Merging of Multistep Predictors for Decentralized Adaptive Control

Václav Šmídl, Josef Andrýsek

***Abstract*— Decentralized adaptive control is based on the use of many local controllers in parallel, each of them estimating its own local model and pursuing local aims. When each controller designs its strategy using only its model, the resulting control will be suboptimal since local models do not allow prediction of consequences of actions of the neighbors. We use probabilistic formulation of adaptive control to build predictive densities of future outputs. Mutual exchange of these densities on commonly observed variables is proposed to compensate for incompleteness of the local models. The task is to find a procedure how to use such information withing the control strategy design under the constraint that the resulting design procedure is of the same complexity as the one without the exchange. We present an approximate algorithm and illustrate its performance on a simple example.**

I. Introduction

Decentralized adaptive control is a well developed field for deterministic systems [1], or systems with stochastic inputs [7]. In this work, we are concerned with general stochastic systems that are described by probability density functions. The main task addressed in this paper is what functional forms of uncertain information can be exchanged between the controllers and how such information can be used in design of stochastic control strategy.

A. Centralized adaptive control

Consider probabilistic model of a stochastic system:

$$y_t \sim f(y_t|u_t, d^{t-\partial:t-1}, \theta_t). \tag{1}$$

Here, vector y_t denotes vector of outputs of the system, u_t is the vector of inputs, d_t is their aggregation $d_t = [y_t', u_t']'$, $d^{t-\partial:t-1} = [d_{t-\partial}, \ldots, d_{t-1}]$, and θ_t is an unknown time-variant parameter. The model can be estimated using Bayes' rule with forgetting [4], recursively evaluating posterior density on parameters $f(\theta_t|d^{1:t})$. One-step predictor of y_t is obtained by marginalization:

$$f(y_{t+1}|u_{t+1}, d^{1:t}) = \int f(y_{t+1}|u_{t+1}, d^{1:t}, \theta_{t+1}) f(\theta_{t+1}|d^{1:t}) d\theta_{t+1}. \tag{2}$$

Control strategy for system (1) can be designed using Fully Probabilistic Design (FPD) [2]. FPD minimizes future expected loss in the form of Kullback-Leibler divergence between the predicted and the target pdf of future trajectory. As a result it generates stochastic control law:

$$f(u_{t+1}|d^{1:t}) = \arg \min_{f(u^{(h)})} \mathsf{E}\left[\mathsf{D}\left(f(d^{(h)}|d^{1:t}) \middle\| g(d^{(h)})\right) \middle| d^{1:t}\right]. \tag{3}$$

Here, $d^{(h)} = d^{t+1:t+h}$ denotes data on the predicted horizon h, $\mathsf{E}[\cdot|\cdot]$ denotes conditional expected value, $\mathsf{D}(\cdot||\cdot)$ is the Kullback-Leibler divergence. $f(d^{(h)}|d^{1:t})$ is predictive pdf of future outputs, and $g(d^{(h)})$ is the target pdf of the future outputs. This non-standard technique of control strategy design reduces to linear quadratic (LQ) control for linear Gaussian model (1) and specific choice of Gaussian target pdf [2].

Rigorous evaluation the required multistep predictor

$$f(d^{(h)}|d^{1:t}) = \prod_{\tau=1}^{h} f(d_{t+\tau}|d^{1:t+\tau-1}), \tag{4}$$

is typically intractable and it is often approximated by a product of one-step ahead predictors (2), using $f(d_{t+\tau}|d^{1:t+\tau}) \approx f(d_t|d^{1:t})$. Under this approximation, all steps of the associated design procedure are of the same complexity, allowing evaluation for large h.

B. Extension to decentralized stochastic control

In decentralized control, each controller is assigned to control only a sub-set of all considered inputs using only local models. For simplicity, we will consider only two controllers, C_1 and C_2. Data space of the ith controller, $d_{[i],t}$, is divided into its 'private' variables $d_{i\rceil,t}$ and commonly available variables $d_{\cap,t}$. The full data set is then $d_t = [d_{1\rceil t}, d_{\cap t}, d_{2\rceil t}]$. Input spaces of both controllers are non-overlapping, $u_t = [u_{1,t}, u_{2,t}]$.

Naive decentralized control can be achieved by running two adaptive controllers in parallel, each of them using local model and minimizing local loss function. Such approach can yield mutually adverse individual control strategies. This can be due to: (i) inconsistent loss functions, and/or (ii) inconsistent predictions of future behavior due to uncertainty in parameter estimates and model incompleteness. We will assume that the target densities are compatible and (i) does not arise. We propose a mechanism how to reduce inconsistency in (ii).

II. Merging of multistep predictors

The constrained data-spaces allows each controller to work with reduced models only and thus to produce only

Both authors are with Institute of Information Theory and Automation, Prague, Czech Republic. Support of grants AV ČR 102/08/P250 and MŠMT 1M0572 is gratefully acknowledged. smidl@utia.cas.cz

marginal predictors of the future trajectory, $f_i(d_{[i]}) \equiv f_i(d_{[i]}^{(h)}|d_{[i]}^{1:t})$. The task is to find the best local control strategies using only this information. We propose the following approach: (i) construct a hypothetical global controller, (ii) design the best possible conditionally independent control strategy, and (iii) replicate design of such control strategy on local level. Construction of (i) requires to combine mutually incompatible posterior pdfs from the local controllers. For this task, we use the merging procedure [3] where global predictor is constructed as a probabilistic mixture. Key advantage of this approach is that the merged predictor for each controller requires only marginal density on the predicted data of its neighbors:

$$\tilde{f}(d_\cap, d_{i\rceil}) = f_i(d_{i\rceil}|d_\cap)\left(\alpha f_1(d_\cap) + (1-\alpha) f_2(d_\cap)\right), \quad (5)$$

where $0 \leq \alpha \leq 1$ is the chosen importance weight of the first controller. Major disadvantage is that (5) is a complex function of control strategies and predictors preventing direct design of the control strategy. The problem was considered in [5] and the following approximate algorithm was developed. For all i in time t do:
/0/ Collect realizations $\boldsymbol{d}_{[i],t}$ of $d_{[i],t}$, build predictors (2).
/1/ Design control strategies $f_i(u_{1\rceil t+1}|d^{1:t})$ using (3).
/2/ Using predictors (2) and data $\boldsymbol{d}^{t-\partial:t}$, construct

$$f_i(d_{[i]}^{(h)}|\boldsymbol{d}_{[i]}^{1:t}) = \prod_{\tau=1}^{h}\left[f(y_{[i]t+\tau}|u_{i,t+\tau}, d_{[i]}^{1:t+\tau}) \times f_i(u_{i,t+\tau}|d_{[i]}^{1:t+\tau})\right]\delta(d_{[i]}^{t-\partial:t} - \boldsymbol{d}_{[i]}^{t-\partial:t}). \quad (6)$$

Here, $\delta()$ denotes Dirac delta function.
/3/ Split (6) into marginal $f_i(d_\cap^{(h)})$ and conditional $f_i(d_{i\rceil}^{(h)}|d_\cap^{(h)})$. Send the marginal to all neighbors.
/4/ Build merged predictor (5). Project it into a Gaussian pdf using geometric combination. Decompose it into a product of one-step predictors (4).
/5/ Design a new control strategy $f_i(u_{1\rceil t+1}|d^{1:t})$ using FPD with one-step predictors from step /4/.
/6/ If required, go to /2/, else $t \leftarrow t+1$.

III. Experiment

The following 3-output 2-input system was simulated:

$$f(y_t|\psi_t, \Sigma) = \mathcal{N}(\theta\psi, \Sigma), \quad (7)$$

where $y_t = [y_{1,t}, y_{2,t}, y_{3,t}]'$,
$\psi_t = [y_{1,t-1}, y_{2,t-1}, y_{3,t-1}, u_{1,t}, u_{1,t-1}, u_{2,t}, u_{2,t-1}]'$,

$$\theta = \begin{bmatrix} 0.8 & 0.2 & 0 & -0.3 & 0.4 & 0 & 0 \\ -0.2 & 0.5 & -0.8 & 0.2 & 0.5 & -0.2 & -0.5 \\ 0 & 1.1 & -0.5 & 0 & 0 & -0.2 & 0.3 \end{bmatrix}$$

and $\Sigma = \text{diag}([0.1, 0.1, 0.1])$. The system was controlled to reach $g(y_t) = \mathcal{N}([0,1,0]', \text{diag}([0.1,0.1,0.1]))$. Target pdf for input was chosen as $g(u_t) = \mathcal{N}([0,0]', \text{diag}([0.1, 0.1]))$. This choice correspond to the quadratic criteria with inverses of variances as kernel matrices. A Monte Carlo simulation of (7) controlled by adaptive strategies designed on horizons $h = [1,4,7]$ was simulated for 200 steps for the following methods: **CAC**, centralized adaptive control using model (7); **DAC**, naive adaptive control for C_1 with $d_{[1],t} = [y_{1,t}, y_{2,t}, u_{1,.t}]$, C_2 with $d_{[2],t} = [y_{2,t}, y_{3,t}, u_{[2],t}]$, and accordingly adapted $\psi_{[i],t}$; **DACwM**, the proposed method for the same decentralization as in DAC. Results of the relative improvement in performance of the proposed algorithm over the naive approach are displayed in Fig 1 as function α and h. After one iteration of the algorithm the resulting control can be worse than that of the naive approach, however, with increasing number of iterations, the proposed method outperforms the naive approach [5].

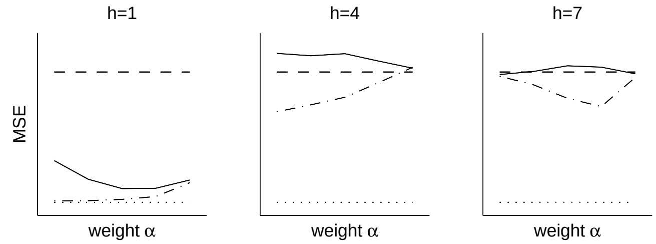

Fig. 1. Relative performance of the the proposed DACwM method after one iteration (full) and 10 iterations (dash-dotted) with respect to CAC (dotted line) and DAC (dashed line). Different columns correspond to strategies designed on indicated horizons h.

IV. Discussion and Conclusion

Preliminary results indicate that merging of multi-step predictors can improve performance of decentralized adaptive controllers. On such a simple example, the computational overhead associated with merging is significantly higher that computational overhead of centralized control. However, this overhead will be smaller on larger systems with minor overlap, such as distributed adaptive control of urban surface traffic using traffic lights [6]. Moreover, the proposed merging operation is easily scalable to multiple neighboring controllers which again is important in such a large systems as city traffic network.

References

[1] P. Ioannou. Decentralized adaptive control of interconnected systems. *Automatic Control, IEEE Transactions on*, 31(4):291–298, Apr 1986.
[2] M. Kárný. Towards fully probabilistic control design. *Automatica*, 32(12):1719–1722, 1996.
[3] J. Kracík. On composition of probability density functions. In J. Andrýsek, M. Kárný, and J. Kracík, editors, *Multiple Participant Decision Making*, pages 113–121, Adelaide, May 2004. Advanced Knowledge International.
[4] R. Kulhavý and F. J. Kraus. On duality of regularized exponential and linear forgetting. *Automatica*, 32:1403–1415, 1996.
[5] V. Šmídl and J. Andrýsek. Distributed Bayesian decision-making: Further experiments. Technical Report 16, DAR–ÚTIA AV ČR, 2007.
[6] V. Šmídl and J. Přikryl. Distributed Bayesian decision-making for urban traffic control. In *Proceedings of IEEE IECON'06*, pages 4695–4700, Paris, France, @nov 2006. IEEE.
[7] T. Xiaobin, X. Hongsheng, W. Gang, and S. Demin. Decentralized robust adaptive controller of modular stochastic control systems. In *Intelligent Control and Automation, 2000. Proceedings of the 3rd World Congress on*, pages 3092–3096, 2000.