Analytical and LMI based design for the Acrobot traking with aplication to robot walking

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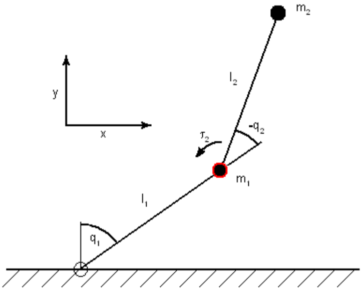
The model of the acrobot
Acrobot

- underactuated mechanical system
- the acrobot is a special case of n-link with $n-1$ actuators
- underactuated angle is at the pivot point



The model of the acrobot

Euler-Lagrange theory

- The acrobot can be modelled by usual Lagrangian approach

$$\mathcal{L}(q,\dot{q}) = K - V = \frac{1}{2}\dot{q}^T D(q)\dot{q} - V(q)$$

- The resulting Euler-Lagrange equation

$$\begin{bmatrix} \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_1} - \frac{\partial \mathcal{L}}{\partial q_1} \\ \vdots \\ \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_n} - \frac{\partial \mathcal{L}}{\partial q_n} \end{bmatrix} = u = \begin{bmatrix} 0 \\ \tau_2 \\ \vdots \\ \tau_n \end{bmatrix}$$

The model of the acrobot

Euler-Lagrange theory

- The Euler-Lagrange equation leads to a dynamic equation

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = u$$

$D(q)$ is the inertia matrix, $C(q,\dot{q})$ contains Coriolis and centrifugal terms, $G(q)$ contains gravity terms, u is vector of external forces

- Kinetic symmetry

$$D(q) \equiv D(q_2)$$

The model of the acrobot
Partial exact feedback linearization

- System transformation into a new system of coordinates that display linear dependence between some output and new input
- Two independent function with relative degree 3

$$\sigma = \frac{\partial \mathcal{L}}{\partial \dot{q}_1} = (\theta_1 + \theta_2 + 2\theta_3 \cos q_2)\dot{q}_1 + (\theta_2 + \theta_3 \cos q_2)\dot{q}_2$$

$$p = q_1 + \frac{q_2}{2} + \frac{2\theta_2 - \theta_1 - \theta_2}{\sqrt{(\theta_1 + \theta_2)^2 - 4\theta_3^2}} \arctan \left(\sqrt{\frac{\theta_1 + \theta_2 - 2\theta_3}{\theta_1 + \theta_2 + 2\theta_3}} \tan \frac{q_2}{2} \right)$$

The model of the acrobot

Partial exact feedback linearization

- The transformation

$$T: \quad \xi_1 = p, \xi_2 = \sigma, \xi_3 = \dot{\sigma}, \xi_4 = \ddot{\sigma}$$

- Connection σ and p with $\mathcal{L}$

$$\dot{p} = d_{11}(q_2)^{-1}\sigma,$$
$$\dot{\sigma} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_1} = \frac{\partial \mathcal{L}}{\partial q_1} = -\frac{\partial V}{\partial q_1}$$

- Acrobot's dynamics in partial exact linearized form

$$\dot{\xi}_1 = d_{11}(q_2)^{-1}\xi_2,\ \dot{\xi}_2 = \xi_3,\ \dot{\xi}_3 = \xi_4,$$
$$\dot{\xi}_4 = \alpha(q,\dot{q})\tau_2 + \beta(q,\dot{q}) = w$$

- Reference system

$$\dot{\xi}_1^r = d_{11}^{-1}(q_2^r)\xi_2^r,\ \dot{\xi}_2^r = \xi_3^r,\ \dot{\xi}_3^r = \xi_4^r,\ \dot{\xi}_4^r = w^r$$

The model of the acrobot

Partial exact feedback linearization

- Denoting $e := \xi - \xi^r$

$$\dot{e}_1 = d_{11}^{-1}(\phi_2(\xi_1, \xi_3))\xi_2 - d_{11}^{-1}(\phi_2(\xi_1^r, \xi_3^r))\xi_2^r$$
$$\dot{e}_2 = e_3, \quad \dot{e}_3 = e_4, \quad \dot{e}_4 = w - w^r$$

- Computations based on the Taylor expansions

$$\dot{e}_1 = \mu_1(t)e_1 + \mu_2(t)e_2 + \mu_3(t)e_3 + o(e)$$
$$\dot{e}_2 = e_3, \quad \dot{e}_3 = e_4, \quad \dot{e}_4 = w - w^r$$

- To ensure $e(t) \rightarrow 0$ for $t \rightarrow \infty$ we use feedback

$$w = w^r + \overline{K}_1(t)e_1 + \overline{K}_2(t)e_2 + \overline{K}_3(t)e_3 + \overline{K}_4(t)e_4$$

- State feedback controller $\overline{K}_{1,2,3,4}(t)$ for the reference trajectory tracking

LMI based design for the Acrobot walking

LMI design of gains $K_{1,2,3,4}$

- Open-loop continuous-time and time-varying linear system, state feedback controller

$$\dot{e} = A(t)e + Bu, \quad u = Ke$$

- Closed-loop system

$$\dot{e} = (A + BK)\, e = \begin{pmatrix} \mu_1(t) & \mu_2(t) & \mu_3(t) & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ K_1 & K_2 & K_3 & K_4 \end{pmatrix} e,$$

- Bounds for $\mu_1(t), \mu_2(t), \mu_3(t)$ are known

- Lyapunov equation is solved for all values of $\mu_1(t), \mu_2(t), \mu_3(t)$

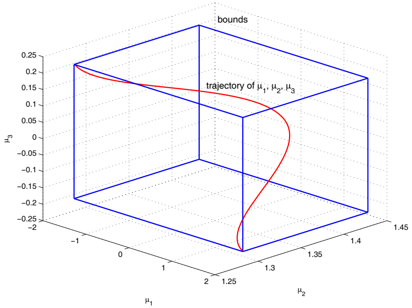
$$(A(\mu) + BK)^{\mathrm{T}} S + S\,(A(\mu) + BK) \preceq 0, \quad S = S^{\mathrm{T}} \succ 0$$

LMI based design for the Acrobot walking

Bounds for LMI

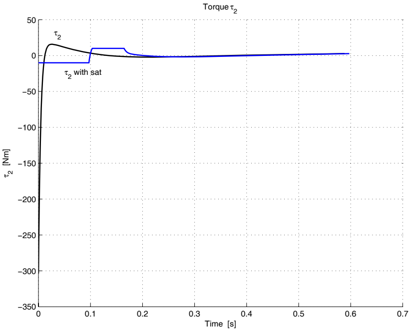
Convex set is defined in the form

- rectangular box
- prismatic box



LMI based design for the Acrobot walking

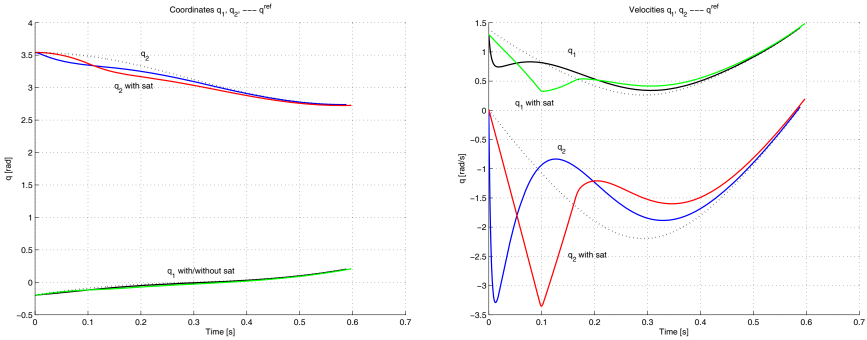
Simulations - Torque



- Yalmip and SEDUMI
- $(K_1, K_2, K_3, K_4) = -10^4 \times (1.9087, 1.2097, 0.1781, 0.0090)$
- saturation limit in the range $\pm 10\,\mathrm{Nm}$

LMI based design for the Acrobot walking

Simulations - Coordinates and Velocities



LMI based design for the Acrobot walking

Animations with saturation $\pm 10\,\mathrm{Nm}$

(Loading movie...)

Analytical design of the Acrobot exponential tracking

Analytical desig of gains $K_{1,2,3,4}$

- Using the following notation

$$\begin{aligned} \overline{e}_1 &= e_1 - \mu_3(t)e_2, \quad \overline{\mu}_2(t) = \mu_2(t) + \mu_1(t)\mu_3(t) - \dot{\mu}_3(t) \\ \widetilde{K}_1 &= \overline{K}_1(t) \\ \widetilde{K}_2 &= \overline{K}_2(t) + \mu_3(t)\overline{K}_1(t) \\ \widetilde{K}_3 &= \overline{K}_3(t) \\ \widetilde{K}_4 &= \overline{K}_4(t) \end{aligned}$$

- The previous system takes the following form

$$\begin{aligned} \dot{\overline{e}}_1 &= \mu_1(t)\overline{e}_1 + \overline{\mu}_2(t)e_2 \\ \dot{e}_2 &= e_3, \quad \dot{e}_3 = e_4, \\ \dot{e}_4 &= \widetilde{K}_1\overline{e}_1 + \widetilde{K}_2 e_2 + \widetilde{K}_3 e_3 + \widetilde{K}_4 e_4 \end{aligned} \tag{1}$$

Analytical design of the Acrobot exponential tracking

Theorem

Theorem

Suppose $\forall t\ \mu_1(t) \in \left[{\mu_1}^{min}, {\mu_1}^{max}\right]$, $0 < {\mu_2}^{min} \leq \mu_2(t) \leq {\mu_2}^{max}$ *and let* K_1, K_2, K_3, K_4 *are such that*

- $K_1 < \frac{K_2 \mu_1(t)}{\overline{\mu}_2(t)}$,
- $\lambda^3 + K_4\lambda^2 + K_3\lambda + K_2$ *is Hurwitz.*

Then $\exists\,\Theta$ *such that (1) is exponential stable for*

$$\widetilde{K}_1(t) = \Theta^3 K_1,\ \ \widetilde{K}_2(t) = \Theta^3 K_2,\ \ \widetilde{K}_3(t) = \Theta^2 K_3,\ \ \widetilde{K}_4(t) = \Theta K_4$$

Analytical design of the Acrobot exponential tracking

Summarizing

Summarizing

The system

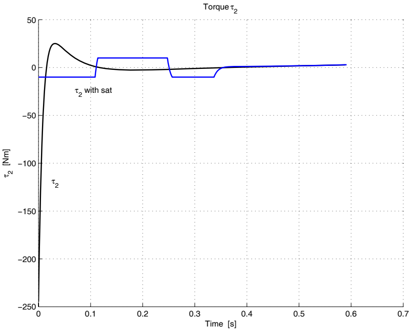
$$\begin{aligned}\dot{\bar{e}}_1 &= \mu_1(t)\bar{e}_1 + \bar{\mu}_2(t)e_2\\ \dot{e}_2 &= e_3\\ \dot{e}_3 &= e_4\\ \dot{e}_4 &= w - w^r\end{aligned}$$

is exponential stable for

$$w = w^r + \Theta^3 K_1\bar{e}_1 + \Theta^3\left(K_2 + \mu_3(t)K_1\right)e_2 + \Theta^2 K_3 e_3 + \Theta K_4 e_4$$

Analytical design of the Acrobot exponential tracking

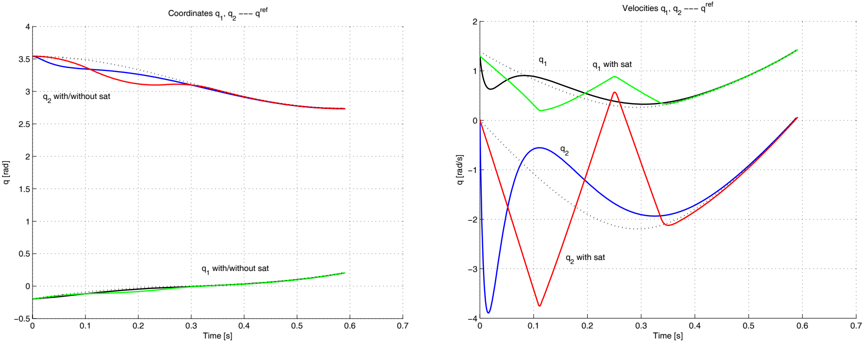
Simulations - Torques



- $(K_1, K_2, K_3, K_4) = -(1.5 \times 6, 6, 12, 8)$
- $\Theta = 20$
- saturation limit in the range $\pm 10\,\mathrm{Nm}$

Analytical design of the Acrobot exponential tracking

Simulations - Coordinates and Velocities



Analytical design of the Acrobot exponential tracking

Animations with saturation $\pm 10\,\mathrm{Nm}$

(Loading movie...)

Impact model for Acrobot

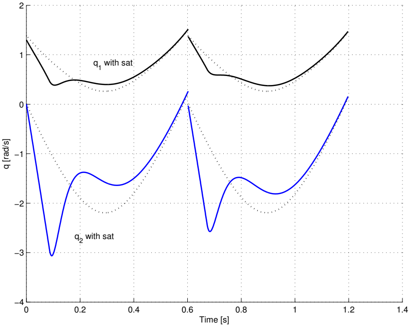
Impact model

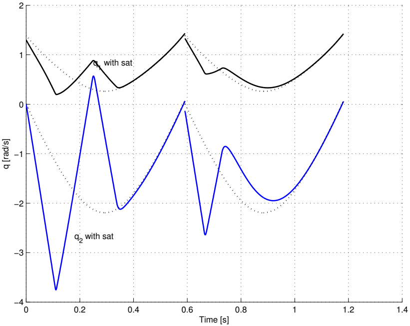
- Occurs when the swing leg touches the walking surface
- The impact between the swing leg and the ground is modeled as a contact between two rigid bodies
- The positions q do not change during the impact $q^+ = q^-$
- Dynamic model of the Acrobot has to be enlarged by reaction force effects

$$D_e(q_e)\ddot{q}_e + C_e(q_e, \dot{q}_e)\dot{q}_e + G_e(q_e) = B_e u + \delta F_{ext}$$

$q_e = (q_1, q_2, {p^h}_H, {p^v}_H)$,
δF_{ext} the vector of external forces

Impact model for Acrobot
Simulations





Conclusions and outlooks

Conclusions

- Two methods for the Acrobot exponential tracking compared
- Both methods give quite large torques but saturation to realistic
- Impact model for the Acrobot presented values works perfectly in simulations

Outlooks

- Propose the reference trajectory that the initial conditions of new step after impact are equal to initial conditions of the reference step

Conclusions and outlooks

Thank you for your attention