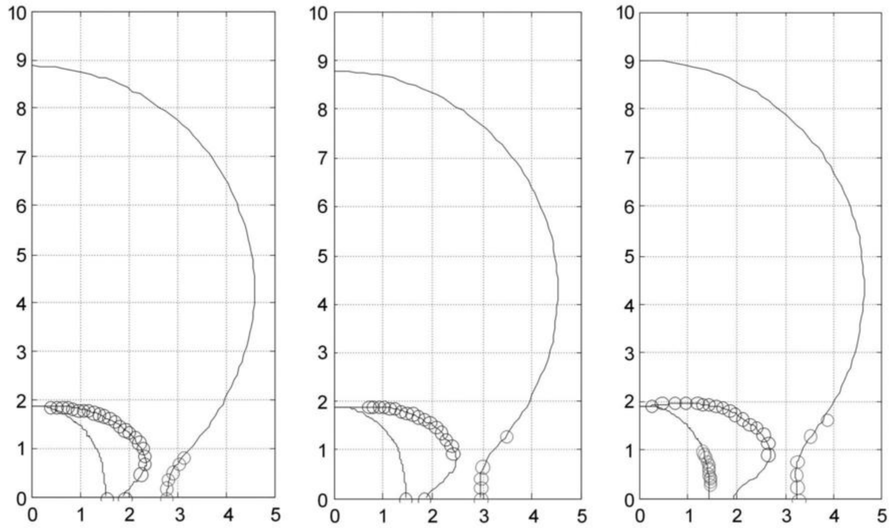

Figure 4: Plots of the phase velocities [mm/μs] propagating in the wave normal directions through CFRP plate specimens of 2, 4 and 8 mm thickness.

Circles - experimentally obtained data, Solid lines - phase velocities for calculated cij. Horizontal axis - phase velocity in x1 direction (perpendicular to plate), Vertical axis - phase velocity in x3 direction (fiber direction)

4 Results

4.1 Etalon sample

	specimenn 2 mm	specimen 4 mm	specimen 8 mm
thickness d [mm]	1.95	3.65	7.51
$\mathbf{c_{11}}$**[GPa]**	12.43±0.43	13.87±0.34	15.981±0.22
$\mathbf{c_{12}}$**[GPa]**	5.07±0.74	7.05±0.51	9.620±0.22
$\mathbf{c_{13}}$**[GPa]**	8.23±0.99	7.07±0.51	7.09±0.39
$\mathbf{c_{33}}$**[GPa]**	125.63±8.77	123.67±7.13	123.20±3.35
$\mathbf{c_{44}}$**[GPa]**	5.63±0.05	5.67±0.06	5.62±0.06
density ρ**[g/cm^3]**	1.6	1.6	1.52

Table 1: Resultant elastic coefficients c_{ij} of plate etalon specimens in form (21)

The normal surfaces (plot of phase velocity $v_\varphi(\mathbf{n})$ versus the wave normal direction) of measured phase velocities of acoustic waves propagating through CFRP plate specimens of different thicknesses in the x_1x_3 plane are demonstrated in figure 4. This figure represents experimentally obtained phase velocities and their fitting to the normal surfaces evaluated for c_{ij} resulting from the optimization (18). The values of c_{ij} expressed in form (21) calculated from optimization (18) are presented in the table 1. The PS/PR and immersion technique measured and calculated phase velocities of wave propagating through 8 mm thick CFRP specimen are compared in figure 5. Then, a similar experiment was performed in an axial plane of an isotropic plexiglass tube (fig 6). The results (fig 7) are in a good agreement with the considered isotropic model of propagation, being not influenced by the cylindrical shape of the specimen at all.

thickness d [mm]	**Bovine specimenn** 8.75±0.5	**Pithioux et al. [18, 21]**
c_{11}[GPa]	27.39±1.8330	23.50
c_{33}[GPa]	34.11±2.1969	34.60
c_{44}[GPa]	13.09±3.3756	9.20
c_{13}[GPa]	9.07±5.7503	8.35
density ρ[g/cm^3]	1.8±0.1	1.8

Table 2: Resultant elastic coefficients c_{ij} of bovine bone specimens in form (21) and comparison with [18, 21].

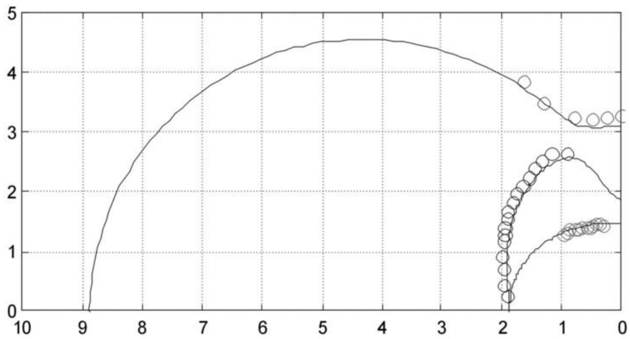

Figure 5: Comparison of immersion measurement and PS/PR measurement [26, 27] of CFRP plate specimen(thickness 8 mm)

Circles - immersion measurement, Solid lines - PS/PR measurement Horizontal axis - phase velocity in x3 direction, Vertical axis- phase velocity in x1

4.2 Bone Sample

The dry bovine cortical bone sample, described in details in paragraph 3.2., was used for our experimental study. The acoustic measurements of qL and qT waves propagating through the specimen immersed in water were performed on ultrasonic immersion scanner (paragraph 3.3.) with following configuration. The bone was horizontally oriented in fiber direction and fastened in the rotational stage between fixed transducer and acoustic mirror. The transducer served as a transmitter and receiver (pulse/echo arrangement). Measurements were performed in one particular place of the bone, approximately in the median part of bone with known shape and thickness (CNC milling machine measurement). The phase velocities of qL and qT waves propagating through the rotating specimen were detected for a broad range of directions. The elastic coefficients were calculated by inverse optimization (18) from the set of measured phase velocities and the stability of resulting data was evaluated by the statistical method (21). As mentioned above, only some of the nine independent elastic coefficients can be determined by this method, namely c_{11}, c_{33}, c_{55}, and c_{13}. The figure 8 shows experimentally obtained phase velocities. Calculated elastic coefficients and its comparison with literature [18] are mentioned in table 2.

The experimental configuration with bone fibers oriented vertically and estimation of remaining coefficients will be shortly performed in the future.

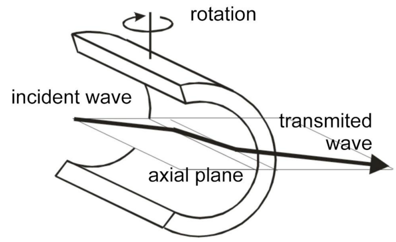

Figure 6: Measurements in an axial plane of a cylindrical specimen.

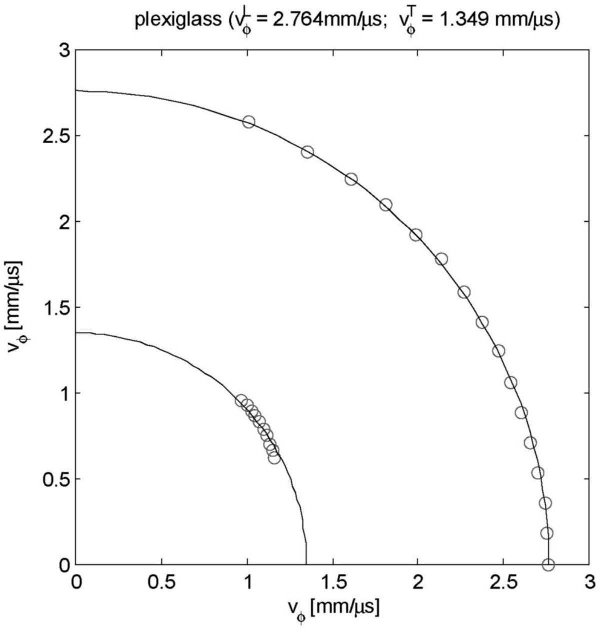

Figure 7: Plots of the phase velocities [mm/μs] propagating through cylindrical specimen. Circles - experimentally obtained data, Solid lines - phase velocities for calculated c_{ij}.

5 Conclusion

In this paper, the ultrasonic immersion scanning technique was used for the determination of elastic coefficients of the orthographically considered bone samples. This technique makes possible to monitor quasi-longitudinal (qL) and quasi-transverse (qT) waves generated from the wave diffraction on liquid/sample interface for a wide range of sample rotations. The measured set of qL and qT phase velocities is used as an input to inversion procedure for the determination of all elastic constants. The stability of inversion optimization is estimated by the Monte-Carlo based statistical method. One of the advantages of the present method is an accuracy of results, while the rough preparation of specimens is sufficient; samples are not required to have precise dimensions and perfectly parallel faces (in comparison with the pulse-transition or pulse-echo contact techniques). The experimental methodology was verified on etalon samples (transversely isotropic composite plates, isotropic cylinder) with known elastic coefficients.

The dry bovine bone sample (femoral diaphysis, assumption of orthotropic material symmetry) was used in this study to eliminate the attenuation characteristics of the sample, which are strongly dependent on temperature, age, blood flow etc. The dried bone specimen is suitable for an evaluation of elastic coefficients of elastic but geometrically complex body. In the experiments, only one measurement point and one sample orientation was measured, so only four of nine elastic coefficients was determined. However, the obtained results are in a good agreement with the

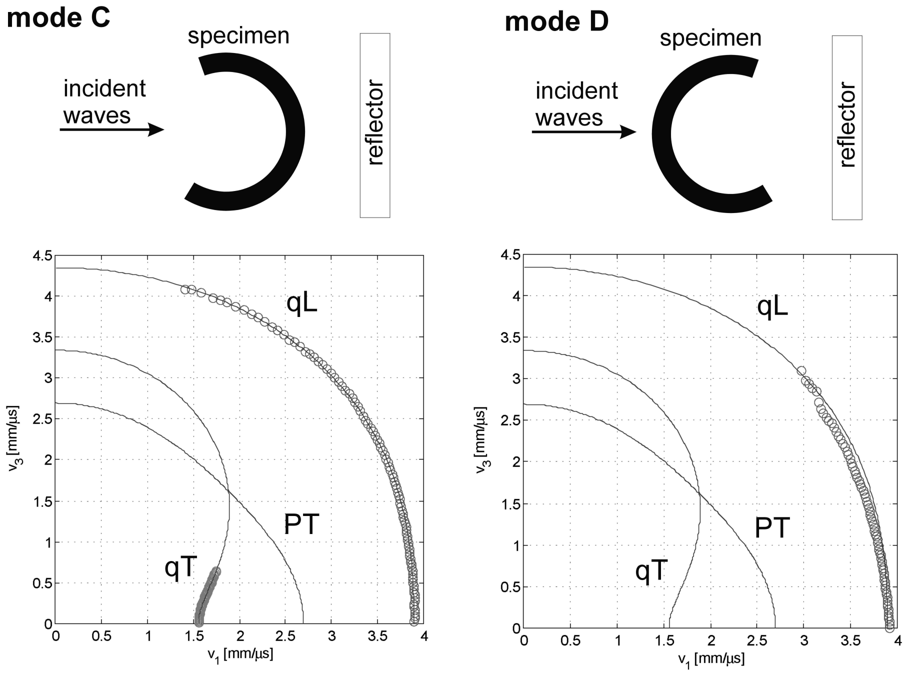

Figure 8: Figure 8: Plots of the phase velocities [mm/μs] propagating through bone specimen
Circles - experimentally obtained data, Solid lines - phase velocities for calculated c_{ij}. Horizontal axis - phase velocity in x_3 direction (fiber direction), Vertical axis- phase velocity in x_1.

literature [18, 21, 17]. The remaining constants and different bone locations will be measured in the closest future.

Moreover, the viscoelastic behavior as well as the influence of fluid components on attenuation of compact bone should be investigated via measuring the velocities and attenuation at various frequencies. The heterogeneous characteristic such an age, weight, sample preparation and storage should be taken into the account. And finally, the influence of surrounding soft tissues together with the recognition of some optimal etalon case should be researched.

6 Acknowledgements

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