arXiv:astro-ph/0205271 v1 16 May 2002

Space Time Fluctuations and Ultra High Energy Cosmic Ray Interactions

R. Aloisio [a], P. Blasi [b], A. Galante [c], P.L. Ghia [d,e], A.F. Grillo [a]

[a] *INFN - Laboratori Nazionali del Gran Sasso, SS. 17bis Assergi (L'Aquila) - Italy*

[b] *INAF - Osservatorio Astrofisico di Arcetri, Largo E. Fermi 5 50125 Firenze - Italy*

[c] *Dipartimento di Fisica, Università di L'Aquila, Via Vetoio 67100 Coppito (L'Aquila) - Italy*

[d] *CNR - IFSI, Sezione di Torino, Corso Fiume 4, 10133 Torino - Italy*

[e] *INFN - Sezione di Torino, Via P. Giuria 1, 10125 Torino - Italy*

Abstract

The intimate geometry of space-time is expected to suffer stochastic fluctuations as a result of quantum gravitational effects. These fluctuations may induce observable consequences on the propagation of high energy particles over large distances, so that the strength and the characteristics of these fluctuations may be constrained, mainly in the range of energies of interest for cosmic ray physics. While invoked as a possible explanation for the detection of the puzzling cosmic rays with energies in excess of the threshold for photopion production (the so-called super-GZK particles), we demonstrate here that lower energy observations may provide strong constraints on the role of a fluctuating space-time structure.

1 Introduction

In the recent past it has been pointed out [1–5] that observations in cosmic ray physics, and in particular Ultra High Energy Cosmic Ray (UHECR) experiments can be used to constrain to high precision the possibility that the fundamental Lorentz invariance (LI) may be broken to some level.

In fact, violations of LI have been invoked by many authors [3,4] to explain puzzling observations related to the detection of cosmic rays with energy in excess of $\approx 10^{20}$ eV, the so-called Greisen-Zatsepin-Kuzmin (GZK) [6] cut-off, and the more uncertain excess of photons in the TeV energy region from distant sources. The two phenomena have in common that in both cases the detected particles should instead suffer significant attenuation since their energy is above the threshold for photopion production (in the first case) and pair production [7] (in the second case). In particular the GZK cutoff has been expected for a long time because of the dramatic energy losses of high energy protons with the $3^o K$ Cosmic Microwave Background Radiation (CMBR). Violations of LI in both cases may act on the kinematics of the processes so to move the thresholds in regions which are possibly outside the observational window. In the following we mainly concentrate upon the case of UHECRs, pointing out, when necessary, the differences with respect to the case of pair production.

The recipes for the violations of LI that have been suggested insofar generally consist of requiring an *explicit* modification of the dispersion relation of UHE particles, due to their propagation in the "vacuum", now affected by quantum gravity (QG). This effect is generally parametrized by introducing a typical mass, expected to be of the order of the Plank mass (M_P), that sets the scale for QG to become effective.

In fact, explicit modifications of the dispersion relation are not really necessary, as was recently pointed out in Refs. [8–11] for the case of propagation of UHECRs. There is an intrinsic uncertainty in the process of measurement of physical quantities on time (space) scales comparable with the Planck time (length). It is generally believed that coordinate measurements cannot be performed with precision better than Planck distance (time) $\delta x \geq l_P$, since such a measurement would result in the production of a black hole. This means that the metric of space-time must feature quantum fluctuations on the Planck scale. A similar line of thought implies that an uncertainty in the measurement of energy and momentum of particles must occur, as described by the relation $\delta p \simeq \delta E \simeq E^2/M_P$. According to the authors of Refs. [9,10] the apparent problem of super-GZK particles may find a solution also in the context of this uncertainty approach.

We discuss here this appealing approach more in detail, by taking into account the effects of the propagation of CRs in the QG vacuum in the presence of the universal microwave background radiation. A fluctuating metric implies that different measurements of the particle energy or momentum may result in different outcomes. Therefore it becomes important to define the probability that the *measured* energy (momentum) of a particle is above some fixed value. Note that averaging over a large number of measurements would yield the *classical* values for the energy and momentum. The process of measurement mentioned above, during the propagation of particles over cosmological distances occurs at each single interaction of the particle with the environment. At each interaction vertex, the fluctuating energy/momentum of the particle is compared with the kinematic threshold for the occurrence of some physical process (in our case the photopion production). A clear consequence of this approach is that particles with classical energy below the standard Lorentz invariant threshold have a certain probability of interacting. In the same way, particles above the classical threshold have a finite probability of evading interaction.

We show here that the most striking consequences of the approach described above derive from low energy particles rather than from particles otherwise above the threshold for photopion production.

2 The effect of Space-Time fluctuations on propagation of UHE particles.

While electroweak and strong interactions propagate through space-time, gravity turns out to be a property of the space-time itself. This simple statement has profound implications in that our belief that gravity can be turned into a quantum theory immediately implies that the structure of space-time has quantum fluctuations. Another way of rephrasing this concept is that space-time is expected to have a granular (or foamy) structure, where however the size of space-time cells fluctuates stochastically, thereby causing an intrinsic uncertainty in the measurements of space-time lengths, and indirectly of energy and momentum of a particle moving through space-time. The uncertainty appears on scales comparable with the Planck scale.

It is generally argued that measurements of distances (times) smaller than the Planck length (time) are conceptually unfeasible, since the process of measurement collects in a Planck size cell an energy in excess of the Planck mass, hence forming a black hole, in which information is lost. This can be traslated in different ways into an uncertainty on energy-momentum measurements ([9,10]). The Planck length is a good estimate of the uncertainty in the De Broglie wave-length λ of a particle with momentum p. Therefore $\delta\lambda \approx l_P$, and $\delta p = \delta(1/\lambda) \approx (p^2 l_P) = (p^2/M_P)$.

Speculating on the exact characteristics of the fluctuations induced by QG is beyond the scope of the present paper, and it would probably be useless anyway, since the current status of QG approaches does not allow such a kind of knowledge. We decided then to adopt a purely phenomenological approach, in which some reasonable assumptions are made concerning the fluctuations in the fabric of space-time, and their consequences for the propagation of high energy particles are inferred. Comparison with experimental data then possibly constrains QG models.

Following [9], we assume that in each measurement:

- the values of energy (momentum) fluctuate around their average values (assumed to be the result theoretically recoverable for an infinite number of measurements of the same observable):

$$E \approx \bar{E} + \alpha \frac{\bar{E}^2}{M_P} \tag{1}$$

$$p \approx \bar{p} + \beta \frac{\bar{p}^2}{M_P} \tag{2}$$

  with α, β normally distributed variables and p the modulus of the 3-momentum (for simplicity we assume rotationally invariant fluctuations);
- the dispersion relation fluctuates as follows:

$$P_\mu g^{\mu\nu} P_\nu = E^2 - p^2 + \gamma \frac{p^3}{M_P} = m^2 \tag{3}$$

  and γ is again a normally distributed variable.

Ideally, QG should predict the type of fluctuations introduced above, but, as already stressed, this is currently out of reach, therefore we assume here that the fluctuations are gaussian. Our conclusions are however not sensitive to this assumption: essentially any symmetrical distribution with variance ≈ 1, within a large factor, would give essentially the same results. Furthermore we *assume* that α, β and γ are uncorrelated random variables; again, this assumption reflects our ignorance in the dynamics of QG [1].

Our interest will be now concentrated upon processes of the type

$$a + b \to c + d$$

where we assume that a kinematic threshold is present; in the realm of UHECR physics (a,b) is either (γ, γ_{3K}) or (p, γ_{3K}) and (c,d) is (e^+, e^-) or (N, π).

To find the value of initial momenta for which the reaction occurs we write down energy-momentum conservation equations and solve them with the help of the dispersion relations, as discussed in detail in [5].

The energy momentum conservation relations are (in the laboratory frame, and specializing to the case in which the target (b) is a low energy background

[1] The fluctuations described above will in general derive from metric fluctuations of magnitude $\delta g^{\mu\nu} \sim h^{\mu\nu} \frac{l_P}{l}$ [3,10]. Our assumption reflects the fact that, while the magnitude of the fluctuation can be guessed, we do not make any assumption on its tensorial structure $h^{\mu\nu}$.

photon for which fluctuations can be entirely neglected)

$$E_a + \alpha_a \frac{E_a^2}{M_p} + \omega = E_c + \alpha_c \frac{E_c^2}{M_p} + E_d + \alpha_d \frac{E_d^2}{M_p} \tag{4}$$

$$p_a + \beta_a \frac{p_a^2}{M_p} - \omega = p_c + \beta_c \frac{p_c^2}{M_p} + p_d + \beta_d \frac{p_d^2}{M_p}. \tag{5}$$

These equations refer to head-on collisions and collinear reaction products, which is appropriate for threshold computations. Together with the modified dispersion relations, these equations, after some manipulations, lead to a cubic equation for the initial momentum as a function of the momentum of one of products, and, after minimization, they define the threshold for the process considered. In figure 1 we report the distribution of thresholds in the $\approx 70\%$ of cases in which the solution is physical; in the other cases the kinematics does not allow the reaction.

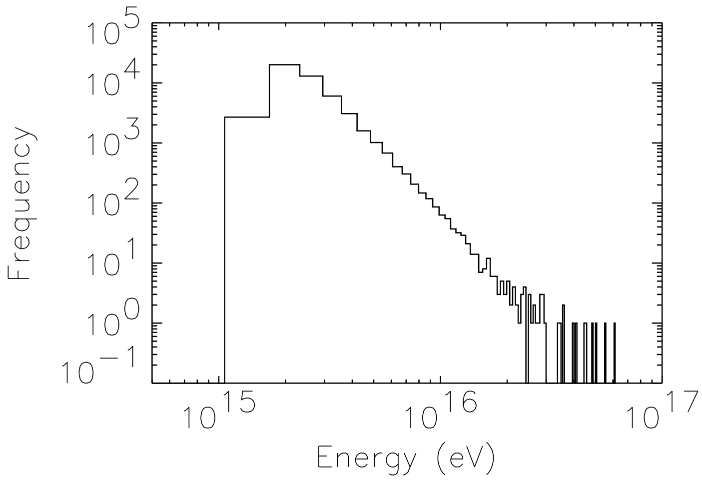

Fig. 1. *Threshold distribution for* $p\gamma_{3^\circ K} \rightarrow N\pi$. *In the* 30% *of cases the reaction is not allowed.*

This threshold distribution can be interpreted in the following way: a particle with energy above $\sim 10^{15}$ eV has essentially 70% probability of being above threshold, and therefore to be absorbed. In the other 30% of the cases the protons do not interact.

In (4,5) the fluctuations are taken independently for each particle, which is justified as long as the energies are appreciably smaller than the Planck energy. At that point it becomes plausible that different particles experience the same fluctuations, or more precisely fluctuations of the same region of space-time. It is instructive to consider this case in some more detail: we introduce then the four-momenta (and dispersion relations) of *all* particles fluctuating in the same way. Specializing to proton interaction on CMBR, the equation which defines the threshold p_{th} is [5]:

$$\eta \frac{2p_0^3}{(m_\pi^2 + 2m_\pi m_p)M_P} \frac{m_\pi m_p}{(m_\pi + m_p)^2} \left(\frac{p_{th}}{p_0}\right)^3 + \left(\frac{p_{th}}{p_0}\right) - 1 = 0 \tag{6}$$

where η is a gaussian variable with zero average and variance of the order of (but not exactly equal to) one, and p_0 is the L.I. threshold (GZK). The threshold is the positive solution of this equation.

The coefficient of the cubic term is very large, of the order of 10^{13} in this case, so that unless η is $O(10^{-13})$, we can write, neglecting pion mass

$$p_{th} \approx p_0 \left(\frac{m_p^2 M_P}{\eta p_0^3}\right)^{\frac{1}{3}}. \tag{7}$$

When η becomes negative, the above equation has no positive root; this happens essentially in 50% of the cases. Since the gaussian distribution is flat in a small interval around zero, the distribution of thresholds for positive η peaks around the value for $\eta \approx 1$, meaning that the threshold moves almost always down to a value of $\approx 10^{15}$ eV [5]; essentially the same result holds for fluctuations affecting only the incident (higest energy) particle. For *independent* fluctuations of final momenta, the asymmetry in the probability distribution of allowed thresholds arises from the fact that even exceedingly small negative values of the fluctuations lead to unphysical solutions.

Building upon our findings, we now apply the same calculations to the case of UHECR protons propagating on cosmological distances. An additional ingredient is needed to complete the dynamics of the process of photopion production, namely the cross section. The rather strong assumption adopted here is that the cross section remains the same as the Lorentz invariant one, provided the reaction is *kinematically* allowed. This implies that the interaction lengths remain unchanged.

In order to assess the situation of UHECRs, we first consider the case of particles above the threshold for photopion production in a Lorentz invariant world. According with eqs. (4, 5), in this case particles have a probability of $\approx$ 30% of being not kinematically allowed to interact inelastically with a photon in the CMBR. Therefore, if our assumption on the invariance of the interaction length is correct, then each proton is still expected to make photopion production, although with a slightly larger pathlength.

The situation is however even more interesting for particles that are below the Lorentz invariant threshold for the process of photopion production. If the energy is below a few 10^{18} eV, a galactic origin seems to be in good agreement with measurements of the anisotropy of cosmic ray arrival directions [12,13]. We will not consider these energies any longer. On the other hand, at energies in excess of 10^{19} eV, cosmic rays are believed to be extragalactic protons, mainly on the ground of the comparison of the size of the magnetized region of our Galaxy and the Larmor radius of these particles. We take these pieces of information as the basis for our line of thought. If the cosmic rays observed in the energy range $E > 10^{19}$eV are extragalactic protons, then our previous calculations apply and we may expect that these particles have a $\sim 70\%$ probability of suffering photopion production, even if their energy is below the classical threshold for this process. Note that the pathlength associated with the process is of the order of the typical pathlength for photopion production (a few tens of Mpc), therefore we are here discussing a dramatic process in which the absorption length of particles drops from Gpc, which would be pertinent to particles with energy below $\sim 10^{20}$ eV in a Lorentz invariant world, to several Mpc, with a corresponding suppression of the flux. What are the consequences for the observed fluxes of cosmic rays? The above result implies that *all* with $E > 10^{15}$ eV are produced within a radius of several tens of Mpc, and above this energy there is no dramatic change of pathlength with energy. There is no longer anything special about $E \sim 10^{20}$ eV, and any mechanism invoked to explain the flux of super-GZK particles must be at work also at lower energies.

The basic situation remains the same in the case of pair production as the physical process under consideration. For a source at cosmological distance, a cutoff is expected due to pair production off the far infrared background (FIR) or the microwave background. Using the results in [5] we expect that the modified thresholds are a factor 0.06 (0.73) lower than the Lorentz invariant ones for the case of interaction on the CMBR (FIR). There is also a small increase in the pathlengths above the threshold, which would appear exponentially in the expression for the flux. Therefore there are two effects that go in opposite directions: the first moves the threshold to even lower energies, and the second increases the flux of radiation at Earth because of the increase of the pathlength. It seems that geometry fluctuations do not provide an immediate explanation of the possible detection of particles in excess of the expected ones from distance sources in the TeV region. In any case the experimental evidence for such an excess seems at present all but established.

3 Discussion, conclusions and perspectives.

The investigation of the dependence of the kinematic thresholds for physical processes has been shown to be a powerful tool to study the Physics of space-time at extremely high energies, close to that Planck scale where the fabric of space-time is expected to change, from the flat and well-behaved sheet that we experience in everyday life to a complicated and unpredictable foam of probability that, till now, evaded any kind of direct investigation.

In this paper we related this foamy structure of space-time with the process of measurement of the physical properties of particles, in particular their energy and momentum. We found that the fluctuating metric may induce a violation of Lorentz invariance that changes the thresholds for the photopion production of a very high energy proton off the photons of the CMBR, or for the pair production of a high energy gamma ray in the bath of the FIR or CMBR photons.

For the case of UHECRs interacting with the CMBR, we obtained a picture that changes radically our view of the effect of QG on this phenomenon, as introduced in previous papers: not only particles with energy above $\sim 10^{20}$ eV are affected by the fluctuations in space-time, but also particles with lower energy, down to $\sim 10^{15}$ eV seem to be affected by such fluctuations. In fact the latter, as a result of a fluctuating space-time, may end up being above the threshold for photopion production, so that particles may suffer significant absorption. Our conclusion is that all particles with energy in excess of $\sim 10^{15}$ eV eventually detected at Earth would be generated at distances comparable with the pathlength for photopion production ($\sim$ 100 Mpc). A consequence of this is that there is no longer anything special characterizing the energy $\sim 10^{20}$ eV.

Since the conclusion of our work are quite strong, it is important to summarize in detail the assumptions involved in the calculations:

1) space-time fluctuations follow from Quantum Gravity. This is a rather mild statement and essentially accepted on the basis of the "space-time foam" approach [14]. Following [9,10] we further assume that these fluctuations are at the level of the Planck scale [2].

2) Fluctuations in the momentum and in the dispersion relation are induced by the fluctuations in the metric of space-time. Our assumption is that the fluctuations in the dispersion relation and in the energy and momenta of the particles involved can be considered as independent. This is an assumption usually shared by most literature in the field.

3) The cross section for photopion production at threshold is assumed to be the same as in the Lorentz invariant case. The cross section can be considered as a combination of a matrix element and a phase space factor. While the former

[2] We assume that this is true in the comoving reference frame, in which the CMB radiation is isotropic. If this is assumed to be true in *all* reference frames, than it is necessary to modify the Lorentz transformations [15,16]

may well be affected by violation of Lorentz invariance, it can be demonstrated that the phase space does not change appreciably once the incident particle has energy above the threshold for photopion production. This may be not true in other situations, as discussed in the literature [17,18].

We list below some tests that may allow to understand whether the current or future observations are compatible with the scenario discussed in this paper.

a) Future experiments [19] dedicated to the detection of UHECRs will provide a substantial increase in the statistics, so that the spectral features of the UHECRs in the energy region $E > 10^{19}$ eV can be resolved, and further indications on the nature of primaries and their possible extragalactic origin will be obtained. In particular the present possible disgreement between AGASA [20] and HiRes [21] will be clarified.

One should also keep in mind that an evaluation of the expected flux in terms of sources distributed as normal galaxies is in contradiction with AGASA data by an amount ranging from 2 to 6σ depending on the assumed source spectrum [22]. Since the nature of the sources is not known, it is not clear if their abundance within the absorption pathlength is sufficient to explain the observed flux in presence of space-time fluctuations, nor if they can induce observable anisotropies.

In any case, in a Lorentz invariant framework a suppression in the flux at $\sim 10^{20}$ eV is expected. If such a feature is unambiguously detected in the UHECR spectrum, no much room would be left for the fluctuations of space-time discussed in this paper, since in this scenario nothing special happens around 10^{20} eV. In quantitative terms [5] this would imply a phenomenological bound on l_P now interpreted as a parameter: $l_p < 10^{-46}$ cm instead of $l_p \approx 10^{-33}$ cm; in other words, only fluctuations with variance $\approx 10^{-13}$, instead of 1, would be allowed [3].

b) According with our findings, all particles with energy in excess of $\sim 10^{15}$ eV lose their energy by photopion production on cosmological spatial scales, as a result of the metric fluctuations. This energy ends up mainly in gamma rays, neutrinos and protons. The protons pile up in the energy region right below $\sim 10^{15}$ eV. The gamma ray component actually generates an electromagnetic cascade that ends up contributing low energy gamma rays, in the energy band accessible to instruments like EGRET and GLAST. This cascade flux cannot be larger than the measured electromagnetic energy density in the same band $\omega_{cas}^{exp} = 10^{-6}\ eV/cm^3$ [23]. The cascade flux in our scenario can be estimated as follows. Let $\Phi(E) = \Phi_0(E/E_0)^{-\gamma}$ be the emissivity in UHECRs

[3] Alternatively, one can assume a more general form of fluctuations, i.e. $\delta E \approx E(E/M_P)^\alpha$ and similar for momentum and dispersion relations. In this case a bound $\alpha > 2.3$ would follow.

(particles/cm^3/s/GeV). Let us choose the energy $E_0 = 10^{10}$ GeV and let us normalize the flux to the observations at the energy E_0. The total energy going into the cascade can be shown to be

$$\omega_{cas} \approx \frac{5 \times 10^{-4}}{\gamma - 2} \; x_{min}^{2-\gamma} \; \xi \; eV \; cm^{-3},$$

where ξ is the fraction of energy going into gamma rays in each photopion production, and $x_{min} = (E_{th}/E_0) = 10^{-4}$ for $E_{th} = 10^{15}$ eV. It is easy to see that, for $\gamma = 2.7$, the cascade bound is violated unless $\xi \ll 10^{-3}$.

One note of warning has to be sent concerning the development of the electromagnetic cascade: the same violations of LI discussed here affect other processes, as stressed in the paper. For instance pair production and pion decay are also affected by violations of LI [17]. Therefore the possibility that the cascade limit is exceeded concerns only those scenarios of violations of LI that do not inhibit appreciably pair production and the decay of neutral pions.

The protons piled up at energies right below 10^{15} eV, would be a nice signature of this scenario, but it seems difficult to envision a way of detecting these remnants. In fact, even a tiny magnetic field on cosmological scales would make the arrival time of these particles to Earth larger than the age of the universe. Moreover, even assuming an exactly zero extragalactic magnetic field, these particles need to penetrate the magnetic field of our own Galaxy and mix with the galactic cosmic rays, making their detection extremely problematic if not impossible.

We believe that the results presented here are robust, in the sense that the qualitative features are rather insensitive to modifications of the theoretical scheme. Clearly a more detailed flux computation, taking into account propagation of primaries as well as generation and propagation of the secondaries is needed in order to assess in a more quantitative way observable effects of possible metric fluctuations on UHECRs.

Acknowledgements We are grateful to Giuseppe Di Carlo for very useful and stimulating discussions.

Note Added

Soon after the completion of the present work, a paper [24] appeared in the preprint archives which addresses the problem of the fluctuations in the metric of space-time and the corresponding violations of LI. Our results include the results of that paper, but also represents a generalization to the particles with energy below the classical threshold for photopion production, that in our opinion imply a possibly stronger constrain on the effect of fluctuations of space-time on cosmic rays propagation.

References

[1] D.A. Kirzhnits and V.A. Chechin, Sov. Jour. Nucl. Phys. **15**, 585 (1971).

[2] L. Gonzalez-Mestres, Proc. 26th ICRC (Salt Lake City, USA), **1**, 179 (1999).

[3] G. Amelino Camelia, J. Ellis, N.E. Mavromatos and S. Sarkar Nature **393** (1998) 763.

[4] S. Coleman, S.L. Glashow, Phys. Rev. **D59**, 116008 (1999).

[5] R. Aloisio, P. Blasi, P.L. Ghia and A.F. Grillo, Phys. Rev. **D62**, 053010 (2000).

[6] K. Greisen, Phys. Rev. Lett. **16**, 748 (1966); G.T. Zatsepin and V.A. Kuzmin, Pis'ma Zh. Ekps. Teor. Fiz. **4**, 114 (1966) [JETP Lett. **4**, 78 (1966)].

[7] A.I. Nikishov, Sov. Phys. - JETP **14**, 393 (1962); P. Goldreich and P. Morrison, Sov. Phys. - JETP **18**, 239 (1964); R.J. Gould and G.P. Schreder, Phys. Rev. Lett. **16**, 252 (1966).

[8] L.H. Ford Int. J. Theor. Phys. **38** 2941 (1999).

[9] Y.J. Ng, D.S. Lee, M.C. Oh and H. van Dam, Phys. Lett. **B507** 236 (2001).

[10] Y.J. Ng, *preprint* astro-ph/0201022.

[11] R. Lieu, Astrophys. J. **568** L67 (2002).

[12] N.Hagashida et al., Astrop. Phys. **10** 303 (1999).

[13] D.J. Bird et al. Ap. J. **511** 739 (1999).

[14] J.A. Wheeler, Annals of Physics **2**, 604 (1957).

[15] G. Amelino-Camelia, *preprint* gr-qc/0012051, hep-th/0012238

[16] J. Maguejo and L. Smolin, *preprint* hep-th/0112090

[17] G. Amelino-Camelia, Phys. Lett. **B528** (2002) 181.

[18] H. Vankov and T. Stanev, *preprint* astro-ph/0202388.

[19] J. W. Cronin, Nucl. Phys. B Proc. Suppl. **28B**, 213 (1992).

[20] N. Sakay et al. Proceedings of 2001 ICRC.

[21] C.C.H. Jui et al. Proceedings of 2001 ICRC.

[22] M. Blanton, P. Blasi and A.V. Olinto, Astrop. Phys., 15, 275 (2001).

[23] P. Sreekumar et al. (EGRET collaboration), Astroph. J. **494** (1998) 523.

[24] G. Amelino-Camelia, Y.J. Ng and H. Van Dam, *preprint* gr-qc/0204077.