Strategy design for futures trading*

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Abstract. The paper addresses the design of trading strategy for futures markets. The problem is formulated as dynamic decision making task and as such is solved. Iterations-spread-in-time and Monte Carlo methods are employed to the solution. The results of off-line real-data experiments are presented.

Abstrakt. Text popisuje návrh obchodní strategie určené pro trhy s futures kontrakty. Návrh se sestává z definice úlohy jako problému dynamického rozhodování. Následně je úloha řešena pomocí iterací rozložených v čase a metody Monte Carlo. Text obsahuje výsledky experimentů prováděných na reálných datech.

1 Introduction

The paper describes a part of research aiming to design automatic trading system for futures markets. The trading on exchanges is based on knowledge and prediction of the price of given commodity, which represents a very complex task.

The futures trading problem is formulated as a particular decision making (DM) task. DM reformulates the task as mathematical problem, which leads to integral equations. We need to solve the equations, but to find the analytical solution is almost impossible and the numerical calculation leads to bad conditioned or long calculated solutions. DM task is necessary to solve in given time, e.g. when the trader on exchange needs the solution each day, the calculation cannot take 3 days and is restricted by 24 hours. Although the reformulation like a DM task is good, we need feasible solution, which calls for an approximation. This paper considers by task redefinition and introduces the approximations.

The paper's outline is as follows. Section 2 introduces terminology of futures exchange, recalls main terms of DM theory and reformulates futures trading problem as dynamic DM task. Section 3 contains approximation of DM. Section 4 presents the experimental results obtained on real data. Section 5 addresses open questions as well as possible directions to approach's improvement.

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2 Preliminaries

2.1 Trading futures

The following definition by of the futures exchange is proposed by [2]. A *futures exchange* is a central financial exchange where people can trade standardized futures contracts; that is, a contract to buy specific quantities of a commodity (basic resources and agricultural products such as iron ore, coal, sugar, coffee beans, wheat, gold, etc) or financial instrument (cash, evidence of an ownership interest in an entity) at a specified price with delivery set at a specified time in the future. A futures contract gives the holder the obligation to buy or sell.

The term position means a commitment to buy or sell a given amount of commodities. The basic types of position are distinguished: *short*, *long* and *flat*.

A *long position* yields a trader's benefit when the price increases, and trader's loss otherwise. This position refers to the situation when

- a trader buys an option contract that he has not already written (i.e. sold), he is said to be opening a long position.
- a trader sells an option contract that he already owns, he is said to be closing a long position.

A *short position* yields a trader's profit when the price decreases, and trader's loss otherwise. This position refers to the situation when

- a trader sells an option contract that he does not already own, he is said to be opening a short position.
- a trader buys an option contract that he has written (i.e. sold), he is said to be closing a short position.

A *flat position* denotes the state when no other type of position is active. Flat position means neither trader's profit nor trader's lose with any price change.

The aim of trader is design such a strategy of positions selecting, which ensures trader's profit with minimal risk. The strategy design is based on prediction of price behavior and is very sensitive i.e. the small impreciseness in strategy make big change of profit.

2.2 Decision making under uncertainty

Decision maker is either human being or device aiming to influence a part of the World he is interested in (so called *System*) The influence desired is expressed by DM aim. To reach this DM aim a decision maker designs and applies a *DM strategy*, R_t. This strategy maps observations of the system's behavior $y_1, \ldots, y_t$ available to decision maker and past decisions $x_1, \ldots x_{t-1}$ to *decisions* x_t:

$$R_t : [y_1, \ldots, y_t, x_1, \ldots x_{t-1}] \rightarrow x_t.$$

The available knowledge grows with time, because it is extended each time step by new system output y_t and also by new decision x_t. The decision typically influences the system, therefore decision maker works with respect to closed loop 'decision maker - system'.

All knowledge about system available to decision maker to design decision x_t is called *experience* $\mathcal{P}_t = (y_1, x_1, \dots, y_{t-1}, x_{t-1}, y_t)$. *Ignorance* $\mathcal{F}_t$ is knowledge about system unavailable to decision maker. *System behavior* consists of experience, decision and ignorance $\mathcal{Q} = (\mathcal{P}_t, x_t, \mathcal{F}_t)$.

Gain is mapping of system behavior to real non-negative number $G : \mathcal{Q} \rightarrow [0, \infty]$. Gain express the success of reaching the decision maker aims with given decision making strategy. The gain is not causal and it is necessary to measure the potential strategy success. Therefore the expected value is defined. Conditioned *expected value* $\mathcal{E}(.|.)$ is functional which returns the value of the gain independent on ignorance for the given strategy and conditioned by experience.

The expected gain conditioned by experience is chosen as following integral:

$$\mathcal{E}\left[G(\mathcal{P}_t, x_t, \mathcal{F}_t)\middle|\mathcal{P}_t, x_t\right] = \int_{\mathcal{F}_t} G(\mathcal{P}_t, x_t, \mathcal{F}_t) f(\mathcal{F}_t|\mathcal{P}_t, x_t) d\mathcal{F}_t, \tag{1}$$

where $f(\mathcal{F}_t|\mathcal{P}_t, x_t)$ is probability density function of the ignorance conditioned by experience, this terms stands for the decision makers imagination of the ignorance based on experience. See [3] for general derivation of this equation.

The decision maker chose the decision $x_t \in \mathcal{X}$ to maximize of expected value in each time t:

$$x_t = \arg\max_{x_t \in \mathcal{X}} \mathcal{E}\left[G(\mathcal{Q})\middle|\mathcal{P}_t, x_t\right], \tag{2}$$

which is the idea based on principle of optimality - see [1].

2.3 Futures trades as DM task

This subsection reformulates futures trading task as a decision making problem.

The system is exchange with one kind of futures contract. The system output y_t is a price of the contract. We design the strategy for discrete time starting from 1, finishing by horizon T. The strategy starts and finishes with the flat position.

The decision maker designs in each time t an integer number $x_t \in \mathcal{Z}$ as decision. The decision x_t characterizes traders position, i.e. $|x_T|$ characterizes count of contracts and $\operatorname{sign}(x_T)$ characterizes the type of position 1 long, -1 short and 0 flat. The flat position at the beginning and at the horizon is expresses as: $x_0 = x_T = 0$.

The profit influenced only by the decision x_t is expressed via:

$$g(x_t, x_{t-1}, y_{t+1}, y_t) = (y_{t+1} - y_t)x_t - C|x_{t-1} - x_t|, \tag{3}$$

where $(y_{t+1} - y_t)x_t$ is profit caused by the change of price and C is normalized transaction costs for position change and $|x_{t-1} - x_t|$ is change of position. The gain from the whole trading can be expressed as a sum of partial gain (3) over time $t \in \{1, 2, \dots, T\}$. The gain function $G_t(.)$ expresses the profit caused by decisions $x_t, \dots, x_T$:

$$G_t(x_{t-1}, \dots, x_T, y_t, \dots, y_T) = -C|x_{T-1} - x_T| + \sum_{k=t}^{T-1} (y_{k+1} - y_k)x_k - C|x_{k-1} - x_k|, \tag{4}$$

Easy to see, that the function $G_t(.)$ is additive and backward recursive

$$G_t(x_{t-1}, \dots, x_T, y_t, \dots, y_T) = g(x_t, x_{t-1}, y_{t+1}, y_t) + G_{t+1}(x_t, \dots, x_T, y_{t+1}, \dots, y_T) \tag{5}$$

with initial condition

$$G_T(x_{T-1}, x_T, y_T) = -C|x_{T-1} - x_T|. \tag{6}$$

2.4 Solution of dynamic DM problem

To maximize the profit, the gain over the decisions $x_1, \ldots, x_T$ should be maximized:

$$\max_{\{x_1,\ldots,x_T\}} G_1(x_0, \ldots, x_T, y_1, \ldots, y_T). \tag{7}$$

Using the optimality principle (see [1] for details) and additivity of the gain function the optimal gain in time t can be expressed:

$$B_t(x_{t-1}, \ldots, x_T, y_t, \ldots, y_T) = \max_{x_t} \Big[g(x_{t-1}, x_t, y_t, y_{t+1}) + \max_{\{x_{t+1},\ldots,x_t\}} G_{t+1}(x_t, \ldots, x_T, y_t, \ldots, y_T) \Big].$$

Function $B_t(.)$ is called Bellman's function and hold the following recursive shape:

$$B_t(x_{t-1}, \ldots, x_T, y_t, \ldots, y_T) = \max_{x_t} \Big[g(x_{t-1}, x_t, y_t, y_{t+1}) + B_{t+1}(x_t, \ldots, x_T, y_t, \ldots, y_T) \Big],$$

where the maximal argument is the optimal decision at time t. But to find this argument, the knowledge of future decisions and prices is needed, i.e. $x_{t+1}, \ldots, x_T, y_t, \ldots, y_T$. These variables are the part of ignorance, therefore the expected value must be used:

$$\mathcal{V}_t(x_{t-1}, y_t) = \max_{x_t} \mathcal{E} \Big[g(x_{t-1}, x_t, y_t, y_{t+1}) + \mathcal{V}_{t+1}(x_t, y_{t+1}) \Big| x_0, \ldots, x_t, y_1, \ldots, y_t \Big], \tag{8}$$

where $\mathcal{V}_t(.)$ is called admissible Bellman's function.

3 Approximation of decision making

The substitution (3) into the equation (8) results in more suitable form:

$$\begin{aligned} \mathcal{V}_t(x_{t-1}, y_t) &= \max_{x_k} \Big[- y_t x_t - C|x_{t-1} - x_t| + x_t \underbrace{\mathcal{E}(y_{t+1}|x_0, \ldots, x_t, y_1, \ldots, y_t)}_{(*)} \\ &\quad + \underbrace{\mathcal{E}\Big(\mathcal{V}_{t+1}(x_t, y_{t+1}) \Big| x_0, \ldots, x_t, y_1, \ldots, y_t \Big)}_{(**)} \Big]. \end{aligned} \tag{9}$$

This paragraph concerns expressing the term $(*)$, which characterizes expected value of future price y_{k+1} conditioned by the experience.

The probability density function $f(y_{k+1}|x_0, \ldots, x_t, y_1, \ldots, y_t)$ is required to express the expected value $(*)$. The probability density function can be written in the parameterized form:

$$f(y_{t+1}|x_0, \ldots, x_t, y_1, \ldots, y_t) = \int_\theta f(y_{t+1}|\theta, x_0, \ldots, x_t, y_1, \ldots, y_t) f(\theta|x_0, \ldots, x_t, y_1, \ldots, y_t) d\theta \tag{10}$$

The last expression consists of two density functions: $f(\theta|x_0,\ldots,x_t,y_1,\ldots,y_t)$ is the density of model parameters conditioned by experience, where θ is vector of the parameters. $f(y_{t+1}|\theta,x_0,\ldots,x_t,y_1,\ldots,y_t)$ is density of price y_{t+1} conditioned by model parameters and experience.

The assumed model is autoregressive and has following shape:

$$y_t = a_1 y_{t-1} + a_2 y_{t-2} + \ldots + a_N y_{t-N} + b + e_t, \tag{11}$$

where $\theta = (a_1,\ldots,a_N,b)$ are model parameters, N denotes model's order and e_t is white noise with distribution $N(0,\sigma^2)$, therefore the model prediction is normally distributed:

$$f(y_{t+1}|\theta,x_0,\ldots,x_t,y_1,\ldots,y_t) = N(a_1 y_t + a_2 y_{t-1} + \ldots + a_N y_{t-N+1} + b, \sigma^2). \tag{12}$$

The density function of model parameters $f(\theta|x_0,\ldots,x_t,y_1,\ldots,y_t)$ is estimated using software MIXTOOLS [4], which works with the distribution $f(\theta|x_0,\ldots,x_t,y_1,\ldots,y_t)$ and generates samples of model parameters.

This scheme corresponds with principles of Monte Carlo method and the expected value of the future price can be calculated using the following formula:

$$\hat{y}_{k+1} = \sum_{i\in S}(a_{1,i}y_k + a_{2,i}y_{k-1} + \ldots + a_{N,i}y_{k-N+1} + b_i)p_i, \tag{13}$$

where S is a set of samples, i is an index of sample, $(a_{1,i},\ldots,a_{N,i},b_i)$ is a sample vector and p_i is probability of the sample i.

Let approximate the term $(**)$ of the equation (9). The main problem of calculating the term is backward character of equation (8), where the future value of Bellman's function $\mathcal{V}_{t+1}(.)$ is needed to calculation the $\mathcal{V}_t(.)$. This problem is solvable two ways: expressing the generalized shape of Bellman's function or approximation by suitable shape.

We need to find formal solution of equation (9) to express the generalized shape of Bellman's function. The desired solution must be valid for all sequences $y_1,\ldots,y_T$. However this task is very complex and it seems impossible to find the formal solution.

The approximation of Bellman's function is more promising way. The approximation must be suitable for further computing, but at the same time contains the parameters of Bellman's function, therefore the following shape has been chosen:

$$\mathcal{V}_t(x_{t-1},y_t) \approx V_t(x_{t-1},y_t) \equiv p(x_{t-1})y_t + q(x_{t-1}), \tag{14}$$

where $p(.)$ and $q(.)$ are real functions. The approximation does not depend on ignorance, therefore the expected value in term $(**)$ is expressed as follows:

$$\mathcal{E}\Big(\mathcal{V}_{k+1}(x_k,y_{k+1})\Big|x_0,\ldots,x_t,y_1,\ldots,y_t\Big) \approx V_{k+1}(x_k,y_{k+1}). \tag{15}$$

The tasks is to design algorithm how to find functions $p(.)$ and $q(.)$ in definition (14). The approximation generates a non-preciseness in equation (8):

$$V_k(x_{k-1},y_k) + e_k = \max_{x_k}\mathcal{E}\Big[g(x_k,x_{k-1},y_{k+1},y_k) + V_{k+1}(x_k,y_{k+1})\Big|x_0,\ldots,x_t,y_1,\ldots,y_t Big], \tag{16}$$

where e_k is introduced non-preciseness, which is restricted by constant.

All terms in equation (16) are known or calculable. The design assumes, that Bellman's function shape does not vary. Therefore if the tth approximation of Bellman's function is $\hat{V}_t(x_{t-1}, y_t)$, the non-preciseness of approximation in time t can be expressed via:

$$e_t = \max_{x_t} \mathcal{E}\left[g(x_t, x_{t-1}, y_{t+1}, y_t) + \hat{V}_{t+1}(x_{t-1}, y_t) \middle| x_0, \ldots, x_t, y_1, \ldots, y_t \right] - \hat{V}_t(_{t-1}, y_t). \quad (17)$$

Then we minimize the sum of squares $\min_{\hat{V}_t} \sum_{k=1}^{t} e_k^2$ and arguments of minimum are the best approximation of the function $\hat{V}_t(.)$. The minimization leads to least squares method.

4 Experimental part

This section describes the experimental setup, data and results obtained. The designed trading strategy is defined at discrete time $t \in \{1, 2, \ldots, T\}$. The time step corresponds with interval of 24 hours. The trading period is given by available data.

The data used for design of the strategy are so-called close prices, which are collected once a day. It is the last price, when the exchange closes trading. The economic specialists grant that close price is the most stable price. The close price y_t is assumed to be known in time t, i.e. y_t is available to design the decision x_t.

The part of data sets is transaction costs c_t. Moreover the price changes during the day and the close price represent the best approximation, but the risk constant is demanded. Therefore the slippage constant c_s, which characterizes typically change of the price in delay between decision and real trading is employed. This constant is used as penalization for each action in design. And the whole transaction costs C (firstly used in the equation (3)) is defined as $C = c_t + c_s$.

The general equations used in this paper do not specify the restriction to decision x_t. The restrictions depend on the trader's account, because traders must own money to buy or sell contract at an exchange and the range of contracts to position is limited by owned money. We use following values of decision $x_t \in \{-1, 0, 1\}$. This three values are enough for experiments, because the wider range of actions leads only to use the extremal values of decision. This phenomenon is caused by the shape of gain function (3), which is partially linear function of decision x_t. The strategy starts and ends with flat position, therefore $x_0 = x_T = 0$.

The order of model (see equation (11)) is set to $N = 2$, because this value gives the best profit of strategies in the previous research. Predictions are generated by Monte Carlo method. The count of Monte Carlo samples is chosen dynamically: The decision is final, when it is not influenced by new Monte Carlo samples.

4.1 Used data

There are 35 available price sequences for the experiments. The sequences contain prices for more than 15 year, i.e. about 3900 trading days in each sequence. The experiment set is too wide to present all results here, therefore the following five futures contracts were chosen as reference markets.

Ticker	Description
CC	Cocoa - CSCE
CL	Petroleum-Crude Oil Light
FV2	5-Year U.S. Treasury Note
JY	Japanese Yen - FOREX
W	Wheat - CBT

The reference markets were chosen by economic specialist to include all typical kind of markets - i.e. cocoa and wheat are typical agriculture product, petroleum-crude oil is mined material, Japanese Yen is typical foreign currency and treasury note stands for bond markets.

4.2 Results

There are many ways, how evaluate the quality of designed strategy. The net profit calculated by (4) is the main criterion, secondary criteria are gross loss (sum of loss trades profit), gross profit (sum of win trades profit), count of winning and losing trades. By using these criteria it is possible to calculate sum of the transaction cost and sum of slippages.

The main non-quantitative pointer is the plot of cumulative gain depending on time. It is difficult to analyze it but it gives important information about the strategy. In ideal case, the plot increases.

	CC	**CL**	**FV2**	**JY**	**W**
Net profit	-40530.00	29390.00	-26368.75	-76992.50	-13210.00
Gross profit	23020.00	120360.00	52692.50	180000.00	54707.50
Gross loss	-63550.00	-90970.00	-79061.25	-256992.50	-67917.50
Transaction cost	-1780.00	-1580.00	-1900.00	-3080.00	-2060.00
Slippages	-8900.00	-6320.00	-17812.50	-38500.00	-15450.00
Trades	89	79	95	154	103
Wining trades	24	42	31	50	39
Losing trades	65	37	64	104	64

Table 1: Result overview

The results overview is in Table 1. The system designed good strategy for exchange with oil futures (CL), where the net profit is positive and the profit grows almost all the time (see Figure 1). Worst results were at cocoa future market, where the profit decreases in time. Other markets finished with negative profit, but the curve of cumulative gain shows only local decreasing, e.g. the FV2 curve decreases only at interval [1000,2500] and the other parts stagnate (see Figure 2).

The practical approach of presented design is good, because the algorithm works at one of reference markets. And three reference markets seems that the better settings or small algorithm changes can improve them to positive results.

Although the results do not suffice the requirements to usage at real trading, the theoretical results brought improvements. The methods of Monte Carlo and iterations-spread-in-time were applied and tested to new task, where the properties of both methods can be explored.

5 Future work

The main directions of the further research are:

Bellman's function - the used approximation is oversimplified. A more complex approximation is typically used to reach better results. The analytical properties of Bellman function should be explored to find the better approximation, which should lead to higher profit.

High dimensional model - present model uses only the close price to prediction, but other data channels are available too. The usage of the high dimensional model is traditional way, how obtain better results. Additional channels contain new prices, information about traders positions etc., which brings the new important information for decision maker.

Prediction quality influenced indirectly the trading system quality. Testing of prediction quality is related with model and settings of Monte Carlo method, which can be innovated by knowledge about prediction behavior.

The listed open problems should lead to improve the results and better knowledge about the approximate dynamic programming. The further approach should support the usage of this design to trading in markets as fully automatic system.

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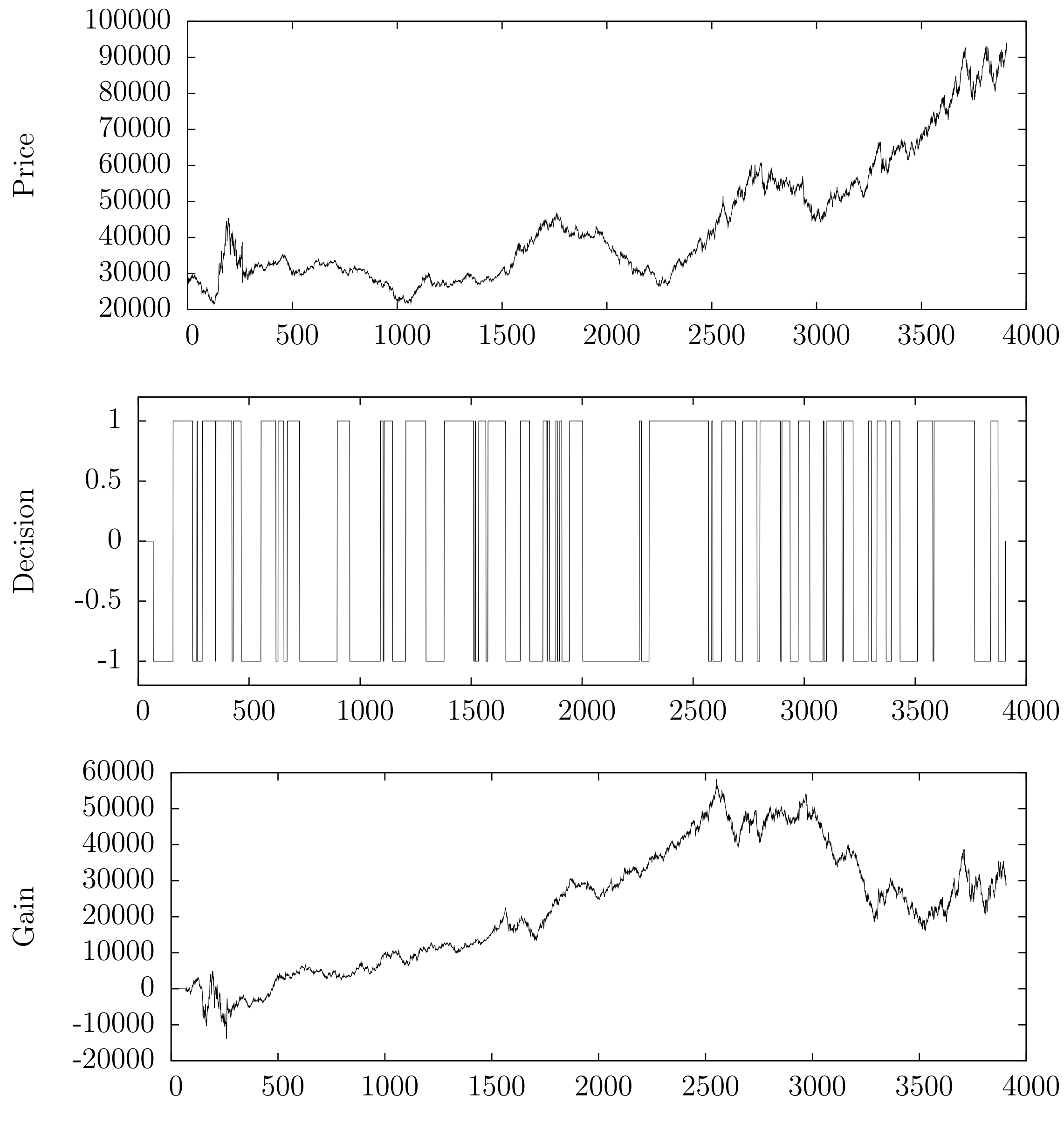

Figure 1: Results on CL

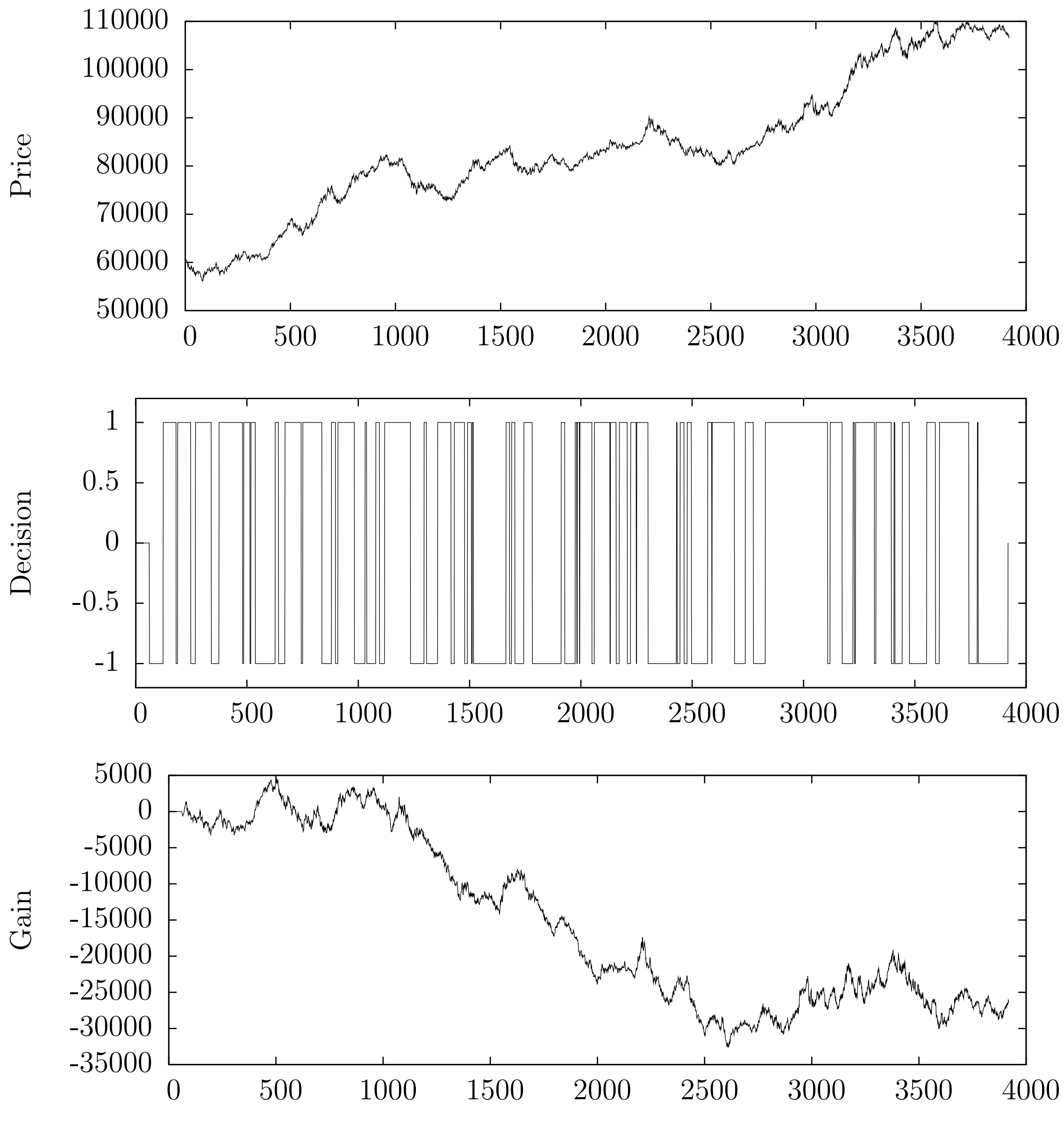

Figure 2: Results on FV2