# On Time Parameterizations of User Demands in Mechatronics

10th PhD Workshop - Hluboká nad Vltavou



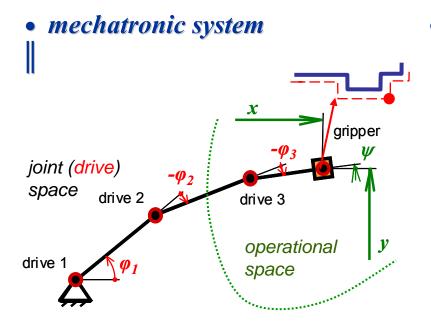
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#### **Outline:**

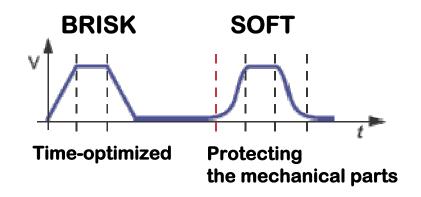
- **Ⅲ User Demands in Mechatronics (Intro)**
- **├** Concepts of Time Parameterization
- □ Dynamical point of view of time parameterization
  - Range-Space Modification example
  - **□** End-Point Modification example
- | Kinematical point of view of time parameterization
- Several Notes as Conclusion

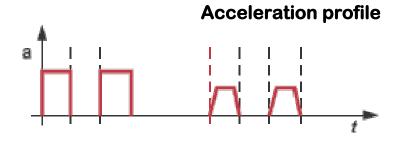
#### **User Demands in Mechatronics**



- # mechanical elements
  (beams, joints, gears, grippers)
- | electro-mechanical (drives, sensors)
- electrical
   (control units)

 combination of technological demands and construction limits of the system: to accomplish <u>assigned path</u> or to reach <u>predetermined key point</u> with certain <u>velocity</u> and acceleration profiles . . .





#### **Concepts of Time Parameterization**

- kinematical point of view
  - ⇒ curve-based planning
- length of path
- approximate time for execution of path
- geometrical parameter

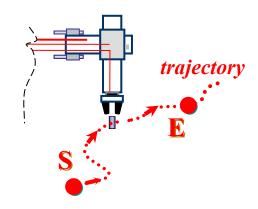
(selection of acceleration polynomial with border conditions:

$$\begin{array}{c}
p = s(t) \\
\dot{p} = v(t) \\
\ddot{p} = a(t)
\end{array}$$
 $s, v, a \rightarrow 3^{rd} \ order / s, v, a, a' \rightarrow 5^{th} \ order)$ 

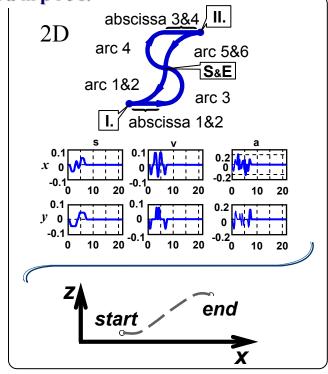
$$in 1D - parameter \ t \ (time)$$

- coordinates and their time derivatives (param. p(t))

  - 2D, 3D general curves (selection from table by parameter)
- dynamical point of view
  - ⇒ range space design
  - *⇒* point-to-point design
- free ranges and Start and End points are given or
- only Start and End points are given
- timing via specific modifications
   of predictive control algorithms
   ⇒ coordinates and their time derivatives



Examples:



#### Dynamical point of view of time parameterization via Predictive Control:

#### Model **Cost function (criterion):**

$$\hat{\mathbf{x}}_{k+1} = \mathbf{A} \, \mathbf{x}_{k} + \mathbf{B} \, \mathbf{u}_{k} \\
\hat{\mathbf{y}}_{k+1} = \mathbf{C} \, \hat{\mathbf{x}}_{k+1}$$

$$J = \sum_{j=k}^{k+N} \left( (\hat{\mathbf{y}}_{j+1} - \mathbf{w}_{j+1})^{T} \, \mathbf{Q}_{y} \, (\hat{\mathbf{y}}_{j+1} - \mathbf{w}_{j+1}) + (\mathbf{u}_{j} - \mathbf{u}_{j-1})^{T} \, \mathbf{Q}_{u} \, (\mathbf{u}_{j} - \mathbf{u}_{j-1}) \right)$$

#### Equations of predictions $\Rightarrow \hat{y} = f + G \Delta u$

$$\hat{\mathbf{y}} = [\hat{y}_{k+1}, \dots, \hat{y}_{k+N+1}]^T, \quad \mathbf{u} = [u_k, \dots, u_{k+N}]^T$$

### $\hat{y}$ k, k+1, k+2

#### Minimization of quadratic criterion:

$$\min_{\mathbf{u}} \mathbf{J} = \min_{\mathbf{u}} \mathbf{J} \mathbf{J}, \quad (\min_{\mathbf{u}} (quadratic \ form)), \quad u_{opt} = \arg\min_{\mathbf{u}} \mathbf{J}$$

#### **Range-Space Modification**

$$J_{k} = \sum_{j=k}^{k+N} \{ \| (\hat{y}_{j+1} - r_{a_{j+1}}) \mathbf{Q}_{ra} \|^{2}$$

$$J_{k} = \sum_{j=N_{O}+1}^{k+N} \{ \| (\hat{y}_{k+j} - w_{k+j}) \overline{\mathbf{Q}}_{y} \|^{2}$$

$$+ \| (\hat{y}_{j+1} - r_{b_{j+1}}) \mathbf{Q}_{rb} \|^{2} + \| \mathbf{u}_{j} \mathbf{Q}_{u} \|^{2} \}$$

$$+ \| \mathbf{u}_{k+j-1} \overline{\mathbf{Q}}_{u} \|^{2} \}$$

$$+ \| \mathbf{u}_{k+j-1} \overline{\mathbf{Q}}_{u} \|^{2} \}$$

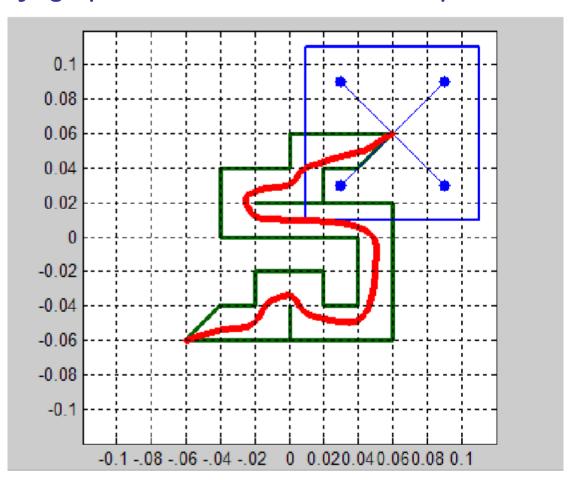
#### **End-Point Modification**

$$J_{k} = \sum_{j=No+1}^{k+N} \{ \| (\hat{y}_{k+j} - w_{k+j}) \overline{\mathbf{Q}}_{y} \|^{2} + \| \mathbf{u}_{k+j-1} \overline{\mathbf{Q}}_{u} \|^{2} \}$$

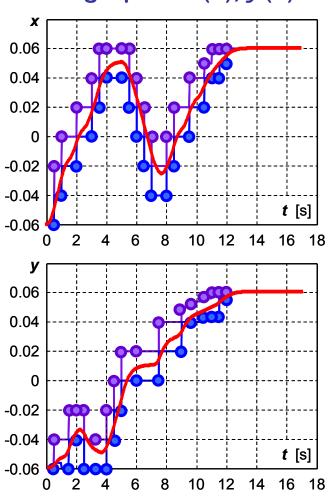
$$= \text{const.}$$

#### Range-Space Modification - example

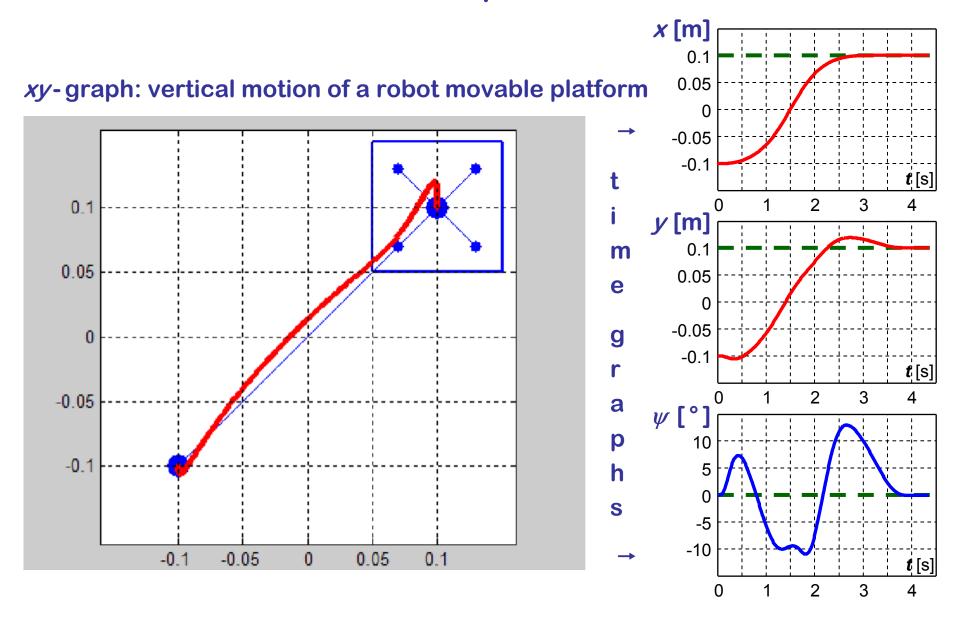
#### xy-graph: motion of a robot movable platform



#### time – graphs: x(t), y(t)



#### **End-Point Modification - example**



#### **Several Notes as Conclusion**

- Suitable time parameterization can provide:
  - safe utilization of machinery
  - optimal duration of the motion process
  - optimal path of the machinery motion
- In the presentation, there were addressed:
  - kinematical approach based on analytical geometry and kinematical laws
  - dynamical approach realized by specific control tasks and using appropriate dynamical models of systems
- The procedures of time parameterization together with path optimization are continuously under development due to continuous efforts to minimize production costs at keeping the profit from number of products
  - ⇒ minimize operational time (path optimization ...)
  - ⇒ maximize machinery use (velocity and acceleration profile optimization)

# On Time Parameterizations Tof User Demands in Mechatronics

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, Thank you

for your

attention

