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On energy spectra of UHE cosmic rays accelerated in supergalactic accretion flows

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Abstract. Some ultra-high energy (UHE) cosmic ray (CR) events may be correlated with the Local Supercluster plane. We consider acceleration of such particles at the large-scale accretion flows of matter towards such plane and/or accretion flows onto galaxy clusters.

The formed shocks and the general compression flow are expected to allow for CR diffusive acceleration of UHE particles. For a simplified flow geometry we consider a stationary acceleration of such CRs and we discuss influence of model parameters on a particle spectral index. We show that the general convergent flow pattern leads naturally to very flat proton spectra with the phase-space spectral indices $\sigma \approx 3.0$, as compared to the 'canonical' shock value of 4.0 .

Key words: cosmic rays – acceleration of particles – shock waves – accretion

1. Introduction

Recent studies of the ultra high energy (UHE) cosmic rays' (CRs) generation in large supergalactic structures are supported by presumable detection of extragalactic magnetic fields (Kim et al. 1991) and theoretical modelling of large scale accretion flows (Ryu et al. 1993, Bertschinger 1985). Typical energies of UHE particles $E \geq 10^{18}$eV and the remarkable change in the observed spectrum in this energy range indicate their extragalactic origin. Additionally, the galactic magnetic fields in the range of a few μG, with a typical Larmor radius of an UHE particle $>$ 1kpc, essentially exclude the acceleration sites located in our Galaxy. On the other hand the sources of particles with $E \sim 10^{20}$ eV cannot be more distant than $50-100$ Mpc due to particle interactions with the microwave background radiation.

Several authors indicate the Local Supercluster (LSC) as a main candidate to provide the sources of UHE CRs (Stanev et al. 1995; Sigl et al. 1999). This idea is clearly motivated by two facts. First, this is a natural site of all potential UHE sources. Second, there are indications (Cen et al. 1994) that the large-scale accretion shocks can be a generic feature of gravitational structure formation.

Below, after a short discussion of physical parameters typical for LSC, we analyze some aspects of the acceleration process of energetic protons at a simplified supergalactic and/or galaxy cluster structure, with the use of the diffusive acceleration model. We show that the general convergent flow pattern leads naturally to very flat proton spectra with the phase-space spectral indices $\sigma \approx 3.0$, as compared to the 'canonical' shock value of 4.0 . The actual value of the index depends on a number of parameters including the involved spatial scales of the flow, the involved velocity of accreting flow and diffusive properties of the medium, depending on the magnetic field structure.

2. Physical conditions in supergalactic accretion flows

In analytic and numerical studies of the evolution of the mass distribution one can see the hierarchy going from galaxy clusters up to two-dimensional structures called *walls* or *sheets* and *filaments* being at the intersections of such walls. The same picture is revealed by observations of the distribution of luminous matter (e.g. Lapparent et al. 1991).

Below we would like to recall the present status of the diffusion acceleration in large-scale shocks accompanying a structure formation in the universe. Thus, in the following we briefly review the constraints for the main parameters – the strength of extragalactic magnetic field, and the accretion flow velocity – which are required to accelerate protons to energies beyond the EeV scale. Cosmic magnetic fields beyond the Galactic disk are poorly known. There are however some observational indications for its existence in galaxy cluster cores as well as in their outer regions (Kronberg 1994; Kim et al.1991; Ensslin et al. 1998a; Valee 1990, 1997). The observation of diffuse ra-

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dio emission from galaxy clusters provides evidence that magnetic fields and relativistic electrons are distributed there on megaparsec scales. Typical μG fields and the 100 kpc scales were detected by Faraday rotation measurements, but magnetic fields are still undetectable for larger structures. Nevertheless, recently, existence of large-scale magnetic field correlated with large-scale structure of the universe is often hypothesized (Ryu et al. 1998; Kulsrud et al. 1997; Medina Tanco 1998). There are X-rays from the hot gas detected outside clusters (Soltan et al. 1996), which allow to assume that the significant magnetic field occur there on scales typical for galaxy superclusters. Unfortunately, unless we directly see the magnetic fields at radio or γ-ray secondaries to UHE CRs (which spectral shape is sensitive to cosmological magnetic fields; see discussion Lee et al. 1995), they must serve only as a postulate.

The nonuniform extragalactic magnetic field associated with the large-scale filaments and sheets is supposed to be responsible for the Faraday rotation of extragalactic sources (Ryu et al. 1998). In contrast to the upper limit $B_{ext} \leq 10^{-9}$G derived from RM of quasars based on the assumption of magnetic field uniformity, in the case of non-uniform fields, the field strength is expected to be in the range $10^{-9} - 10^{-6}$G. This high strength inside the cosmological walls ($\sim 0.1\mu$G) substantially decreases in the surrounding voids. According to the simulations the field can be well ordered along the structure for several megaparsecs. The relatively high strength of magnetic field in the walls could be due to turbulent amplification associated with the of large-scale structure formation (Kulsrud et al. 1997).

On the other hand, both numerical simulations (Ryu et al. 1993) and theoretical modelling (Bertschinger 1985) of structure formation points out that large scale accretion shocks must occur, when the diffuse matter falls down to generated deep potential wells. These shocks amplify and order the magnetic field. The increase of the strength and the field coherence length is suggested to be limited by the energy equipartition state, which in the case of large scale sheets should be smaller than the value quoted by Ryu et al. for filaments. Thus the typical upper value of B_{sheet} is expected to be $\sim 0.1\mu$G at 10 Mpc coherence length scale. Although one cannot yet confirm the existence of large-scale flows with direct observations it has been suggested (Ensslin et al. 1998b) that the shocks coupled with galaxy clusters may be responsible for acceleration of electrons which we observe in the so-called 'cluster radio relics'. The regions of cluster relics which show diffuse radio emission and do not coincide with any host-galaxy are treated as tracers of accretion shock waves developing at large-scale plasma inflows onto galaxy clusters (Ensslin et al. 1998a).

In spite of lack of evidence of shocks associated with sheets and filaments, one believes, according to hydrodynamical simulation and theoretical investigations, that they are formed on the border of the largest scale structure.

The properties of the considered shocks like the shock position and its velocity, u, can also be derived through numerical simulations. In particular, shocks around clusters are typically described by $u \sim 1000$ km/s (Kang et al. 1996). In the case of larger structure, one only assumes that the accretion velocity onto a galaxy wall should be consistent with galaxy streaming velocity. Thus, for equipartition $\sim 0.1\mu$G magnetic field the accreting matter velocity $u \sim 400$ km/s is comparable to the characteristic turbulent velocity (Kulsrud et al. 1997). This value is consistent with the bulk flow motion of field spiral galaxies (Giovanelli et al. 1998; Dekel 1994). Determined simultaneously by observations, the gravitational instability theory and the numerical simulations, the coherent estimate of streaming motion velocities give values of 250 ± 40km/s at a distance of 10 Mpc from LSC plane. The simulations and analytic approach suggest their increase at the smaller scales of order of a few Mpc (R. Juszkiewicz, private comm.). Thus the value of 400 km/s at the shock location seems to be reasonable. Below, for the discussed accretion flow/shock structures we will derive spectra of accelerated high energy protons with the use of a simplified one dimensional model.

3. A model of stationary acceleration

Let us consider a 1D steady-state symmetric model for UHE CRs acceleration (Fig. 1). We assume that seed particles are provided for the cosmic rays acceleration mechanism by the galaxies concentrated near the central plane of a flattened supergalactic structure. On both sides this structure is accompanied by planar shock waves (Fig. 1). For numerical estimates we locate the shocks at the distance $x_0 \sim 3$ Mpc from the supergalactic plane and for the accreting matter velocity we assume $u \sim 400$ km/s. Then, for an order of 0.1 μG supergalactic magnetic field, the diffusive description can be still valid for particles with energies reaching 10^{20} eV (Blasi et al. 1999; Sigl et al. 1999). For a finite extent of an acceleration region, in order to obtain the power-law particles' distributions, one can assume a particle spatial diffusion coefficient κ to be a constant. As one cannot expect this simplification to hold in real objects, the obtained solutions should be treated as approximations valid in a limited energy range.

The considered diffusive transport equation for cosmic ray phase space distribution function, $f(x,p)$, can be written in the form

$$u(x)\frac{\partial f}{\partial x} - \kappa\frac{\partial^2 f}{\partial x^2} - \frac{1}{3}\frac{\partial u(x)}{\partial x}p\frac{\partial f}{\partial p} = Q(x,p) \quad , \tag{1}$$

where

– a monoenergetic source is given in the central plane $x = 0$ by $Q(x,p) = Q_0\delta(x)\delta(p-p_0)$,

– the velocity $u(x) = u_1(x) = -u_1 sgn(x)$ of accreting matter in both regions upstream of the shock ($| x | > x_0$) is assumed to be constant,
– the velocity $u(x) = u_2(x)$ in the internal region of the structure – downstream of both shocks ($| x | < x_0$) – is Linearly decreasing towards the central plane to model the plasma compression process accompanying the structure formation inside the supercluster. With the normalization constant C defined by the shock compression ratio r – $| u_2(x_0) | \equiv Cx_0 = u_1/r$ – it can be written as

$$u_2(x) = -C\,x \quad ,$$

– κ is the CR diffusion coefficient. It is taken constant in the whole space to enable an analytic solution of Eq. 1 , which is the power-law in particle momentum.

The free-escape boundaries are given at $x = \pm L$:

$$f(-L,p) = 0 = f(L,p) \quad . \tag{2}$$

One requires continuity of both, the proton distribution $f(x,p)$ and the differential particle flux

$$S(x,p) \equiv -4\pi p^2 \left\{ \frac{u_i}{3} p \frac{\partial f}{\partial p} + \kappa \frac{\partial f}{\partial x} \right\} \tag{3}$$

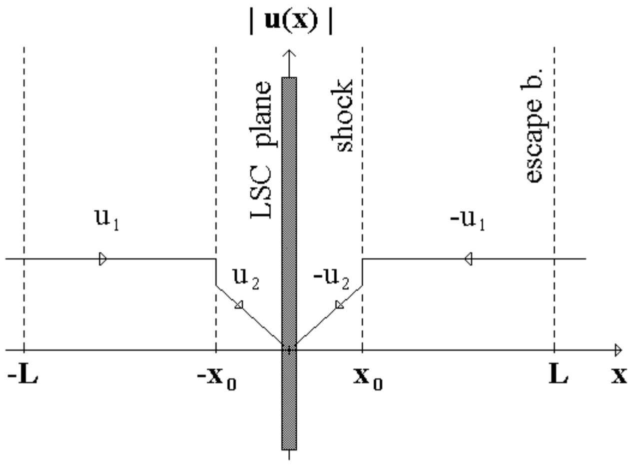

Fig. 1. A simplified symmetric supergalactic accretion flow model considered in the present paper. The involved spatial positions of accretion shocks at $\pm x_0$ and of particle escape boundaries at $\pm L$ are indicated with dashed lines. An inflow velocity plot is superposed on this spatial structure.

at the shocks ($x = \pm x_0$) and continuity of $f(x,p)$ in the central plane:

$$[f]_{x=\pm x_0} = 0 = [f]_{x=0} \quad , \tag{4}$$

$$[S]_{x=\pm x_0} = 0 \quad . \tag{5}$$

At the central plane a flux discontinuity occurs due to the source term

$$[S]_{x=0} = -4\pi p_0^2 Q_0 \delta(p - p_0) \quad . \tag{6}$$

For positive x a solution of Eq. 1 can be found in a separable form $f(x,p) = G_i(x)A(p)$, where we put $i = 1$ (2) for $x > x_0$ ($x < x_0$), through the succesive application of the above conditions. For negative x the solution is provided by the symmetry $f(-x,p) = f(x,p)$ (cf. Fig. 1). In the upstream region, $x_0 < x < L$, we obtain

$$G_1(x) = \exp\left[\frac{u_1}{\kappa}(L - x)\right] - 1 \quad . \tag{7}$$

In the downstream region ($0 < x < x_0$) the solution is expressed by the hypergeometric confluent function (Abramovitz & Stegun 1965)

$$G_2(x) = {}_1F_1\left[-\frac{\sigma}{6}, \frac{1}{2}, \frac{Cx^2}{2\kappa}\right] \quad . \tag{8}$$

The resulting particle momentum spectrum for $p > p_0$ has the power-law form

$$A(p) \propto \left(\frac{p}{p_0}\right)^{\sigma} \quad . \tag{9}$$

In order to derive the spectral index σ, one has to match (Eq-s 4 and 5) the above solutions at the shock. After introducing the new dimensionless variables ρ and ψ,

$$\rho = \frac{Cx_0^2}{2\kappa} \quad , \quad \psi = \frac{L}{x_0} \quad ,$$

one gets a non-algebraic equation for σ,

$$\frac{\sigma}{3} + \left[\frac{r}{(1+r)} \frac{\exp[2r\rho(\psi-1)]}{\exp[2r\rho(\psi-1)] - 1} - \frac{\frac{\sigma}{3}}{(1+r)} \frac{{}_1F_1[1-\frac{\sigma}{6}, \frac{3}{2}, \rho]}{{}_1F_1[-\frac{\sigma}{6}, \frac{1}{2}, \rho]}\right] = 0 \quad , \tag{10}$$

to be solved numerically. Apart from the compression r ($\equiv u_1/Cx_0$) the solution $\sigma(\rho, \psi, r)$ is parametrized by the two additional parameters. The quantity $\rho = Cx_0 \frac{x_0}{2\kappa} = \frac{u_2(x=x_0)}{u_{diff}(x_0)}$ may be regarded as the ratio of the flow velocity to the CR 'diffusion' velocity at the shock spatial scale. The ψ variable gives the confinement volume 'size' measured in the shock distance units.

In order to visualize the resulting spectrum inclination we have plotted the spectral index as a function of ψ for a few values of the compression ratio r and the parameter ρ (Fig. 2). For the size of the particle confinement volume of order of a few shock distances x_0, the spectral changes are insignificant, provided the diffusive approach is valid $\rho\psi \sim u_1 L/\kappa \gg 1$. The main observed feature is a rapid spectral index flattening to its limiting value $\sigma = -3.0$ due to inefficient particle escape from the 'acceleration volume'.

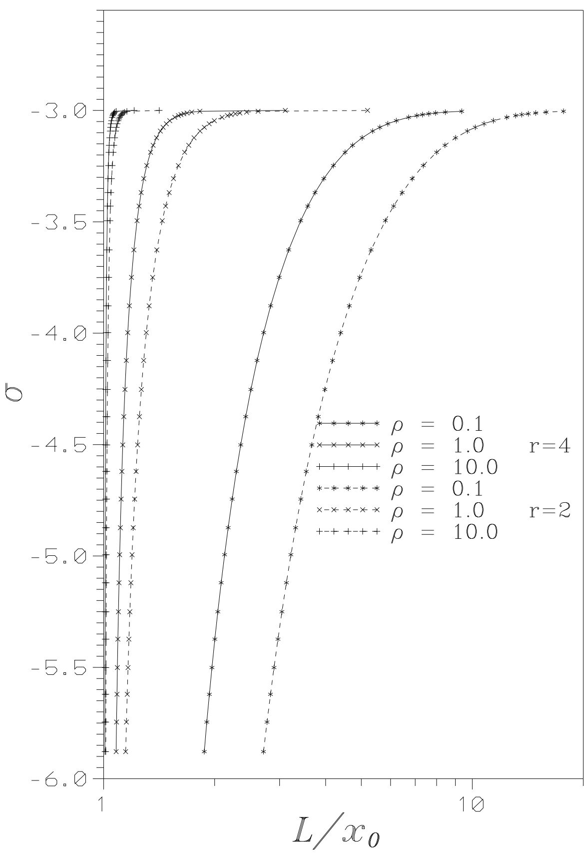

Fig. 2. The spectral index σ versus the spatial scale L/x_0. The results are presented for various $\rho = Cx_0\frac{x_0}{2\kappa}$ and two compressions, $r = 4$ (solid lines) and $r = 2$ (dashed lines), as indicated in the figure.

The shock compression is less significant in determining σ (!). On the other hand one has to note its essential dependence on ρ, in particular for small values of ρ (< 1) admitting for noticeable spectral changes.

In the asymptotic regime $\rho \gg 1$, i.e. when the particle advection term dominates over the diffusive one at $| x | < x_0$, Eq. 10 reduces to

$$\sigma \approx -3\left[\frac{r}{r+1}\,\frac{\exp\left(\frac{u_1}{\kappa}(L-x_0)\right)}{\exp\left(\frac{u_1}{\kappa}(L-x_0)\right)-1}+\frac{1}{1+r}\right] \quad . \qquad (11)$$

In this limit, with diffusive particle escape against the flow substantially reduced, the resulting index value is close to -3.0, as the acceleration results both due to the shock compression and the plasma convergent flow toward the supergalactic plane. On the other hand, if $\rho \ll 1$, Eq. 10 leads to

$$\sigma \approx -3\,\frac{\exp[2r\rho(\psi-1)]}{\exp[2r\rho(\psi-1)]-1} \approx -3\,\frac{\kappa}{u_1(L-x_0)} \quad , \qquad (12)$$

provided that $2r\rho(\psi - 1) \equiv u_1(L - x_0)/\kappa \ll 1$.

Requirement of the diffusive description validity yields the lower limit for the considered ρ. Since the mean free path for proton, $l < x_0$, the inequality holds $\kappa < \frac{1}{3}cx_0$. Thus $\rho > \frac{u_1}{c\,r} \sim \frac{1}{3}10^{-3}$, where c is the light velocity.

4. Discussion

The spectrum of accelerated CR protons may either become very flat or steepen according to the value of ρ parameter. As it was seen in Fig. 2 which represents the solution of Eq. 10 or directly, through its asymptotic solutions, Eq. 11 and Eq. 12; ρ controls the spectrum inclination. On the other hand it depends on the diffusion coefficient. The critical value of $\rho = 1$ separates two distinct spectra regimes. For $\rho \geq 1$ the spectral index saturates rapidly at the value of -3 (see Fig. 2) due to the compressive accretion flow predominance. Below the critical value, i.e. in the case of "diffusion velocity" greater than the inflow motions, the spectral changes can be attributed mainly to diffusion coefficient value. For the above considered parameters of $u_1 = 400$ km/s, $x_0 \approx 3$ Mpc, $r = 2$, the critical value of the diffusion coefficient corresponding to ρ_{cr} is $\kappa^* = 10^{32}$ cm^2/s. For $\kappa < \kappa^*$, we get the hard spectrum with $\sigma = -3$, while for $\kappa > \kappa^*$ the spectrum steepens.

To estimate the maximum energy possible to achieve, let us first remind the rough dimensional restriction demanding the particle orbit should be smaller than the acceleration size L. In fact the realistic spectrum requires the diffusion length $\frac{\kappa}{u}$, is smaller than L. For Bohm diffusion it gives the limitation for Larmor radius $r_g \leq 3L(u/c)$, equivalent to $r_g \leq 50$ kpc which implies $E \leq$ few 10^{18} eV. Here, we considered the totaly chaotic magnetic field, where the Bohm diffusion gives the appropriate description and took for turbulent magnetic field strength $B \approx 0.1\ \mu$G and $L = 10$ Mpc. This random magnetic field component is associated with turbulent motion which occurs simultaneously with streaming accretion motion at the shock vicinity, which in turn generates its ordered component.

The planar symmetry of the model, with correlated magnetic field inside the large scale cosmic structure, can make the diffusion highly anisotropic. Therefore for B aligned with supercluster plane, as required by Ryu at al. (1998), the cross-field diffusion for a quasi-perpendicular shock should be considered. The minimum diffusion coefficient in the perpendicular shocks, derived by Jokipii (1987), is $\kappa_J = 3\,\frac{u_1}{c}\,\kappa_B$. Without entering into the topological characteristics of magnetic fields near the cosmic structure we only put here the value of critical diffusion coefficient referring separately to both magnetic field components and then compare the respective maximum energy. Above, it was clear that for entirely turbulent field, the UH energies can be hardly achieved. Contrary to that, for B strongly aligned with the structure, the diffusion can be reduced up to $(\frac{u_1}{c})^{-1} \sim 10^3$ times with respect to the Bohm diffusion. Thus, for critical diffusion with $\kappa_J = \kappa^*$ one obtains for the Larmor radius $r_g \sim 10^{25}$ cm, which corresponds to $E_{max} \sim 10^{21}$ eV. The latter case gives also the flatter spectrum.

The application of this model to galaxy cluster inflow is even more suitable, since its spherical accretion symmetry will cause the acceleration process to be more efficient than in the planar case. Adopting the same argument as used above for supercluster case, let us consider the typical physical parameters for a galaxy cluster: accretion velocity $u_1 = 2 \cdot 10^3$ km/s, an upper limit for magnetic field $B \approx 1$ μG and $x_0 \approx 3$ Mpc. Thus, for the Bohm diffusion one obtains for particle gyroradius $r_g \leq 1.8 \cdot 10^{23}$ cm. The maximum energy can reach the value of 10^{20} eV and even larger for the Jokipii diffusion model.

Finally, to make sure that such scenario may serve as a viable acceleration process, let us confront the resulted maximum proton energy with that, when the energy losses are included. For UHE protons, acceleration in the astrophysical shock is governed by the equation $dE/dt = E/\tau_{acc}$. The losses above 10^{19} eV are mainly due to pair ($e^{\pm}$) and photomeson production. Both, the mean acceleration time and the timescale for losses, τ_{loss}, has been considered in many papers. Their equality gives the maximum energy up to which the particles can be accelerated. Here, we use the results calculated and plotted by Kang et al. (1997). In their Fig. 2 the intersection point of the curves determines the maximum energy achievable in acceleration process. Taking into consideration the diffusion in a quasi-perpendicular case, the acceleration time τ_{acc} scales like (Kang et al. 1997) $\tau_{acc} \propto u^{-1}B^{-1}$, to yield the maximum energy of $10^{19.6}$ eV.

We have to note that in our model the acceleration time will be smaller than the one estimated by Kang et al. (1997) and the the maximum energy can be greater. This is due to the presence of two shocks associated with the compression inflow structure making the particle escape more difficult. In fact, we should consider the acceleration time estimated as $\tau_{acc}^{-1} = {\tau_s}^{-1} + {\tau_c}^{-1}$, where $\tau_c = \frac{p}{\frac{1}{3}p\frac{\partial u(x)}{\partial x}} = \frac{3x_0}{u_2}$ is the acceleration time scale due to adiabatic acceleration in the compressive flow and τ_s is the scale for the shock acceleration. Here, with the numerical parameters given above, we ignored the second term since it is comparable to the age of universe. However, it must be included when both τ_s and τ_c are of the same order.

5. Conclusions

We demonstrated that the diffusive acceleration in the accretion flow onto the galaxy supercluster can provide an extremely hard spectrum of accelerated UHE protons. It is a consequence of particle confinement in the converging flows, involving the plasma inflow towards the structure central plane with embedded shocks. One should note that in such convergent flows the particle acceleration process can proceed even without shocks. Thus a possibility of a significant deviation of the UHE CR spectral index from the often considered shock index, $\sigma_0 = 3r/(r-1)$, arises in a natural way. This fact should be included in modelling – based on acceleration in supergalactic accretion flows – of the most energetic cosmic rays' component observed at Earth.

Of course, the considered above a symmetric planar model is a substantial simplification. A divergence of the spectrum from the derived $\sigma \approx -3.0$ may arise if particles easily escape from the structure, $u_1 L < \kappa$, or additional particle sinks appear. The latter may be a result of particle escape from the accretion flow along the supergalactic structure, extending to the sides at distances larger or comparable to its vertical scale L. In both cases, the generated spectral index – the one in the acceleration site – is expected to grow with particle energy and efficiency of the acceleration process decreases.

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