Vertex-Based Features for Recognition of Projectively Deformed Polygons

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Abstract

The paper deals with features of a general polygon which are invariant with respect to projective transform. First, some properties of the area of a triangle under projective transform are discussed. New projective triangular invariants of polygons are derived as the quotient of two different products of the areas of triangles formed by the vertices of the polygon. The features are proved to be invariant to numbering of the vertices of the polygon. The number of projective triangular invariants for polygons with given number of vertices is derived. A method for reduction of the number of invariants in case of polygons with high number of vertices is presented. Numerical experiments dealing with three octagons and one polygon with 18 vertices deformed by projective transforms are described.

Keywords: Projective transform, polygon recognition, point-based invariants, triangular invariants

1 Introduction

One of the important tasks in image processing and computer vision is a recognition of objects on images captured under different viewing angles. However, this problem is unsolvable in general case, see [1]. Nevertheless, if we accept restriction only to planar objects, then the distortion between two frames can be described by *projective transform* (sometimes called *perspective projection*)

$$\begin{aligned} x' &= (a_0 + a_1 x + a_2 y)/(1 + c_1 x + c_2 y) \\ y' &= (b_0 + b_1 x + b_2 y)/(1 + c_1 x + c_2 y), \end{aligned} \tag{1}$$

where x and y are the coordinates in the first frame and x' and y' are the coordinates in the second one.

Feature-based recognition of such objects requires features that are invariant to projective transform. Several different approaches to this problem have been published in the recent works. One of them is based on the assumption that non-linear term of the perspective distortion is relatively slight and projective transform can be approximated by an affine transform. This assumption is true, if the distance from the sensor to the object is much greater than the size of the object. In such cases, various affine invariants can be applied such as moment invariants [(2–4)] or Fourier descriptors [(5,6)].

However, in some cases the distortion cannot be approximated by the affine transform and therefore the use of exact projective invariants is required. The invariants which have been developed for this purpose can be categorized into two groups: differential invariants and point-based ones.

Differential invariants are applicable only if the object boundary is a smooth continuous curve. A set of invariants based on boundary derivatives up to the sixth order was presented by Weiss in [(7)]. Unfortunately, these invariants are not defined for such important curves as straight lines or conics. Weiss's invariants are numerically unstable because of the higher order derivatives. To overcome this fact, several improvements were presented in [(8–11)].

The second group of invariants is based on the properties of point sets [(12)], sets composed both from points and straight lines [(13,14)] and triangle pairs [(15)]. A detailed survey of the point-based methods can be found in [(1)].

Among others, projective five-point invariants invariant to point labelling are presented in [(2)] and [(16)]. They can be used not only for the vertices of a polygon, but for arbitrary sets of five points, because they are invariant to any permutation of the points. Nevertheless, they are more complicated and their computation complexity is higher.

The goal of this paper is to derive projective invariants of a general polygon, which are also invariant to the starting point of vertices numbering. The presented method is based on investigation of the properties of a projectively transformed triangle area. These properties are referred in Section 2. In Section 3, the derivation of the invariants is given and the number of invariants is discussed in Section 4. Some numerical experiments are shown in Section 5.

2 The area of a transformed triangle

Let's denote (i,j,k) the area of a triangle with vertices $[x_i,y_i]$, $[x_j,y_j]$ and $[x_k,y_k]$, i.e.

$$\begin{aligned}(i,j,k) &= (x_iy_j - x_jy_i + x_jy_k - x_ky_j + x_ky_i - x_iy_k)/2 \\ &= \frac{1}{2}\begin{vmatrix} x_i & y_i & 1 \\ x_j & y_j & 1 \\ x_k & y_k & 1 \end{vmatrix}. \end{aligned} \tag{2}$$

If the vertices are numbered clockwise, we will consider the area negative.

If $J(x,y)$ is a Jacobian of the projective transform (1) at the point $[x,y]$

$$J(x,y) = \frac{\begin{vmatrix} a_0 & a_1 & a_2 \\ b_0 & b_1 & b_2 \\ 1 & c_1 & c_2 \end{vmatrix}}{(1+c_1x+c_2y)^3}, \tag{3}$$

then the area of the triangle after the transform can be expressed as

$$
\begin{aligned}
(i,j,k)' &= \frac{\begin{vmatrix} a_0 & a_1 & a_2 \\ b_0 & b_1 & b_2 \\ 1 & c_1 & c_2 \end{vmatrix}}{(1+c_1x_i+c_2y_i)\,(1+c_1x_j+c_2y_j)\,(1+c_1x_k+c_2y_k)} \cdot (i,j,k) = \\
&= (i,j,k)\,\sqrt[3]{J(x_i,y_i)\,J(x_j,y_j)\,J(x_k,y_k)}. \qquad (4)
\end{aligned}
$$

The proof follows

$$
\begin{aligned}
(i,j,k)' &= \frac{1}{2}\begin{vmatrix} \frac{a_0+a_1x_i+a_2y_i}{1+c_1x_i+c_2y_i} & \frac{b_0+b_1x_i+b_2y_i}{1+c_1x_i+c_2y_i} & 1 \\ \frac{a_0+a_1x_j+a_2y_j}{1+c_1x_j+c_2y_j} & \frac{b_0+b_1x_j+b_2y_j}{1+c_1x_j+c_2y_j} & 1 \\ \frac{a_0+a_1x_k+a_2y_k}{1+c_1x_k+c_2y_k} & \frac{b_0+b_1x_k+b_2y_k}{1+c_1x_k+c_2y_k} & 1 \end{vmatrix} \\
&= \frac{1}{2\prod\limits_{\ell\in\{i,j,k\}}(1+c_1x_\ell+c_2y_\ell)}\begin{vmatrix} a_0+a_1x_i+a_2y_i & b_0+b_1x_i+b_2y_i & 1+c_1x_i+c_2y_i \\ a_0+a_1x_j+a_2y_j & b_0+b_1x_j+b_2y_j & 1+c_1x_j+c_2y_j \\ a_0+a_1x_k+a_2y_k & b_0+b_1x_k+b_2y_k & 1+c_1x_k+c_2y_k \end{vmatrix} \\
&= \frac{1}{2\prod\limits_{\ell\in\{i,j,k\}}(1+c_1x_\ell+c_2y_\ell)}\det\left(\begin{bmatrix} a_0 & a_1 & a_2 \\ b_0 & b_1 & b_2 \\ 1 & c_1 & c_2 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ x_i & x_j & x_k \\ y_i & y_j & y_k \end{bmatrix}\right) \\
&= \frac{1}{\prod\limits_{\ell\in\{i,j,k\}}(1+c_1x_\ell+c_2y_\ell)}\begin{vmatrix} a_0 & a_1 & a_2 \\ b_0 & b_1 & b_2 \\ 1 & c_1 & c_2 \end{vmatrix}(i,j,k)
\end{aligned}
$$

Another proof of eq. (4) can be found in [17].

As can be seen from (4), the area of the triangle after the transform is given by the area of the original triangle multiplied by some power of Jacobian of the projective transform. Therefore the triangle area is so called *relative invariant* [18]. This property can be utilized for a construction of absolute projective invariants, i.e. invariants, which don't change value under projective transform at all.

3 Projective triangular invariants of polygons

Our task is to find features of planar polygons which are invariant with respect to projective transform. Having invariants of polygons, we can use them in case of more general shaped objects, because the polygons can approximate a number of objects in practice.

A polygon is given by n vertices, where each vertex is given by its coordinates $[x_i, y_i]$. Line segments linking adjacent vertices shouldn't be crossed, but if this condition doesn't hold, it will cause only problems with defining order of vertices, but not with finding the invariants.

Although the order of the vertices is given, the choice of the starting vertex is arbitrary and therefore our features should be invariant to the choice of the starting vertex.

Equation (4) implies that each product of the areas of triangles is a relative invariant to projective transform. If a point $[x_a, y_a]$ occurs n_a times in the product, then the product is only multiplied by the coefficient $(\sqrt[3]{J(x_a, y_a)})^{n_a}$ after the projective transform.

To ensure the invariance to the choice of the starting point, the following algorithm is applied:

1. Choose arbitrary three vertices of the polygon and denote them as $[x_\alpha, y_\alpha]$, $[x_\beta, y_\beta]$ and $[x_\gamma, y_\gamma]$;
   set $P = 1$.

2. **for** $i = 0$ **to** $n - 1$
   $[x_p, y_p] = [x_{\alpha+i}, y_{\alpha+i}]$;
   $[x_q, y_q] = [x_{\beta+i}, y_{\beta+i}]$;
   $[x_r, y_r] = [x_{\gamma+i}, y_{\gamma+i}]$;
   (vertex indexing is considered as cyclic, i.e. $x_{n+k} = x_k$);
   calculate the area (p, q, r) according to (2);
   set $P = P \cdot (p, q, r)$;
   end

It is easy to prove that P is a relative invariant, which does not depend on the choice of the starting point.

Denotation of these invariants issues from the order of the vertices. R_{k_1,k_2,k_3} denotes the invariant constructed from the triangles, whose second vertex has the index greater by k_1 than the first one (i.e. $k_1 - 1$ vertices of the original polygon was omitted between the first and second vertices), the third one greater by k_2 than the second one and the first one greater by k_3 than the third one. If the vertices are numbered from 1 to n, we can write

$$R_{k_1,k_2,k_3} = \prod_{i=1}^{n} (i, (i + k_1 - 1) \bmod n + 1, (i + k_1 + k_2 - 1) \bmod n + 1), \tag{5}$$

where $n = k_1 + k_2 + k_3$ and "mod" denotes the modulo operation. The commas between the indices will be omitted, if it doesn't lead to misunderstanding.

The invariants (5) are called relative projective triangular invariants (RPTI).

Only one RPTI exists for $n = 3$

$$R_{111} = (1, 2, 3)^3$$

and for $n = 4$

$$R_{112} = (1, 2, 3)(2, 3, 4)(3, 4, 1)(4, 1, 2).$$

Any absolute invariants cannot be constructed from R_{111} and R_{112}.

There are two RPTIs for $n = 5$

$$\begin{aligned} R_{113} &= (1, 2, 3)(2, 3, 4)(3, 4, 5)(4, 5, 1)(5, 1, 2), \\ R_{122} &= (1, 2, 4)(2, 3, 5)(3, 4, 1)(4, 5, 2)(5, 1, 3) \end{aligned}$$

and four RPTIs for $n = 6$

$$\begin{aligned}
R_{114} &= (1,2,3)(2,3,4)(3,4,5)(4,5,6)(5,6,1)(6,1,2),\\
R_{123} &= (1,2,4)(2,3,5)(3,4,6)(4,5,1)(5,6,2)(6,1,3),\\
R_{132} &= (1,2,5)(2,3,6)(3,4,1)(4,5,2)(5,6,3)(6,1,4),\\
R_{222} &= (1,3,5)^3(2,4,6)^3.
\end{aligned}$$

The RPTI is not affected by the cyclic shift of the indices, i.e. $R_{123} = R_{231} = R_{312}$, but the interchange of two indices gives a different RPTI; R_{123} is generally different from R_{132}.

The quotient of two general relative triangular invariants is the absolute invariant, if each vertex occurs with the same frequency in the numerator and in the denominator. It is satisfied for the RPTIs defined by (5) ever, when they are calculated on the same polygon. The proof is implied by (4). If a point $[x_a, y_a]$ occurs n_a times both in the numerator and in the denominator, the coefficient $(\sqrt[3]{J(x_a, y_a)})^{n_a}$ is eliminated by reducing. Note: in our case $n_a = 3$.

In the denominator it is suitable to use $R_{1,1,n-2}$, which is constructed from the areas of the triangles from the adjacent vertices of the polygon. If some three adjacent vertices lie on one straight line, the medium vertex can be omitted and the polygon has got less than n vertices. If no three adjacent vertices lie on one straight line, $R_{1,1,n-2}$ is different from zero and it can be used for the normalization of the invariants.

The absolute projective triangular invariant (PTI) after this normalization is denoted as I_{k_1,k_2,k_3}:

$$I_{k_1,k_2,k_3} = \frac{R_{k_1,k_2,k_3}}{R_{1,1,n-2}}. \tag{6}$$

All PTIs for a given polygon can be found by finding all partitions of the number n to a sum of three positive integers k_1, k_2 and k_3, i.e. $n = k_1 + k_2 + k_3$.

Each of such triplets of the numbers corresponds with one PTI excluding the "normalization" triplet $1, 1, n-2$. If $k_1 \neq k_2 \neq k_3$, another PTI can be found by interchanging two of them.

Any PTI does not exist if $n < 5$. If n=5, only one PTI exists (I_{122}). Three PTIs exists if n=6: I_{123},I_{132} and I_{222}.

4 The number of the invariants

The number of PTIs for given n can be derived by the following way. In the construction of RPTIs each triangle from the vertices of the polygon appears just once. If n is not dividable by 3, then the number of the triangles is $\binom{n}{3}$. If n is dividable by 3, then $n/3$ triangles are repeated three times, therefore the number of the triangles in the RPTIs is equal to $\binom{n}{3} + 2n/3$. Since each RPTI is constructed from n triangles, the number of the invariants is equal to the number of the triangles divided by n:

$$\begin{array}{ll} \binom{n}{3}/n = (n-1)(n-2)/6 & \text{or} \\ \binom{n}{3}/n + 2/3 = (n-1)(n-2)/6 + 2/3 & \text{respectively.} \end{array} \tag{7}$$

One RPTI is ever used for the normalization, therefore the number ℓ of the PTIs is less by 1 than the number of the RPTIs

$$\begin{array}{rcll} \ell & = & (n-1)(n-2)/6 - 1 & \text{if} \quad n \bmod 3 \neq 0, \\ \ell & = & (n-1)(n-2)/6 - 1/3 & \text{if} \quad n \bmod 3 = 0. \end{array} \tag{8}$$

The eqs.(8) mean that the number of the PTIs grows quadraticly with n, but the polygon with n vertices has $2n$ degrees of freedom and the projective transform has 8 parameters. Therefore the polygon can have only $m = 2n - 8$ independent invariants to the projective transform:

$$\begin{array}{l} \text{if } (n < 11) \text{ then } (\ell < m) \quad , \\ \text{if } (n = 11) \text{ then } (\ell = m) \quad , \\ \text{if } (n > 11) \text{ then } (\ell > m) \quad . \end{array}$$

That is why it is suitable to reduce the number of the invariants for the polygons with more than 11 vertices.

The PTIs can be divided to groups according to the least index g ($g = \min(k_1, k_2, k_3)$). The number of these groups is $[n/3]$ (integer part). If the number of the invariants should be m, then each group can consist of $2n/(n/3)$, i.e. 6 invariants maximally. Only if n is not dividable by 3, then the group with $g = 1$ can consist of 7 invariants. The PTIs in each group can be partially added up to acquire the desirable number of the invariants. In the following text these invariants are denoted I_{gi}, where $i = 1, 2, \cdots$. The original numbers of the PTIs in the groups are shown in the Tab. 1, the numbers of the I_{gi} are shown in Tab. 2.

g	$n \bmod 3 = 0$	$n \bmod 3 = 1$	$n \bmod 3 = 2$
$[n/3]$	1	1	2
$[n/3]-1$	3	4	5
$[n/3]-2$	6	7	8
$[n/3]-3$	9	10	11
$[n/3]-4$	12	13	14
$[n/3]-5$	15	16	17
. . .			
4	$3([n/3]-3)-3$	$3([n/3]-3)-2$	$3([n/3]-3)-1$
3	$3([n/3]-2)-3$	$3([n/3]-2)-2$	$3([n/3]-2)-1$
2	$3([n/3]-1)-3$	$3([n/3]-1)-2$	$3([n/3]-1)-1$
1	$3[n/3]-4$	$3[n/3]-3$	$3[n/3]-2$

Table 1: The original numbers of the PTIs in the groups

g	$n \bmod 3 = 0$	$n \bmod 3 = 1$	$n \bmod 3 = 2$
$[n/3]$	1	1	2
$[n/3]-1$	3	4	5
$[n/3]-2$	6	6	6
$[n/3]-3$	6	6	6
$[n/3]-4$	6	6	6
$[n/3]-5$	6	6	6
. . .			
4	6	6	6
3	6	6	6
2	6	6	6
1	6	7	7

Table 2: The numbers of the invariants I_{gi} in the groups

5 Numerical experiments

In order to demonstrate the behaviour of the PTIs in practical tasks, the following experiments were carried out.

The objects in Figs. 1a, 2a, 3 and 4a were created by a graphic editor and then the objects in Figs. 1a, 2a and 4a were deformed by projective transforms with various parameters (Figs. b, c, d). The objects in Figs. 1, 2 and 3 are octagons. The acquired values of the PTIs are shown in Tab. 3. They show good stability of the PTIs. Another important property of projective invariants is discriminability, what means how the values of the invariants differ for different objects without projective dependency. The objects in Figs. 1 and 2 are similar from the point of view of the projective transform. They both have got four collinear vertices (3,4,7,8 in Fig. 1a and 2,3,6,7 in Fig. 2a) with two vertices between them (5,6 in Fig. 1a and 4,5 in Fig. 2a). The only difference is two other triplets of collinear points in Fig. 1a (1,6,3 and 2,5,8), which are not in Fig. 2a. That is why five values of the PTIs are similar and only the values of I_{233} are distinct. However, the objects can be recognized according to them. The object in Fig. 3 is quite different, it have got no three collinear points, and its values of the PTIs are very distinct. The discriminability is satisfactory.

The objects in Fig. 4 are polygons with 18 vertices and their values of the invariants are shown in Tab. 4. The values of the invariants of the objects a - d in each figure should be constant, but because of discretization slight errors appeared. The most significant error occurs in case of the object in Fig. 4d, the invariant $I_{24} = I_{2,11,5} + I_{2,6,10}$. The values of $I_{2,11,5}$ are zero, but $I_{2,6,10}$ is a product which contains the area of the triangle $(16, 18, 6)$ (see Fig. 4). These vertices of the object in Fig. 4a lie on one straight line, but in Fig. 4d don't and this fact causes error. Other areas of $I_{2,6,10}$ are not zero, that is why the error is so big. However, the stability of the other invariants is satisfactory and the set of invariants can be used for the recognition.

A lot of zero values of the invariants is caused by relatively high degree of symmetry of these objects, many triplets of the vertices lie on one straight line.

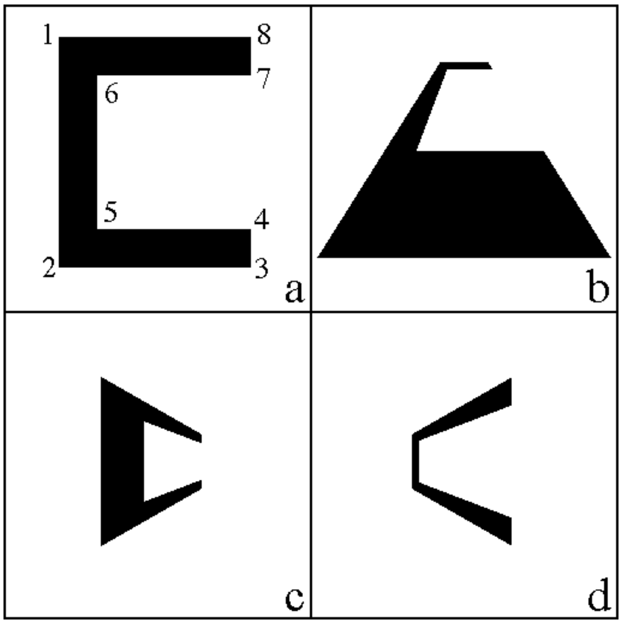

Figure 1: The octagons I, a original, from b to d projective deformations

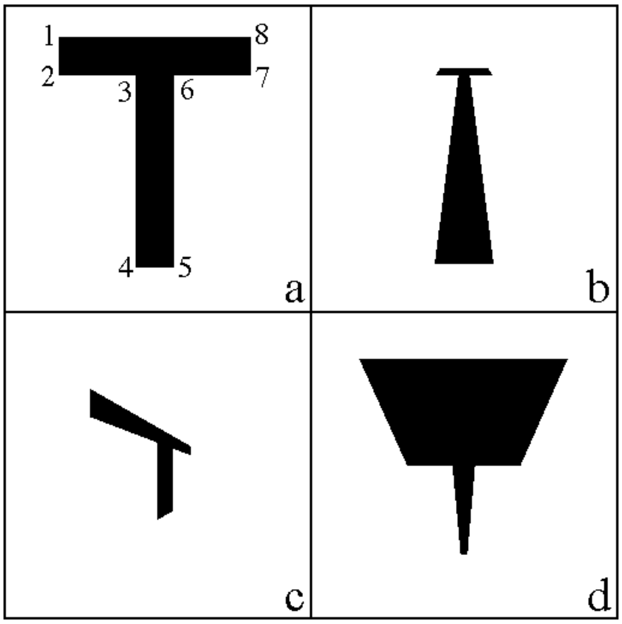

Figure 2: The octagons II, a original, from b to d projective deformations

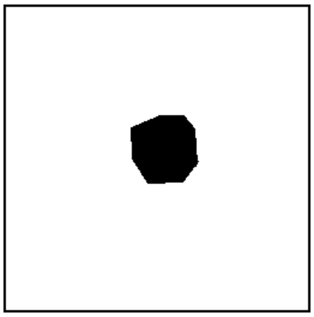

Figure 3: The octagon III

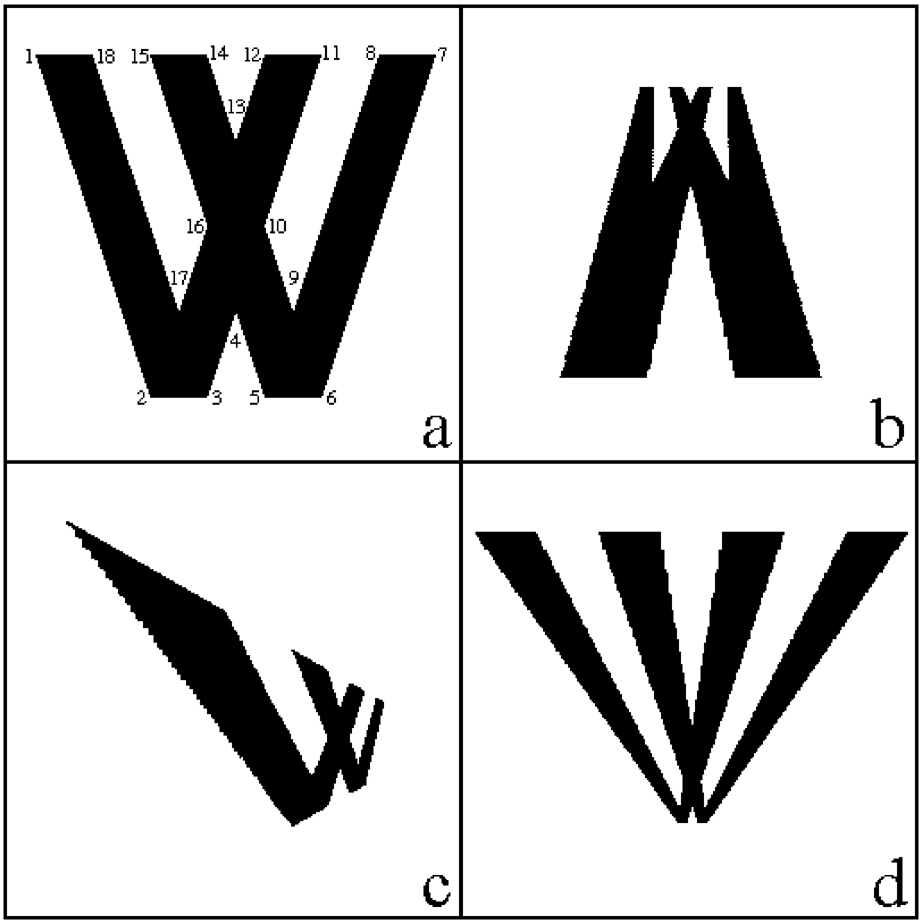

Figure 4: The polygons with 18 vertices, *a* original, from *b* to *d* projective deformations

	Fig.1a	Fig.1b	Fig.1c	Fig.1d	Fig.2a	Fig.2b	Fig.2c	Fig.2d	Fig.3
I_{125}	1	0.927	1.16	0.946	1	0.937	1.27	1.16	2051
I_{152}	1	0.901	1.16	0.946	1	1.26	1.03	1.27	1707
I_{134}	0	-0.001	0	0	0	0	-0.01	0	$3.12 \cdot 10^4$
I_{143}	0	0.001	0	0	0	0	0.02	0	$3.18 \cdot 10^4$
I_{224}	564	425	793	587	478	417	563	654	$5.43 \cdot 10^5$
I_{233}	0	0	0	0	502	640	611	665	$2.42 \cdot 10^6$

Table 3: The values of the PTIs of the objects in Figs. 1, 2 and 3

	Fig. 4a	Fig. 4b	Fig. 4c	Fig. 4d
$I_{11} = I_{1,2,15} + I_{1,15,2}$	0	0	0	0
$I_{12} = I_{1,3,14} + I_{1,14,3}$	0	0	0	0
$I_{13} = I_{1,4,13} + I_{1,13,4}$	0	0	0	0
$I_{14} = I_{1,5,12} + I_{1,12,5}$	-0.002	-0.001	-0.002	-0.001
$I_{15} = I_{1,6,11} + I_{1,11,6} + I_{1,7,10}$	0	0	0	0
$I_{16} = I_{1,10,7} + I_{1,8,9} + I_{1,9,8}$	0.03	0.02	0.02	0.03
$I_{21} = I_{2,2,14} + I_{2,3,13}$	0.03	0.03	0.005	0.09
$I_{22} = I_{2,13,3} + I_{2,4,12}$	-0.006	-0.007	-0.005	-0.009
$I_{23} = I_{2,12,4} + I_{2,5,11}$	0	0	-0.08	0
$I_{24} = I_{2,11,5} + I_{2,6,10}$	0	1.64	-1.83	-9.51
$I_{25} = I_{2,10,6} + I_{2,7,9}$	287	206	266	428
$I_{26} = I_{2,9,7} + I_{2,8,8}$	4775	3595	4562	6146
$I_{31} = I_{3,3,12}$	0	0	0	0
$I_{32} = I_{3,4,11}$	0	0	0	0
$I_{33} = I_{3,11,4}$	0	0	0	0
$I_{34} = I_{3,5,10} + I_{3,10,5}$	0	0	-0.004	0
$I_{35} = I_{3,6,9} + I_{3,9,6}$	10.8	8.34	7.59	18.9
$I_{36} = I_{3,7,8} + I_{3,8,7}$	0	0	-0.02	0
$I_{41} = I_{4,4,10}$	0	0	0	0
$I_{42} = I_{4,5,9}$	22.3	13.1	21.5	25.4
$I_{43} = I_{4,9,5}$	22	13.1	22.3	26.4
$I_{44} = I_{4,6,8}$	0	0	0.2	0
$I_{45} = I_{4,8,6}$	0	0	0.004	0
$I_{46} = I_{4,7,7}$	0	0	0	0
$I_{51} = I_{5,5,8}$	-116	-135	-121	-146
$I_{52} = I_{5,6,7}$	0	0	-1.61	0
$I_{53} = I_{5,7,6}$	0	0	0.3	0
$I_{61} = I_{6,6,6}$	-446	-414	-348	-558

Table 4: The values of the invariants of the objects in Fig. 4 (the values were divided by 10^6)

6 Conclusions

The projective triangular invariants introduced in this paper are defined on objects from straight line segments, i.e. polygons, where the differential invariants fail.

The numerical experiments were carried out to find the numerical stability and discriminability. They both were found satisfactory.

In comparison with previously published point-based methods, our invariants are invariant to the choice of the starting point and they do not need any point matching, on the other side they can only be used for the vertices of the polygons which define the order of the points (they cannot be used for a set of isolated points). The invariants to the choice of labelling presented in [(2)] or in [(16)] can be used for arbitrary sets of points, but they are more complicated and therefore their computing complexity is higher.

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References

1. J. B. Burns, R. S. Weiss, E. M. Riseman, The Non-Existence of General-Case View-Invariants. In J. L. Mundy, A. Zisserman, editors, *Geometric Invariance in Computer Vision*, pp. 120-131, MIT Press (1992).

2. T.H. Reiss, *Recognition Planar Objects Using Invariant Image Features*, Lecture Notes in Computer Science **676**, Springer (1993).

3. J. Flusser and T. Suk, Pattern recognition by affine moment invariants, *Pattern Recognition* **26**, 167-174 (1993).

4. J. Flusser, T. Suk, A Moment-Based Approach to Registration of Images with Affine Geometric Distortion, *IEEE Trans. Geosci. Remote Sensing*, Vol. 32, No. 2, pp. 382-387 (1994).

5. C.C.Lin and R. Chellapa, Classification of partial 2-D shapes using Fourier descriptors, *IEEE Trans. Pattern Anal. Mach. Intell.* **9**, 686-690 (1987).

6. K. Arbter, W.E. Snyder, H. Burkhardt and G. Hirzinger, Application of affine-invariant Fourier descriptors to recognition of 3-D objects. *IEEE Trans. Pattern Anal. Mach. Intell.* **12**, 640-647 (1990).

7. I. Weiss, Projective invariants of shapes, *Proc. Image Understanding Workshop*, Cambridge, Mass., U.S.A., 1125-1134 (1988).

8. C.A. Rothwell, A. Zisserman, D.A. Forsyth and J.L. Mundy, Canonical Frames for Planar Object Recognition, *Proc. 2nd ECCV*, 757-772, Springer (1992).

9. A.M. Bruckstein, R.J. Holt, A.N. Netravali and T.J. Richardson, Invariant Signatures for Planar Shape Recognition under Partial Occlusion, *Proc. 11th International Conference on Pattern Recognition*, The Hague, The Netherlands, 108-112, (1992).

10. A.H. Salden, B.M. ter Haar Romeny and M.A. Viergever, Affine and projective differential geometric invariants of space curves *(to appear)*.

11. I. Weiss, Differential Invariants without Derivatives, *Proc. 11th International Conference on Pattern Recognition*, The Hague, The Netherlands, 394-398 (1992).

12. P. Meer and I. Weiss, Point/Line Correspondence under 2D Projective Transformation, *Proc. 11th International Conference on Pattern Recognition*, The Hague, The Netherlands, 399-402 (1992).

13. T.H. Reiss, Object recognition using algebraic and differential invariants, *Signal Processing* **32**, 367-395 (1993).

14. D. Forsyth, J.L. Mundy, A. Zisserman, C. Coelho, A. Heller and C. Rothwell, Invariant Descriptors for 3-D Object Recognition and Pose, *IEEE Trans. Pattern Anal. Mach. Intell.* **10**, 971-991 (1987).

15. S. Linnainmaa, D. Harwood and L.S. Davis, Pose determination for a three dimensional object using triangle pairs, *IEEE Trans. Pattern Anal. Mach. Intell.* **5**, 634-647 (1988).

16. R. Lenz, P. Meer, Point configuration invariants under simultaneous projective and permutation transformations, *Pattern Recognition*, Vol. 27, No. 11, pp. 1523-1532 (1994).

17. T. Suk: *Triangular Projective Invariants*, Internal Res. Rep., UTIA AVCR (1992) (in Czech).

18. I. Shur, *Vorlesungen über Invariantentheorie.* Springer, Berlin (1968) (in German).