INVARIANT SHAPE DESCRIPTION AND MEASURE OF OBJECT SIMILARITY

J Flusser

Institute of Information Theory and Automation, Czechoslovakia

INTRODUCTION

One of the most important problems in computer graphics and image processing is to find rotational, translational and scale invariant description of planar shapes. A number of solutions to that problem were presented: Fourier descriptors by Lin and Chellappa [1], Hadamard coefficients by Kuncheva [2], moment invariants by Hu [3] and by Maitra [4] and boundary representation via chain codes or distance vectors (see for example Peli [5]). In this paper, a new invariant shape description by a binary *shape matrix* is presented. Properties of the shape matrix are discussed and a measure of the shape similarity is introduced. The problem of image matching is also adressed. It is shown that the shape matrix can be successfully used as the feature for object recognition.

SHAPE MATRIX CONSTRUCTION

For given planar object G we define its shape matrix of size $n \times n$ as follows.

ALGORITHM 1

1. Find the centre of gravity $T = (x_t, y_t)$ of object G.
2. Find such point $M = (x_m, y_m)$ that $M \in G$ and $d(M,T) = \max_{A \in G} d(A,T)$, where d is Euclidean distance in R^2.
3. Construct the square with the centre in T and with the size of the side $2 \cdot d(M,T)$. Point M lies in the centre of one side.
4. Divide the square into $n \times n$ subsquares.
5. Denote S_{kj} the subsquares of the constructed grid; $k, j = 1, \dots, n$.
6. Define the $n \times n$ binary matrix B
$$B_{kj} = \begin{cases} 1 & \iff \mu(S_{kj} \cap G) \geq \mu(S_{kj})/2 \\ 0 & \text{otherwise,} \end{cases}$$
where $\mu(F)$ is the area of the planar region F.

The shape matrix exists for every compact planar object G. There is no limit to the scope of the shapes that the shape matrix can represent. It can describe even shapes with holes. It is easy to prove that the shape matrix is invariant under displacement, rotation and scaling of the object. The shape of the object can be reconstructed from the shape matrix. The accuracy of the reconstruction is given by the size of the subsquares S_{kj}, i. e. we can reach more accurate shape description by increasing n. An example of the shape matrix construction is shown in Fig. 1.

OBJECT SIMILARITY MEASURE

Two different shapes are distinguishable by the use of their shape matrixes, i. e. there exists such n that the shape matrixes of size $n \times n$ differ from each other.

To determine the degree of similarity between two objects, their shape matrixes A and B are compared. The dimensions of the matrixes should be equal. The similarity relation $p(A,B)$ is defined by the following formula:

$$p(A,B) = 1 - \frac{1}{n \cdot n} \sum_{j=1}^{n} \sum_{i=1}^{n} |A_{ij} - B_{ij}|. \qquad (1)$$

There exist shapes with more than one maximum radius (i. e. more than one point M is found during the second step of Algorithm 1) which may produce different shape matrixes depending on the maximum radii used. Let the object A be described by s different shape matrixes $A^1, \dots, A^s$ and the object B be described by q different shape matrixes $B^1, \dots, B^q$. We define the similarity $p(A,B)$ by the following relation :

$$p(A,B) = \max_{k,l} p^{kl}(A,B) \qquad k = 1,\dots,s \qquad l = 1,\dots,q \qquad (2)$$

where

$$p^{kl}(A,B) = 1 - \frac{1}{n \cdot n} \sum_{j=1}^{n} \sum_{i=1}^{n} |A_{ij}^k - B_{ij}^l|. \qquad (3)$$

Of course, the similarity $p(A,B)$ between two given shapes A and B depends on the dimension of the shape matrixes. The choice of the appropriate shape matrix dimension is very important step.

Typical behaviour of $p(A,B)$ is shown in Fig. 2. We want to find such n that the value of $p(A,B)$ is close to the ideal value

$$p_0(A,B) = \lim_{n \to \infty} p(A,B). \qquad (4)$$

On the other hand, the construction of the shape matrix is very time-consuming for high n.

In order to determine an optimal value n', the following algorithm can be used :

ALGORITHM 2

1. Define $n = 2$, $\epsilon > 0$.
2. Construct the shape matrixes A and B of the dimension n. Compute $p_n = p(A, B)$ according to (1) or (2).
3. Construct the shape matrixes A and B of the dimension $n+1$. Compute $p_{n+1} = p(A, B)$ according to (1) or (2).
4. IF ($| p_n - p_{n+1} | < \epsilon$) THEN $n' = n + 1$; STOP
   ELSE $p_n = p_{n+1}$; $n = n + 1$; GOTO *Step* 3 .

PATTERN RECOGNITION

The shape matrixes can be used as the features for recognition of shifted, rotated and scaled objects. Suppose that the finite number of shape classes is given. Each class is represented by one template. The similarities between the object and each of templates are computed according to (1) or (2). Object classification is then performed by maximum similarity. If the maximum similarity is little than some threshold t given in advance, the object is labeled as "unclassified".

The performance of the proposed pattern recognition technique is shown in the following experiment. Fig. 3*a* shows the templates, Fig. 3*b* shows the objects that we want to classify. The shape matrixes of the size 16×16 were used to compute the similarities (see Table 1). The final classification is summarized in Table 2 (the rejection threshold $t = 0.85$ was used). It can be seen clearly by visual comparison of both images that the final correspondence determined by the proposed algorithm is correct everywhere.

TABLE 1 - Similarity between objects and templates (all values are multiplied by 100).

b \ a	1	2	3	4	5	6
A	88	61	68	83	45	76
B	62	85	81	69	80	65
C	96	65	68	84	49	78
D	81	73	71	93	59	77
E	77	53	56	72	37	65
F	49	82	73	59	88	58
G	79	66	65	76	60	98

TABLE 2 - Results of classification (u - unclassified)

Templates	1	2	3	4	5	6	u
Objects	C	B	-	D	F	G	A E

REFERENCES

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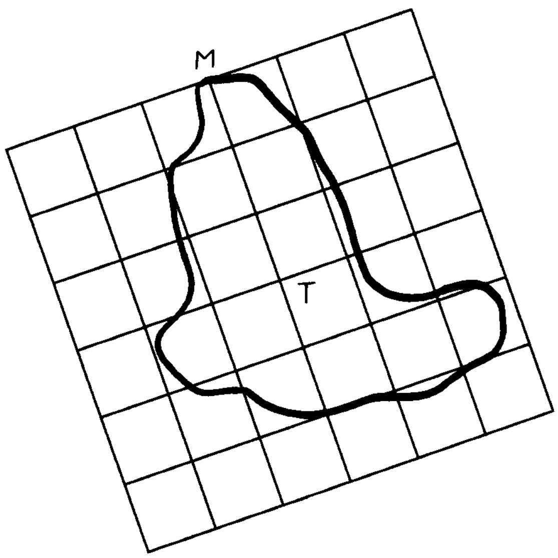
[3] Hu, M.K., 1962, IRE Trans. Inf. Theory, 8, 179-187

[4] Maitra, S., 1979, Proc. of the IEEE, 67, 697-699

[5] Peli, T., 1981, Proc. of the IEEE, 69, 483-485



$$B = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

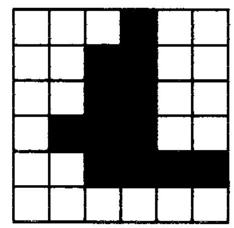


Figure 1: Construction of the shape matrix (left) and reconstruction of the object (right), $n = 6$

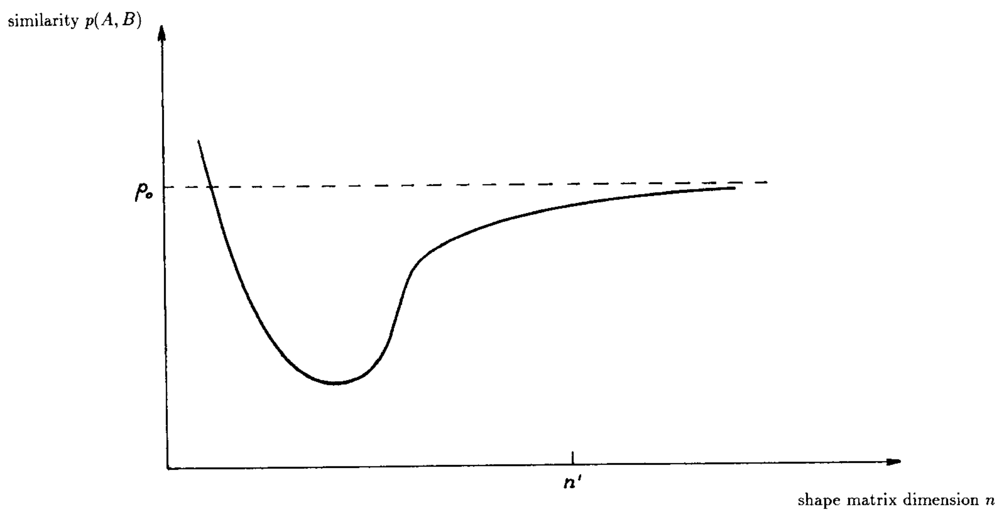


Figure 2: Typical behaviour of object similarity $p(A, B)$

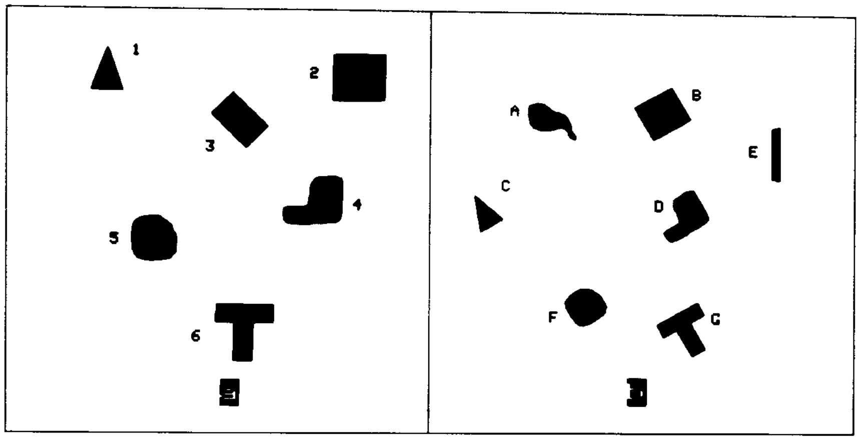


Figure 3: Pattern recognition experiment: a) templates, b) objects to be recognized