Application of Simplified Ray Method for the Determination of the Cortical Bone Elastic Coefficients by the Ultrasonic Wave Inversion

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Abstract— **This work contributes to the methodology of an evaluation of elastic properties of cortical bones by ultrasonic wave inversion, whilst the bone is considered to be a linear elastic anisotropic continuum. Velocities of acoustic waves are used as an input data into inverse problem and they are experimentally detected by means of the ultrasonic based pulse-echo immersion technique. The geometry of bone specimens is also implicated into algorithm by the model of wave propagation through curvilinear anisotropic sample based on the simplified ray method. The stability of resulting data is evaluated by the statistical method based on the Monte-Carlo simulation. The immersion method based on the wave inversion has shown to be a reliable tool for determination of some elastic constants only, the remaining coefficients need to be measured or improved by another experimental method. The ultrasonic contact pulse through transmission technique was rated as an acceptable experimental approach for this purposes. The RUS was found to be an unsuitable method for the measurement of the elastic coefficients of the cortical bone tissue.**

Keywords— **Simplified Ray Method, Cortical Bone, Inverse Problem, Monte-Carlo Simulation, Matrix of Elastic Coefficients.**

I. Introduction

The aim of this study is contribution to the methodology of an evaluation of elastic properties of cortical bone by ultrasonic wave inversion, whilst the bone is considered to be a linear elastic anisotropic continuum. Velocities of acoustic waves are used as an input data into inverse problem and they are experimentally detected by means of the ultrasonic based pulse-echo immersion technique. This method was developed on composite structures such as plates and cylindrical shells. The geometry of bone specimens is also implicated into algorithm by the model of wave propagation through curvilinear anisotropic sample based on the simplified ray method, which is an original approach and its application to the experimental determination of the bovine femoral sample is the main subject of the interest of this work. The stability of resulting data from inverse algorithm is evaluated by the statistical method based on the Monte-Carlo simulation. The suggested approach has a potential for qualify of such measurements performed on fresh bones and also for improvement in-situ ultrasonic techniques.

II. Materials and methods

The main aim of this experiment is to deal with possibilities of the measurement of the matrix of elastic coefficients of the cortical bone by means of the dynamical, ultrasound based, mechanical tests. The methodology should be non-destructive; ultrasound based, appropriates for a rapid measurement and undemanding a sample preparation. The ultrasonic-pulsed through-transmission method with the specimen immersed in a liquid between two opposite transducers has been chosen as a suitable technique (Fig. 1).

Following experiments were performed on the dry bovine femur. Dry bovine bone was used instead of a wet bone [1] for the measurement, because of the independent determination of elastic properties separately, from the natural visco-elastic behaviour of bones. The bone sample was slit into two parts along the bone axis in order to monitor just simple wave propagation through one face of the bone and each part was shape-measured on CNC milling machine (Fig. 4). During the experiment, just one particular place in a middle part of the bone localized on a medial side of the bovine femur sample was examined.

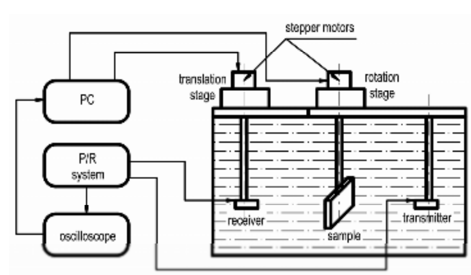

Fig. 1 Experimental set-up for immersion measurement

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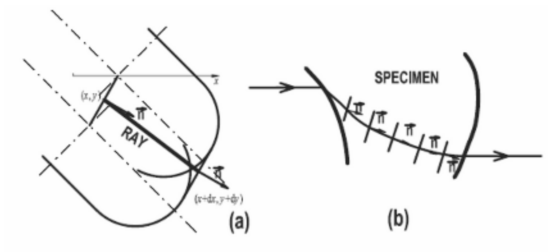

Fig. 2 (a) The ray increment according to the Huygens axiom; (b) A successive ray construction through the specimen thickness

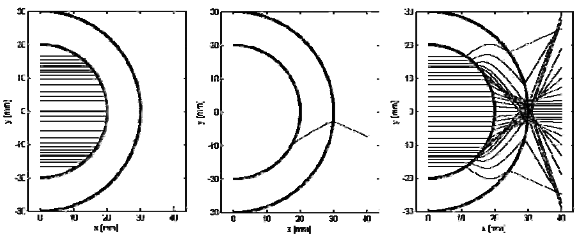

Fig. 3 The interaction of planar wave with a strongly anisotropic tube. The selection of initial rays; the path of one ray; the complete interaction

The three different modes of the measurement, modes C, D and I, were performed. Modes C and D corresponded to the horizontal positioning of the bone between the transducer and the reflector where the wave propagation in an axial plane of the bone was observed. The bone geometry was not solved in these modes, the bone geometry was considered as planar in the surrounding of a measuring position. This mode was appropriate for evaluation of 6 out of 9 elastic coefficients (c_{11}, c_{33}, c_{44}, c_{55}, c_{66} and c_{13}) of the orthotropic material symmetry. These elastic constants were evaluated from measured quasi-longitudinal and quasi-transverse wave velocities via the solution of an inverse problem of the Christoffel equation [2]. The mode I corresponded to the vertical configuration of the measurement. In this mode, the propagation of the planar wave was observed in the plane perpendicular to the long axis of the bone, so the bone curvature needs to be considered. This is resolved by means of Simplified Ray Method [3]. The technique [2] is based on a wavefront substitution by the closely localized energy flow (ray) in every geometrical point. The Christoffel's equation along rays and the ray behaviour at a solid/liquid interface will be solved numerically afterwards. Rays in immersion are lines perpendicular to the wavefront – the planar wavefront is replaced by the set of respectively parallel rays. The anisotropy orientation is included in the model by the definition of the angle of anisotropy orientation in each point of the sample. The rays inside the sample are designed on the basis of Huygens axiom, thus each point of the current wavefront is a new point source and those newly generated wavefronts are superimpose into new wave fronts (Fig. 2). An example of a modelling of the complex interaction of a planar wave in an anisotropic curvilinear specimen is illustrated on Figure 3. The Carbon Fibre Reinforced Plastic (CFRP) tube, the material having the transversely isotropic symmetry, is introduced as a model example.

The mode I was used for the determination of coefficients c_{22} and c_{12}. The remaining coefficient c_{23} was determined by the simple contact pulse-transmission measurement.

To estimate the accuracy of the optimization procedure's results, no appropriate analytical approach is available. The only possible solution is, thus, the Monte Carlo simulation, based on running the whole optimization process several times with randomly distorted input data. The Gaussian statistic made over the set of results is then expected to reveal the reliability of optimized coefficients. In this work, the wave arrival times were determined accurately.

III. RESULTS

The stability of elastic coefficients of the bovine bone sample resulting from an inverse problem optimization was evaluated by the simulation based on the Monte-Carlo statistical method [2]. Input parameters into this simulation were variations of specimen thickness, rotations of a sample (mode C,D) or a reflector (mode I), a temperature of the water bath and a density of the specimen. The Monte-Carlo simulation was repeated 30 times to generate a representative set of output data. The variability of this set is approximately expressed by the usual Gaussian statistic quantities, namely standard deviations. Obviously, the presented standard deviations cannot be treated absolutely, but they bring a valuable insight in how sensitive and stable the optimization procedure is for each particular coefficient. Final resultant coefficients c_{ij} in GPa can be expressed in the following form:

$$c_{ij} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{13} & c_{23} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{pmatrix} = \tag{1}$$

$$\begin{pmatrix} 27.4 \pm 1.6 & 9.1 \pm 3.5 & 8.3 \pm 5.3 & 0 & 0 & 0 \\ 9.1 \pm 3.5 & 30.3 \pm 2.8 & 8.5 & 0 & 0 & 0 \\ 8.3 \pm 5.3 & 8.5 & 34.1 \pm 1.7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 9.3 \pm 0.9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 7.0 \pm 0.4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6.9 \pm 0.5 \end{pmatrix}$$

Fig. 4 The bone specimen and its shape measurement via contact probe on milling machine

Particular coefficients of matrix (1) were evaluated from modes C, D and I (see Materials and Methods). The modes C and D (Fig. 6) were used for the determination of 6 elastic coefficients without considering the general geometry of bone specimen – without solving the ray model. The mode I served for the determination of 2 elastic coefficients. This mode corresponds to the vertical configuration of the measurement, so the wave propagation and the elastic constant evaluation of the bone specimen with the general geometry of a bone specimen must be resolved via the ray method.

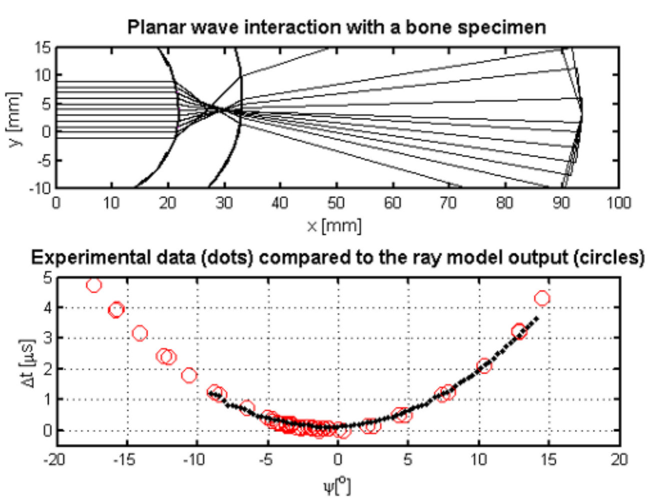

Fig. 5 The diagram of the wave propagation through the bovine bone - mode I. (a) The ray model of the wave interaction with the bovine bone sample; the specimen is stationary, the reflector is rotating, (b) The comparison of the ray model and experimentally obtained data

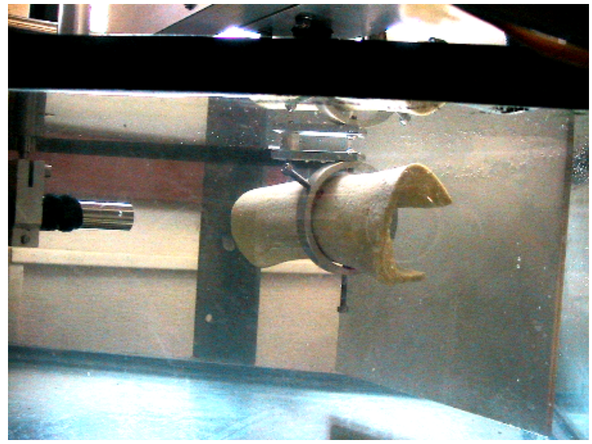

Fig. 6 Example of measuremnty in mode D

The experimental procedure and the elastic constant evaluation of the mode I is subsequent. The input geometry of the bone specimen into the ray algorithm was obtained by the contact probe on milling machine (Fig. 4). During the experiment, the bone sample was rotated into the vertical position, so the wave propagation in the plane perpendicular to the bone axis could be observed. Then, the ray algorithm is solved for a different positioning of the specimen until the ray model is tuned to the measured data. This situation, the final tuning in fact, and agreement of experimental data with the ray model are demonstrated in Figure 5. The Christoffel equation along thereby obtained rays and behaviour of rays at the solid/specimen interface was numerically solved by means of the inverse problem.

The remaining coefficient wasn't possible to determine via immersion technique without additional specimen cutting. This coefficient was evaluated subsequently via simple pulse-echo contact technique.

IV. Discussion and conclusion

The original contribution of this work is an application of the ray method to the evaluation of elastic constants of curvilinear anisotropic bone samples. The inverse problem for phase velocities and the sensitivity analysis of inverse approach based on the Monte-Carlo statistical simulation are also formulated in this contribution.

The propose methodology is usable for the measurement of all 9 elastic coefficients of compact bone, but specimen must be cut, which is at variance with request on non-destructivity of entire process. 8 coefficients can be measured non-destructively, but general specimen shape needs to be considered, which leads to application of ray method. The tuning of ray model to experiment and measurement of specimen shape is quite laborious. This immersion technique is very suitable for quick evaluation of 5 constants of long bone non-destructively without solution of ray model.

The resultant matrix of elastic coefficients (1) of the bovine dry femur evaluated in this work is in line with other data [2] and the original presumption, that the dry bone is known to be stiffer then the wet one was satisfied.

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