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INVARIANCE VIOLATION EXTENDS THE COSMIC-RAY HORIZON?

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ABSTRACT

We postulate that the energy-momentum relation is modified for very high energy particles to violate Lorentz invariance and the speed of photon is changed from the light velocity c. The violation effect is amplified in a sensitive way to detection through the modified kinematical constraints on the conservation of energy and momentum, in the absorption process of γ-rays colliding against photons of longer wavelengths and converting into an electron-positron pair. For γ-rays of energies higher than 10^{13} eV, the minimum energy of the soft photons for the reaction and then the absorption mean free path of γ-rays are altered by orders of magnitude from those conventionally estimated. Consideration is similarly applied to high-energy cosmic-ray protons. The consequences may require the standard assumptions on the maximum distance from which very high energy radiation can travel to be revised.

Subject headings: acceleration of particles — elementary particles — galaxies: individual (Markarian 421, Markarian 501) — relativity — scattering

1. INTRODUCTION

Phenomena that strikingly deviate from the conventional understanding can take place if the speed of photons is allowed to vary from the constant light velocity c. Coleman & Glashow (1997) argued that the velocity c_0 of light in the vacuum in a frame moving relative to the rest frame of the universe can differ from the maximal attainable speed c of a material body. If $c_0 > c$, the photon 4-momentum $(\boldsymbol{E}/c_0, E)$ is timelike, allowing a sufficiently energetic photon to decay into an electron-positron pair. Alternatively, if $c_0 < c$, a charged particle will lose energy via vacuum Čerenkov radiation because the particle speed approaching c at high energies can exceed the light speed c_0. Thus, the distance that energetic γ-rays or charged particles can travel through is shortened for either $c_0 > c$ or $c_0 < c$. However, the violation of relativity may also possibly increase the traveling distance of high-energy radiation rather than decrease it.

The speed of photons can vary because of modifications of the vacuum (Latorre, Pascual, & Tarrach 1995). Different energies excite the vacuum differently; with quantum gravity, Amelino-Camelia et al. (1997) speculated that the energy-momentum relation for photons is modified to violate Lorentz invariance, and the anomalous effect increases as photon energy approaches the Planck energy scale ($\sim 10^{19}$ GeV). Amelino-Camelia et al. (1998) has proposed a test for the modified speed by measuring the arrival time structure of photons from objects at cosmological distances. The propagation speed is $dE/dk = c(1 - \xi_\gamma E/E_0)$ from the modified relation of energy E and momentum k,

$$c^2k^2 = \left(E + \xi_\gamma \frac{E^2}{2E_0}\right)^2 = E^2 + \xi_\gamma \frac{E^3}{E_0}, \qquad (1)$$

with $\xi_\gamma = \pm 1$ and $E_0 \sim 10^{19}$ GeV, according to Amelino-Camelia et al. (1998). By assuming the existence of a preferred frame for Lorenz transformation but preserving other kinds of invariance, Coleman & Glashow (1998) give a relation that is similar to equation (1) and equivalent to $c^2k^2 = E^2 - m_i^2c^4 + (c^2 - c_i^2)E^2$ for $ck \approx E \gg m_ic^2$, where c_i is the maximal attainable speed of particle species i with mass m_i. Our study in what follows will be in an explicit form based on equation (1), but the results are, to some extent, also applicable to the other case by inserting $c^2 - c_i^2$ in place of ξ_iE/E_0.

The electromagnetic radiation of the shortest wavelength so far detected is TeV γ-rays (see, for example, Weekes et al. 1997; Ong 1998), but even for these high-energy photons, the effect of velocity variation remains as small as a fraction $\sim 10^{-15}$ of the light velocity. However, a higher degree of anomalous effect is expected for the absorption process, in which γ-rays are converted into an electron–positron pair when they collide with soft photons of the universal radiation field. Photons of TeV energies react with infrared photons, and PeV γ-rays react with the 2.7 K cosmic microwave background (CMB). From the number density of the photons of the universal infrared and the CMB, the collision mean free path is usually estimated as about 100 Mpc and 10 kpc (e.g., Strong et al. 1974; Stecker, De Jager, & Salamon 1992), respectively, for TeV and PeV γ-rays. The largest distance we can see with the high-energy radiations is thus much shorter than those of X-rays and optical photons of longer wavelengths.

2. KINEMATICAL CONSTRAINTS ON THE PHOTON-PHOTON COLLISION PROCESS

The threshold energy is determined by a function of the total energy W and momentum Q of the initial state, $W^2 - c^2Q^2$, which must exceed $(2mc^2)^2$ for the pair creation, where m is the electron mass. In the initial state of a γ-ray obeying relation (1) and soft photons of energy ϵ, the square of the energy E and the momentum $\sim E/c$ in $W^2 - c^2Q^2$ almost cancel each other to leave $4\epsilon E - \xi_\gamma E^3/E_0$ (the two photons are assumed to be from opposite directions). The remaining two terms are of similar magnitude when ϵ/E is as small as $\xi_\gamma E/E_0$, incurring a considerable degree of anomalous effect. In addition, the two terms become as large as m^2c^4 when $E \approx 10^{12}\xi_\gamma^{-1/3} \approx 10^{12}\epsilon^{-1}$ eV, and the threshold condition is considerably affected by the violation effect for $E > 10^{13}\xi_\gamma^{-1/3}$ eV, which can be much lower than the Planck energy scale.

Let us examine the above speculation by calculating in our rest frame the conservation of momentum of the reaction. By putting $c = 1$ hereafter, the momentum $E - \epsilon + \xi_\gamma E^2/(2E_0)$ of the initial state (we assume for simplicity, but not to lose the

generality, "head-on" collisions of γ-rays and soft photons) must be equal to $p_1 \cos\alpha + p_2 \cos\beta$ of the final state, where $(E_1, \boldsymbol{p}_1)$ and $(E_2, \boldsymbol{p}_2)$ are the energy and momentum of the electron and positron and $E + \epsilon = E_1 + E_2$. The α and β are the angles $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$ make against the direction of the initial γ-ray and are on the order of m/E_i ($i = 1$ and 2). By using the approximation $p_i = E_i - m^2/(2E_i)$ and the condition on the transverse momenta $\alpha p_1 = \beta p_2$, we obtain

$$-2\epsilon + \frac{\xi_\gamma E^2}{2E_0} = -\frac{E+\epsilon}{2}\left(\frac{m^2}{E_1 E_2} + \alpha\beta\right). \quad (2)$$

We use a parameter x to replace E_1 and E_2 by $E_{1,2} = (1 \pm x)(E + \epsilon)/2$ with the condition $|x| \le 1 - 2m/(E + \epsilon)$. Equation (2) is then written as

$$\epsilon \approx \frac{m^2}{E}\,\frac{1}{1-x^2} + \frac{\alpha\beta E}{4} + \xi_\gamma \frac{E^2}{4E_0}. \quad (3)$$

The first and second terms are of the same order of magnitude, the latter term approaching zero at the threshold of the reaction. With increasing E, softer photons can serve as the *target* photons, shortening the absorption mean free path. This conventional view is altered, as E increases further, by the presence of the term $\xi_\gamma E^2/(4E_0)$. However, the electron and positron may also obey a modified relation similar to equation (1). Let us set, similarly to equation (1), the square of the modified momentum q to be equal to

$$q^2 = p^2 + \xi_e \frac{E^3}{E_0} = E^2 - m^2 + \xi_e \frac{E^3}{E_0}. \quad (4)$$

We grant $|\xi_e| \neq 1$ for allowing electrons to have a different degree of the invariance violation. By replacing p by q, a relation similar to equation (3) is obtained:

$$\epsilon = \frac{m^2}{E}\,\frac{1}{1-x^2} + \frac{\alpha\beta E}{4} + \frac{E^2}{4E_0}\left(\xi_\gamma - \xi_e \frac{1+x^2}{2}\right). \quad (5)$$

Since energy E is divided almost equally into two particles (when $x \approx 0$) and the anomalous term has E^2 dependence, the factor 2 appears in the denominator of $\xi_e(1 + x^2)/2$, thus reducing the effect in the final state. The fact might seem to suggest that the single "leading" particle taking almost all the total energy ($|x| \approx 1$) is capable of eliminating the anomalous effect. However, the substitution of $|x| = 1 - 2m/(E + \epsilon)$ into equation (5) increases the energy ϵ to $m \sim 1$ MeV, which is much higher than m^2/E, and then leads to a very large traveling distance, resulting in no less anomalous a consequence. Only if ξ_e is tuned to be $2\xi_\gamma$ for an unknown reason can we recover the ordinary relation for ϵ.

3. COMPARISON WITH OBSERVATIONS

The energy ϵ of the target photon is plotted against the γ-ray energy E in Figure 1. A minimum value of ϵ is taken, if $\xi = 2\xi_\gamma - \xi_e > 0$, at the critical energy $E_c = (4m^2E_0/\xi)^{1/3} \approx 10^{13}$ eV for $\xi = 1$. The minimum value is about 0.03 eV, and ϵ must increase to about 30 eV at $E = 10^{15}$ eV. Thus, the CMB of $\sim 10^{-3}$ eV is never given a chance to contribute to the pair creation process. The traveling length has the shortest value of $\sim$10 Mpc at $E = E_c$ and increases again for $E > E_c$. Since $\sim$30 eV photons are very sparse, γ-rays would travel through distances greater than 1 Gpc. When $\xi < 0$, ϵ must rapidly decrease as E approaches E_c. Before taking negative values, the CMB photons become available as a target. The energetic γ-rays are then efficiently absorbed with a mean free path of $\sim$10 kpc, which is shorter by orders of magnitude than the ordinary case.

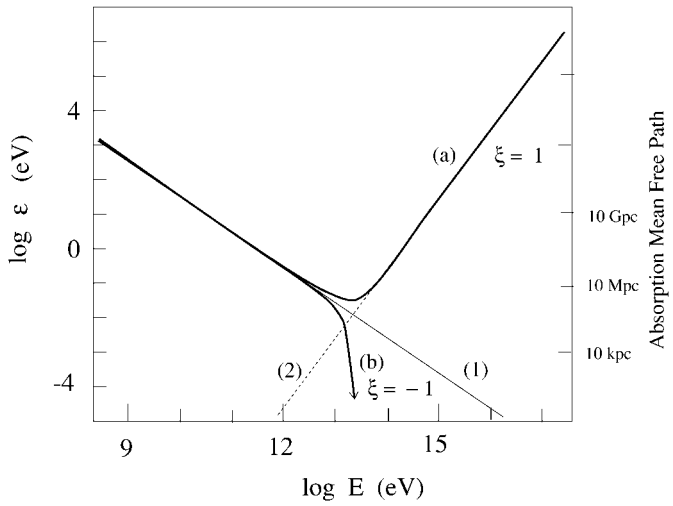

FIG. 1.—Energy of target photons against γ-ray energy. The number density of target photons decreases with their energy ϵ, and thus the mean free path generally increases with ϵ. The traveling distance of γ-rays is indicated on the right-hand side of the vertical axis. The thin line (1) shows the ordinary case of $\epsilon = m^2/E$, and the dashed line (2) represents the anomalous contribution $\xi E^3/(4E_0)$, where $\xi = 2\xi_\gamma - \xi_e$ as shown in eq. (5) of the text. The result of adding the both terms is shown as indicated by the thick lines for $\xi = 1$ (a) and -1 (b) cases. In the model of Coleman & Glashow (1998), the line (a) has a less steep increase with energy because of the factor $c^2 - c_i^2$ instead of $\xi_i E/E_0$.

Among the TeV γ-ray sources are two active galaxies, Mrk 421 and Mrk 501 (Punch et al. 1992; Aharoninan et al. 1997; Catanese et al. 1997). The objects are at distances of $\sim$100 Mpc, and the detection is consistent with the absorption mean free path determined by infrared intensity in the extragalactic space. The energy spectrum of the γ-rays is found to extend up to 10–20 TeV. If the spectrum continues to extend to even higher energies, $\xi < 0$ would be excluded because, if $\xi < 0$, a very sharp cutoff at the common energy E_c is expected in the energy spectrum for any extragalactic objects. The negative ϵ for $E > E_c$ in the case of $\xi < 0$ implies that γ-rays do not need target photons for pair creation. The threshold of γ-rays decaying into an electron-positron pair (Coleman & Glashow 1997) is given by $|E_c|$. By setting $|E_c| = -E_c = [4m^2E_0/(-\xi)]^{1/3} > 20$ TeV, we obtain a limit on ξ to be $\xi > -1.3$ (by setting $E_0 = 1 \times 10^{28}$ eV). On the other hand, when ξ is positive and $\sim$1, the traveling distance turns over to increase above about 10 TeV and to become larger than 1 Gpc for the photons around 100 TeV. Thus, 100 TeV photons would arrive from extragalactic sources, implying that evidence of detection would be strongly in favor of the $\xi > 0$ case. Extragalactic diffuse γ-rays are most likely from distant unresolved active galaxies. The energy spectrum is observed by Sreekumar, Stecker, & Kappadath (1997) to extend to about 50 GeV with a hard power index of ~ -2.3. The spectrum is expected to become steeper at higher energies due to the more intense infrared photons and then the CMB at longer wavelengths. However, in the case of $\xi > 0$, the energy spectrum may have a *turnover* of showing a harder slope above $E = E_c$. Electrons

lose their energy quickly and have difficulty producing such high-energy γ-rays. However, cosmic-ray protons are known to have energy spectra extending to 10^{20} eV and are able to produce γ-rays as high as or larger than 100 TeV. The increase of traveling distance suggests more contribution from more distant sources, and thus we may expect considerably intense, diffuse γ-rays in the higher energy region. From the Crab Nebula, 50 TeV photons have been detected (Tanimori et al. 1998) by using an imaging Čerenkov telescope at large zenith angles. Detection of 100 TeV γ-rays is not unrealistic by the extensive application of such methods.

4. THE CASE OF COSMIC-RAY PROTONS

Our phenomenological consideration can be extended to other processes of soft photons acting as the reaction target. Extremely high energy cosmic-ray protons (e.g., Hill & Schramm 1985; Axford 1994; O'Halloran, Sokolsky, & Yoshida 1998; Takeda et al. 1998) are believed to lose energy by pion production in collisions with the CMB. The number density of the CMB and the photo-pion cross section determine the collision mean free path to be ~10 Mpc. The energy spectrum thus suffers from the GZK cutoff (Greisen 1966; Zatsepin & Kuzmin 1966), a spectral cutoff at about 10^{20} eV, beyond which energy no cosmic-ray protons can arrive if the protons are from distances farther than the mean free path. However, the relation (4) assumed for proton and pion alters the threshold of the energy ϵ of target photons from $\mu M/(2E)$ to

$$\epsilon = \frac{\mu M}{2E} + \Xi \frac{E^2}{4E_0}, \tag{6}$$

where

$$\Xi = \xi_p - \xi_p \left(\frac{M}{M+\mu}\right)^2 - \xi_\pi \left(\frac{\mu}{M+\mu}\right)^2 .$$

The energy E is for the initial proton, M and μ are the proton and pion mass, and ξ_p and ξ_π are for protons and pions, respectively. We have set the parameter x to be $(M-\mu)/(M+\mu)$, corresponding to the case in which the ratio of proton to pion energy in the final state is $(1+x)/(1-x) = M/\mu$ near the threshold of the process. If Ξ is positive and not very far from the order of 1, the second term becomes comparable with $\mu M/(2E)$ at the critical energy $E_{p\pi} = (M\mu E_0/\Xi)^{1/3} \approx 10^{15}$ eV. At $E \approx 10^{20}$ eV, the energy ϵ needs to be, as dominated by $\Xi E^2/(4E_0)$, as large as 10^{12} eV, and the CMB cannot play the role of target photons. Cosmic-ray protons at ~10^{20} eV can then travel without the attenuation from cosmological distances farther than ~10 Mpc. As a result, the energy spectrum of cosmic rays would extend beyond 10^{20} eV, provided that the acceleration to such high energies takes place in celestial objects. There is speculation by Vietri (1995) and Waxman (1995) that γ-ray bursts (GRBs) provide the origin of the highest energy cosmic rays. If so, GRBs at cosmological distances would produce a cosmic-ray spectrum without the GZK cutoff. If $\Xi < 0$, ϵ can be ~10^{-3} eV when E approaches $|E_{p\pi}| = -E_{p\pi} \approx 10^{15}$ eV, implying that the target photons of the CMB give a characteristic attenuation length of ~10 Mpc to 10^{15} eV cosmic rays. For $E > |E_{p\pi}|$, the kinematical condition does not require the target photon for the pion production to occur, and the processes of $p \rightarrow p + \pi^0$ and $p \rightarrow n + \pi^+$ will cause protons to rapidly lose energy. The energy loss due to the spontaneous production of pion is in contradiction with the evidence of cosmic-ray protons up to 10^{20} eV. Thus, we limit $|E_{p\pi}|$ to longer than 10^{20} eV and obtain a bound of $\Xi > -10^{-17}$.

5. DISCUSSION

Collision processes of high-energy radiations with soft photons enable us to test the consequences of violation of Lorentz invariance. In particular, the effect of quantum gravity which Amelino-Camelia et al. (1998) suggest can be made detectable by the reaction process in the energy region much lower than the Planck energy scale, but in the sufficiently high-energy region to which the validity of Lorentz transformation has not been examined yet by accelerator experiments. The violation suggests an interesting possibility of increasing the distance we can reach by observing high-energy γ-rays and cosmic rays. Absence of the GZK cutoff would not necessarily imply a nearby origin of the extremely high energy cosmic rays. Photons of energies larger than 100 TeV from extragalactic objects would provide a clear test because it is hard to explain by mechanisms other than the violation of invariance. It seems impossible, once we admit relation (1) for photons, to totally eliminate the anomalous effect without a specially designed tuning among the invariance violating terms for various particles of different masses. Other ways have been proposed to alter the conventional view on the relativity (Gonzares-Mestres 1997; Sato 1998). The absence of the GZK cutoff can be explained, according to Sato (1998), without abandoning the Lorenz invariance, but by giving a special status to "the universe frame" in which the CMB is isotropic.

Protons are expected to lose energy via vacuum Čerenkov radiation (Coleman & Glashow 1997), when $\xi_\gamma - \xi_p > 0$. The threshold energy $E_{\rm th} = [M^2 E_0/(\xi_\gamma - \xi_p)]^{1/3}$ of the radiation is calculated by equating the speed of photons to that of protons. The detection of the highest energy 10^{20} eV of cosmic rays sets a constraint $E_{\rm th} > 10^{20}$ eV and limits the ξ-parameters to be $\xi_\gamma - \xi_p < 10^{-15}$ or rather $\xi_\gamma \leq \xi_p$ for practical use. The Čerenkov radiation from electrons yields a less tight constraint because of the dependence of ξ on $E_{\rm th}$, i.e., $\xi = \xi_\gamma - \xi_e \propto E_{\rm th}^{-3}$. The highest energy 2×10^{12} eV of cosmic-ray electrons ever detected (Nishiumura et al. 1997) must be less than $E_{\rm th}$, and then we obtain $\xi_\gamma - \xi_e < 10^3$. If we choose, for the bound on $E_{\rm th}$, ~100 TeV electrons which are responsible for TeV γ-rays and which are considered to extend over a spatial scale of at least ~1 pc, the constraint is made somewhat tighter as $\xi_\gamma - \xi_e < 10^{-2}$. We may argue about the allowed region of (ξ_γ, ξ_e)-values by combining the condition $-1.3 < 2\xi_\gamma - \xi_e$ from the decay process $\gamma \rightarrow e^+ + e^-$. The region is, to some extent, in disfavor with negative values. The time structure of the outbursts of TeV γ-rays from Mrk 421 and Mrk 501 sets a constraint on ξ_γ. The delay time of 10 TeV γ-rays relative to photons of lower energies is $(d/c)\xi_\gamma(10^{13}\ {\rm eV}/E_0) \approx 10\xi_\gamma$ s for distance $d = 100$ Mpc. The duration of the outbursts, which has been observed typically as a few hours, is considered to be longer than the delay time, providing a constraint of $\xi_\gamma < 10^3$. We need to know the outburst structure in timescales as short as ~10 s to constrain $|\xi|$ to less than ~1. In order to restrict the $|\xi|$ parameter more tightly, it is very useful to observe photons of higher energies from more distant objects. However, we cannot expect that such γ-rays survive against conversion into an electron-positron pair and reach us, unless we premise that the Lorenz invariance is violated.

We may argue that the extensive air shower events of ~10^{20} eV are in truth not due to protons. Such a claim is relevant to

a "correct" understanding of the complex phenomena of the cascade shower process, which still leaves room for a speculation of "anomalous interaction" of cosmic rays. Thus, it is interesting to indicate that the modified relation of energy and momentum can affect detection of the high-energy radiations. Observation of γ-rays, for example, is based on the pair creation process in the detector material (satellite experiments) or in the atmosphere (ground-based observation for TeV photons and cosmic rays). The recoil momentum δ of atomic nucleus in the material is calculated as twice the right-hand side of equation (5). The ordinary recoil is $c\delta_0 \approx m^2E/(2E_1E_2) = 0.1$ eV at $E \approx 10^{13}$ eV, which corresponds to quite a small energy transfer to the recoiled atomic nucleus of a large mass. When $\xi_\gamma = 1$ and $E > E_c$, δ must be larger than δ_0 but remains reasonably small so that the process would not be largely affected. When $\xi_\gamma = -1$ and $E > E_c$, δ appears negative, the recoil being in the opposite direction to the initial γ-ray. There are generally several processes, such as the production of pions in the γ-ray–nucleon reaction, that compete with the pair creation and for which the invariance violation is of different strengths. The consequence may appear as the observation of an anomalous behavior of very high energy radiations.

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