On Time Parameterizations of User Demands in Mechatronics

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***Abstract*—Time parameterization of user demands (demands on course of path, position etc.) is one of inherent preparative operations before starting real control of any of mechatronic systems. The main objective is to generate the reference inputs i.e. desired, required values with appropriate timing of used control system. In general, the time parameterization itself represents generating a time sequence of the reference values according to some deterministic way defined beforehand, where this time-reference sequence interpolates the initial parameters arising from user demands. This paper addresses optimally-smooth time parameterizations intended for mechatronic systems particularly for machining robotic applications.**

I. Introduction

TIME parameterization of user demands is one of inherent preparative operations of control design of any mechatronic systems [7, 8]. Its main purpose is to generate the reference inputs i.e. desired, required values with appropriate timing of used control system corresponding to user demands. In mechatronic field, these demands are usually given by a set of positions with kinematic parameters as desired velocities or constraints specifying maximal permissible acceleration and jerk [8]. The user demands can follow from:

- technology of production procedures (machining velocities, motion orientation),

but even from:

- system construction (minimum radiuses, shapes, kinematic and dynamic limits).

From control point of view, the time parameterization itself represents generating a time sequence of the reference values according to some deterministic way defined beforehand, where this time-reference sequence interpolates the initial parameters representing user demands.

This paper focuses on optimally-smooth time parameterizations, i.e. parameterization protecting systems e.g. from undue mechanical wear. For simplicity, the explanation will be arranged for mechatronics particularly for robotic systems. These systems combine different elements:

- mechanical (beams, joints, gears, grippers etc.),
- electromechanical (drives, sensors),

and

- electrical (control units).

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In majority cases, the mechatronic systems are highly dynamic systems, therefore the right forming of feasible reference inputs is important. Due to high dynamics, ordered set or pairs of time and reference values (timetable) is considered. The parameterization can be performed either for:

- operational space,

or for:

- joint (drive) space.

Operational space represents the space from view of end-effector, workpiece/tool gripper, i.e. space of resultant movement. In comparison, joint space is mainly a space of actuated joints – i.e. drive space (see Fig. 1). From user point of view, the parameterization in operational space is more natural, therefore it is preferred.

As was mentioned, the result is predominantly represented by timetable, but it can be represented even by some continuous smooth time-parameter functions [3]. The choice of character of representation depends on computation time of on-line generation of the trajectory from the parameter function or on memory space for off-line timetable. In the both cases, a topical time index is determining parameter. It represents a pointer both to predefined parameter functions and to time tables.

The paper is organized as follows. The second section deals with the time parameterization via geometric parameter. This parameterization is based on analytic geometry and basic kinematical laws. The third section outlines the parameterization as specific control tasks using multistep model-based predictive control [2]. It takes into account the kinematics and dynamics of the whole controlled mechatronic system. Finally, the concluding sections demonstrate described approaches to time parameterization by several examples and conclude the paper by practical remarks.

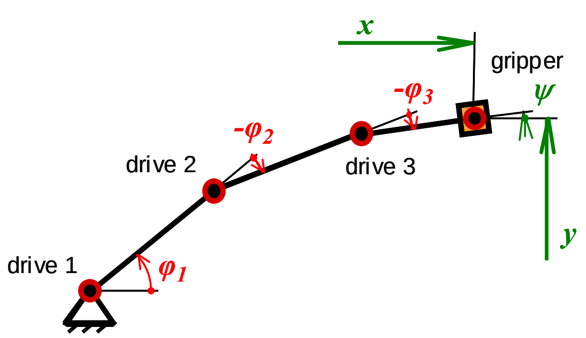

Fig. 1. Operational (x, y, ψ) and joint ($\varphi 1$, $\varphi 2$, $\varphi 3$) oriented coordinates (coordinate spaces)

II. PARAMETERIZATION VIA GEOMETRIC PARAMETER

During the time parameterization, it is necessary to provide smooth and continuous segments or curves, including their smooth transitions among them. In robotics, such segments are defined by sets of positions, velocities and accelerations. The analytical geometry [1] concerns with this task. This section focuses on time parameterization of several main types of curves (trajectory segments) [3]. Let us distinguish the following segment types both for two-dimensional (2D) and three-dimensional (3D) space [4]:

- abscissa segments
- arc (circle) segments
- general curves
  - densely sampled general curves (dense grid – set of geometrical points)
  - densely sampled curves described by parametrical equations
- dwell on place for defined time.

Let us suppose that separation of abscissa and arc segments is due to simple computation of length of these segments. Note, that path (curve) can be defined as cross-section of a general surface or solid. This eventuality leads to segments of type 'general curves'.

From technological user demands, let us consider initial, final, (optimal or maximal) values of the positions and velocities, respectively, which corresponding to key points of geometric path of the robot motion. Then the parameterization of described features is expedient to base on elementary laws of kinematics. It is realized in two phases. The first phase is a computation of geometrical parameter condensed in one-dimensional (1D) space. Then, the parameter serves as a pointer to the real parameterization in 2D or 3D space, which follows just this parameter (second phase). Note, individual parameters of segments are determined by geometry of time parameterized curves.

A. *Computation of geometric parameter*

To determine geometrical time-dependent parameter, it is necessary at first to compute the distance-length of segment and rotation angle, which are performed together. Generally, it is given by expressions

$$\ell = \int_s ds \tag{1}$$

where ds is an element of the segment.

Then, it is necessary to determine orientation time for given segment considering the knowledge of initial and final velocities and accelerations. Let initial and final values of accelerations are equaled zeros, in order to provide smooth transitions among different types of segments. If the initial and final values of the path position and appropriate first and second time derivatives (velocities and accelerations) are known, then for the geometrical parameter determination, the expression for accelerations in basic form can be used:

$$a = \frac{dv}{dt} \tag{2}$$

By double integration of the expressions (2) in a frame of one segment and its defined initial and final conditions, the expression for orientation working time is given:

$$t = \frac{2\,\ell}{v_{initial} + v_{final}} \tag{3}$$

It is labeled as $t_{final} = f$. Thus, the sufficient time for execution of the movement is provided. Furthermore, the value of t_{final} is rounded up to the nearest multiple of sampling Ts to provide uniform trajectory sampling needed for digital control.

Now, the essential time parameterization of 1D parameter can be made. To connect smoothly and to accomplish the movement of a robotic system, the trajectory should have the first derivative continuous and also smooth, and at the same time, the second derivative • *continuous and smooth* or • *at least continuous and segmentally smooth* curves.

In order to fulfill • *continuous and segmentally smooth type*, i.e. for adequate number of initial and final conditions

$$\begin{aligned} &t=0:\ s(0)=0, \quad v(0)=v_{initial}=v_0,\ a(0)=0 \\ &t=f:\ s(f)=\ell, \quad v(f)=v_{final}=v_f,\ a(f)=0 \end{aligned} \tag{4}$$

it is viable to prescribe the equation of acceleration:

$$a(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \tag{5}$$

consecutively velocity and position in the form:

$$\int a\ dt = v(t) = v_0 + a_0 t + a_1 \frac{1}{2} t^2 + a_2 \frac{1}{3} t^3 + a_3 \frac{1}{4} t^4 \tag{6}$$

$$\begin{aligned} &\iint a\ dt = \int v\ dt = s(t) \\ &= s_0 + v_0 t + \frac{1}{2} a_0 t^2 + \frac{1}{6} a_1 t^3 + \frac{1}{12} a_2 t^4 + \frac{1}{20} a_3 t^5 \end{aligned} \tag{7}$$

together, after insertion of initial and final conditions

$$\begin{bmatrix} a_0 \\ v_0 \\ s_0 \end{bmatrix} = \left. \begin{bmatrix} a(t) \\ v(t) \\ s(t) \end{bmatrix} \right|_{t=0} \qquad \Rightarrow a_0, v_0, s_0 \tag{8}$$

$$\begin{bmatrix} t & t^2 & t^3 \\ \frac{1}{2}t^2 & \frac{1}{3}t^3 & \frac{1}{4}t^4 \\ \frac{1}{6}t^3 & \frac{1}{12}t^4 & \frac{1}{20}t^5 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \left. \begin{bmatrix} a(t) - a_0 \\ v(t) - v_0 \\ s(t) - s_0 - v_0 t \end{bmatrix} \right|_{t=f} \Rightarrow \begin{matrix} a_1 \\ a_2 \\ a_3 \end{matrix} \tag{9}$$

Thus, by solution of (8) and (9), the parameters a_0, a_1, a_2, a_3 can be determined.

In order to fulfill • *continuous and really smooth type* even for acceleration, the corresponding initial and final conditions have to be arranges as follows

$$\begin{aligned} &t=0:\ s(0)=0,\ v(0)=v_{initial}=v_0,\ a(0)=0,\ a'(0)=0 \\ &t=f:\ s(f)=\ell,\ v(f)=v_{final}=v_f,\ a(f)=0,\ a'(f)=0 \end{aligned} \tag{10}$$

For given conditions, the new equation of acceleration is defined as:

$$\begin{gathered} a(t)=a_0+a_1t+a_2t^2+a_3t^3+a_4t^4+a_5t^5 \\ \left(a'(t)=a_1+2a_2t+3a_3t^2+4a_4t^3+5a_5t^4\right) \end{gathered} \tag{11}$$

Such selection leads to systems of algebraic equations considering insertion of initial and final conditions

$$\begin{bmatrix} a_0 \\ a_1 \\ v_0 \\ s_0 \end{bmatrix} = \begin{bmatrix} a(t) \\ a'(t) \\ v(t) \\ s(t) \end{bmatrix}\Bigg|_{t=0} \Rightarrow a_0, a_1, v_0, s_0 \tag{12}$$

$$\begin{gathered} \begin{bmatrix} t^2 & t^3 & t^4 & t^5 \\ 2t & 3t^2 & 4t^3 & 5t^4 \\ \frac{1}{3}t^3 & \frac{1}{4}t^4 & \frac{1}{5}t^5 & \frac{1}{6}t^6 \\ \frac{1}{12}t^4 & \frac{1}{20}t^5 & \frac{1}{30}t^6 & \frac{1}{42}t^7 \end{bmatrix} \begin{bmatrix} a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} \\ = \begin{bmatrix} a(t)-a_0-a_1t \\ a'(t)-a_1 \\ v(t)-v_0-a_0t-a_1\frac{t^2}{2} \\ s(t)-s_0-v_0t-a_0\frac{t^2}{2}-a_1\frac{t^3}{6} \end{bmatrix}\Bigg|_{t=f} \Rightarrow \begin{matrix} a_2 \\ a_3 \\ a_4 \\ a_5 \end{matrix} \end{gathered} \tag{13}$$

Thus, by the solution of (12) and (13), the parameters $a_0, a_1, a_2, a_3, a_4, a_5$ are determined.

Finally, the systems of equations (8) and (9) or (12) and (13) define time dependency of geometric parameter. In essence, the parameter and its derivatives – let us mark them $p, \dot{p}, \ddot{p}$ – are represented as follows

$$\begin{aligned} &p=s(t) \qquad p\in\langle 0,\ \ell\rangle \\ &\dot{p}=v(t) \\ &\ddot{p}=a(t) \end{aligned} \tag{14}$$

Finally, as already mentioned, the time (parameter t) occurring in equations above belongs to interval $<0, t_{final\,(=f)}>$, i.e. the time vector can be generated according to equation (15)

$$\boldsymbol{t}=[0,\ Ts,\ 2\cdot Ts,\ \cdots,\ k\cdot Ts],\quad k=\frac{t_{final}}{Ts} \tag{15}$$

For this selection of the time vector, the computation of appropriate vector of values of geometric parameter $p(t)$ is finished. Now, it is possible to continue with real planning of individual trajectory segments, defined at the beginning of the section.

B. *Parameterization of Abscissa Segment*

The first, the simplest one, is an abscissa segment (Fig. 2). It can be characterized by coordinates of initial point, direction vector with distance-length or final point in case of zero initial velocity or distance-length of the abscissa in case of nonzero initial velocity. The distance-length is given by Pythagorean Theorem

$$\ell=\sqrt{(x_f-x_0)^2+(y_f-y_0)^2+(z_f-z_0)^2} \tag{16}$$

Then the determining parameters are the following

$$\begin{aligned} v_0=0:\ &\text{plane } \mathbf{XYZ}_0=[x_0,\ y_0,\ z_0]^T,\ \vec{a} \text{ and } \ell, \\ &\text{or } \mathbf{XYZ}_f=[x_f,\ y_f,\ z_f]^T,\ v_f \end{aligned} \tag{17}$$

$$v_0\neq 0:\mathbf{XYZ}_0=[x_0,\ y_0,\ z_0]^T,\ \ \ell,\ v_f \tag{18}$$

where $\mathbf{XYZ}_0$ and v_0 are taken as values of the first segment of new trajectory or as final values from previous segment ${}_{topical}\mathbf{XYZ}_0 = {}_{prev}\mathbf{XYZ}_f$ and ${}_{topical}v_0 = {}_{prev}v_f$.

Finally, the parametric equations of abscissa segment in condensed matrix form is given for position [1]

$$\mathbf{XYZ}(t)=\mathbf{XYZ}_0+\frac{\vec{a}}{\|\vec{a}\|}\,p(t),\quad p\in\langle 0,\ell\rangle \tag{19}$$

$$\mathbf{XYZ}(t)=\mathbf{XYZ}_0+(\mathbf{XYZ}_f-\mathbf{XYZ}_0)\,p_n(t),\quad p_n\in\langle 0,1\rangle \tag{20}$$

where equation (20) is in a form of uniform parameterization defined as

$$p_n(t)=\frac{s(t)}{\ell},\quad \dot{p}_n(t)=\frac{v(t)}{\ell},\quad \ddot{p}_n(t)=\frac{a(t)}{\ell} \tag{21}$$

Consecutively, as well as equations of positions, the equations for velocity and acceleration are given by equations (22) and (23), respectively

$$\dot{\mathbf{XYZ}}(t)=\frac{\vec{a}}{\|\vec{a}\|}\,\dot{p}(t)=[v_x(t),v_y(t),v_z(t)]^T \tag{22}$$

$$\ddot{\mathbf{XYZ}}(t)=\frac{\vec{a}}{\|\vec{a}\|}\,\ddot{p}(t)=[a_x(t),a_y(t),a_z(t)]^T \tag{23}$$

The equations (19), (22) and (23) define the abscissa segment.

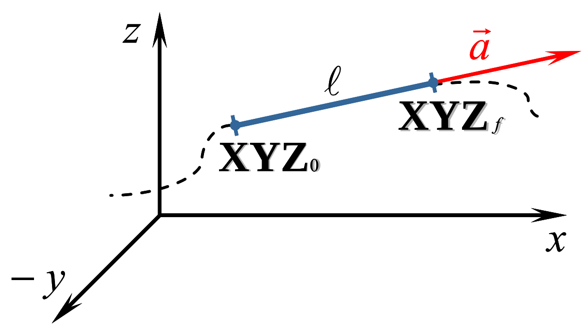

Fig. 2. Abscissa segment

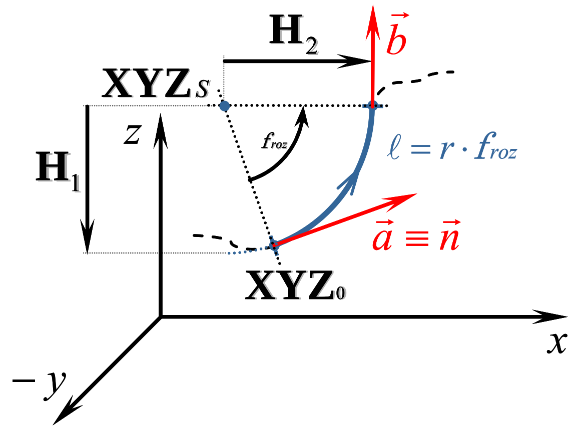

Fig. 3. Arc (circle) segment

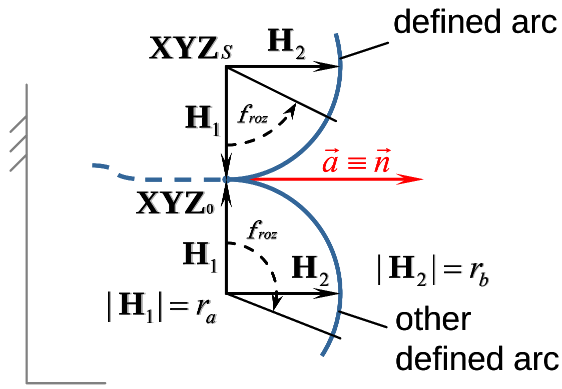

Fig. 4. Arc/elliptical segment – plane of the segment

C. Parameterization of Arc (Circle) Segment

The next of the simplest segments is arc – circle segment (Fig. 3). In general, the time parameterization of arc segment can be utilized also for elliptical segments. Only one difference of parameters is represented by two radiuses r_a and r_b and different determination of segment length. The length of the ellipse can be possibly determined by approximation of the ellipse curve by its splitting to small abscissa elements. Then their length can be determined by Pythagorean Theorem – equation (16).

The circle segment is characterized by coordinates of initial point; tangential vector $\vec{a}$ of circle in this point (it represents, at the same time, normal vector of circle-center plane $\vec{a} = \vec{n}$); radiuses of major r_a and secondary r_b semi-axis ($\|\vec{\mathrm{H}}_1\|, \|\vec{\mathrm{H}}_2\|$); outspreading angle of circle f_{roz}; angle of center position in circle-center plane f_{kr}.

Thus the parameters are given as follows

$$v_0 = 0:\ \mathbf{XYZ}_0 = [x_0,\ y_0,\ z_0]^T,\quad \|\vec{\mathrm{H}}_1\| = r_a, \|\vec{\mathrm{H}}_2\| = r_b (r_a\ circle),\ f_{roz}, f_{kr},\ \vec{a},\ v_f \tag{24}$$

$$v_0 \neq 0:\ \mathbf{XYZ}_0 = [x_0,\ y_0,\ z_0]^T,\quad \|\vec{\mathrm{H}}_1\| = r_a, \|\vec{\mathrm{H}}_2\| = r_b (r_a\ circle),\ f_{roz},\ f_{kr},\ v_f \tag{25}$$

where $\mathbf{XYZ}_0$ and v_0 are taken as values of the first segment of new trajectory or as final values from previous segment ${}_{topical}\mathbf{XYZ}_0 = {}_{prev}\mathbf{XYZ}_f$ and ${}_{topical}v_0 = {}_{prev}v_f$. The same is valid also for tangential vector, when the initial velocity is not zero. After assignment by user, the following values are known:

$$\mathbf{XYZ}_0 = [x_0,\ y_0,\ z_0]^T,\ r_a,\ r_b,\ \ell,\ p_c,\ f_{roz},\ f_{kr},\ \vec{a},\ f_v,\ v_f \tag{26}$$

Now, the parametric equations of the arc (circle)/elliptical segment can be written [1]

$$\mathbf{XYZ}(t) = \mathbf{XYZ}_S + \vec{\mathrm{H}}_1 \cos(p_c) + \vec{\mathrm{H}}_2 \sin(p_c),\ p_c \in \langle 0;\ f_{roz} \rangle \tag{27}$$

where $\mathbf{XYZ}(t)$ is a vector of coordinates of circle points;

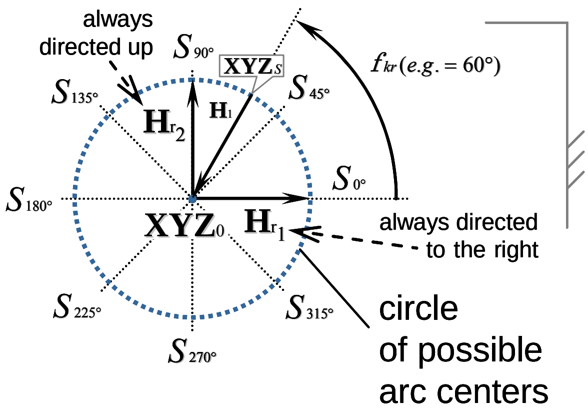

Fig. 5. Arc/elliptical segment – plane of arc centers

$\mathbf{XYZ}_S$ is a vector of arc center; $\vec{\mathrm{H}}_1$ and $\vec{\mathrm{H}}_2$ are vectors of major and secondary semi-axis and p_c is geometrical parameter defined as follows

$$\text{for circle seg. } p_c(t) = \frac{p(t)}{r_a},\ p(t) \in \langle 0;\ \ell \rangle,\ p_c \in \langle 0; f_{roz} \rangle \tag{28}$$

$$\text{for ellipse segment } p_c = f(p(t)),\quad p(t) \in \langle 0;\ \ell \rangle \tag{29}$$

function $f(p(t))$ is defined as a table computed from known ellipse points – linearly distributed parametric angle p_c and appropriate time-parameterized geometrical parameter $p(t)$ representing motion along ellipse curve projected to 1D. Illustrative schemes are shown in Fig. 4 and Fig. 5.

D. Parameterization of General Curve Segment

In engineering practice, there are a lot of general curves either parametric curves as previous cases (abscissa or arc-circle/elliptical segments) or purely general curves, at which it is difficult to compute their length. However, the knowledge of the length is necessary condition, without which the time parameterization is not feasible, in spite of the possibility to compute geometrical points of considered curves. This problem can be solved on engineering level by the following way. At first, let us define possibilities of obtaining of key curve points.

In principle, there are two cases as already mentioned at the beginning of the Section II.:

- densely sampled general curves (dense grid – set of geometrical points)
- densely sampled curves described by parametrical equations.

The first group can represent general curves as contours of the shape of some required product. Such type of curves arises from design procedure of designer, who designs the shape of the final product. Nowadays, in a branch of technical documentation, the very powerful software environment AutoCAD in different forms is used. It enables to export the key coordinates of required curves to other software or directly to provide computations of time parameterization in its environment by some user scripts and functions. (In this work, the former way – handling with exported data – is considered.)

The second group represents curves, which can be parametrically described. However the determination of their length is not simple as well. To this group, a lot of geometrical curves and surfaces belong: ellipse, parabola, hyperbola, Ferguson cubic, Bezier cubic, Coons cubic B-splines, screw line, spiral, cycloids, epicycloids, hypocycloids and cross-sections of the surfaces - plates.

By the way of dense sampling, as already mentioned, it is possible to approximate curves by their splitting to small abscissa elements with possible computation of the element length (see eq. 16).

$$\begin{aligned} \Delta\ell = ds = \left\|\mathbf{XYZ}_{k+1} - \mathbf{XYZ}_k\right\| \le eps \ \ (e.g.\,10^{-6}\,\mathrm{m} = 1\mu\mathrm{m}) \\ \ell = \sum \Delta\ell \end{aligned} \tag{30}$$

Total length is ℓ = number of small abscissa elements $\Delta\ell$, Computation of geometrical parameter $p = p(t)$, $p \in \langle 0; \ell \rangle$; $t := \langle 0 : Ts : \frac{t\max}{Ts} \cdot Ts \rangle$, Then, the time parameterization can be provided by the following algorithm:

```
XT=[];
for i=1:NumOfElements ...% position
  idx = find([l(:)- p(i)]<eps
  XT(i,:)=[X(idx),Y(idx),Z(idx)];
End

dXT=[];
dXT = [vx0,vy0,vz0;   ...% velocity
      (XT(2:end,1)- XT(1:end-1,1))/Ts,
      (•y)/Ts, (•z)/Ts];

ddXT=[];
ddXT = [0,0,0;        ...% acceleration
      (dXT(2:end-1,1)- dXT(1:end-2,1))/Ts,
      (•dy)/Ts, (•dz)/Ts];

CS=[XT,dXT,ddXT];     ...% Cartesian c.
```

where `(•(d).)=((d)XT(·,i.)- (d)XT(·,i.))`.

The vector **CS** contains positions, velocities and accelerations sampled in $k \cdot Ts$. The computation is only approximation. Therefore, the parameterization accuracy depends on the selection of initial length of the abscissa element.

III. Parameterization via Specific Control Task

In industrial applications, there exist a lot of control operations, which provide different changes of working points, movements or interval stabilization, where accurate achievement of some predetermined trajectory is not important, however a fulfillment of some permitted output range or reaching of some point is required. The points, which should be reached, can represent e.g. end-positions of manipulative operations or new working points of controlled system.

This section outlines two specific control tasks, which can realize time parameterization online during real control process. The first is characterized by known permitted ranges (limits) of reference signal and the second task is determined only by defined end point. In general, these tasks represent control task with specific constraints on system output.

The both tasks can be realized by specific modifications of generalized predictive control, which represents multistep model-based control strategy [2]. The multistep attribute is important; its presence provides suitable distribution of user demands within time as presented time parameterization in Section II. The result (output record) of described control tasks can be used as full valuable time parameterization for another control realization.

At first, let us start from introduction of predictive control. It is a multi-step control based on equations of predictions

$$\hat{\mathbf{y}} = \mathbf{f} + \mathbf{G}\mathbf{u} \tag{31}$$

and the local minimization of quadratic criterion

$$J_k = \sum_{j=1}^{N} \left\{ \| (\hat{y}_{k+j} - w_{k+j})\, Q_y \|^2 + \| (u_{k+j-1}\, Q_u \|^2 \right\} \tag{32}$$

where N is a horizon of predictions; Q_y and Q_u are penalizations – tuning parameters; y outputs; w user demands and u are searched inputs, which are determined by minimization of the criterion in every time step.

A. Parameterization for path given by permitted ranges

To parameterize path given by permitted ranges and end-point only, it is necessary to make the following steps:

- define ranges of the workspace or topical domain;
- select penalizations balancing of ranges' importance.

Then, it is possible to minimize modified quadratic criterion, which provides mentioned assignment [5]

$$\begin{aligned} J_k = \sum_{j=1}^{N} \Big\{ & \| (\hat{y}_{k+j} - r_{ak+j})\, Q_{ra} \|^2 + \| (\hat{y}_{k+j} - r_{bk+j})\, Q_{rb} \|^2 \\ & + \| (u_{k+j-1} Q_u \|^2 \Big\} \end{aligned} \tag{33}$$

In the criterion, there occur new terms $r_a(\cdot)$ and $r_b(\cdot)$, which correspond to the ranges. Furthermore, there are also new output penalizations Q_{ra} and Q_{rb}, in usual standard criterion, there is only one penalization Q_y. The minimization of described criterion generates the control actions as usual, only matrices in it have different types.

B. Parameterization for free path and fixed end-point

The task of the reference path given only by end-point can be formulated in that way: “Let two points (start and end point) and presumptive time be given. In this time, the system should move between those points. A path is free of hard constraints; only end-point should be achieved”.

Predictive design can be used again, only quadratic criterion has to be reshaped again, but in different way:

$$J_k = \sum_{j=No+1}^{N} \|(\hat{y}_{k+j} - w_{k+j})\, Q_y\|^2 + \sum_{j=1}^{Nu} \|(u_{k+j-1}\, Q_u\|^2 \qquad (34)$$

The criterion includes more adjustable parameters: horizon of prediction N, horizon of initial insensitivity No and control horizon Nu, penalizations Q_y and Q_u, and also the desired values w, which determine the transition from start to end point in the criterion. In case of end-point modification, the value vector $\mathbf{w}$ corresponds to end-point i.e. $\mathbf{w} = [w_{k+j}, \cdots w_{k+N}] \mid w_{k+j} = w_f,\ j = No+1, \cdots, N$, furthermore with setting: $N := Nmax, \cdots, Nmin,\ Nmin > n$, where $Nmax = T/Ts$, $No = N - n$, $Nu = N$, and n is a system order. The criterion is minimized repeatedly as usual, but with changeable horizons.

IV. Examples

This section demonstrates time parameterized trajectories according to explanation in section II. and III. Table 1 and Fig. 6 show time parameterization via geometric parameter. Fig. 7 and Fig. 8 show parameterization as control tasks: path given by permitted range and free path with fixed end-point.

TABLE 1

DATA FOR CURVE GENERATED IN AUTOCAD

```
AutoCAD:  Command: _list
Select objects: selected: 1
SPLINE: Layout: axes, Length: 126.01 Order: 4
Properties: Planar, No rational, No periodic
Parametric range: First 0.00, Final 120.79
  Key points (Number 12):
  X = -50.00    , Y =   0.00    , Z = 0.00
  X = -50.19    , Y =  -4.26    , Z = 0.00
  X = -50.55    , Y = -12.39    , Z = 0.00
  X = -35.31    , Y = -15.24    , Z = 0.00
  X = -26.57    , Y =  -1.61    , Z = 0.00
  X = -11.70    , Y = -14.70    , Z = 0.00
  X = - 0.09    , Y =   1.27    , Z = 0.00
  X =  12.07    , Y =  10.62    , Z = 0.00
  X =  26.88    , Y =   7.59    , Z = 0.00
  X =  32.45    , Y =  -9.67    , Z = 0.00
  X =  40.60    , Y =  -0.14    , Z = 0.00
  X =  50.00    , Y =   0.00    , Z = 0.00
```

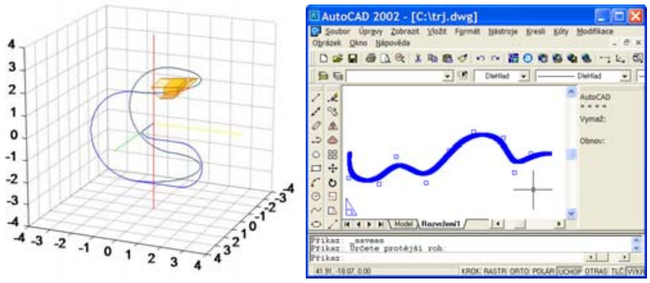

Fig. 6. Analytical curve, general curve from AutoCAD

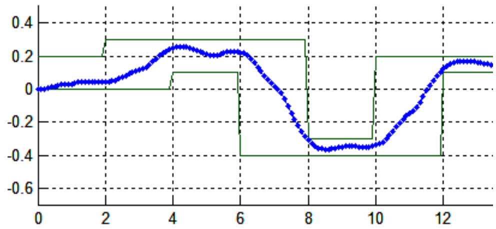

Fig. 7. Parameterized trajectory given by permitted range

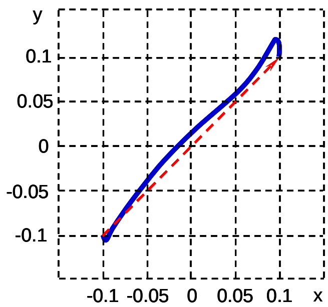

Fig. 8. Parameterized free trajectory (continuous line)

V. Conclusion

The paper investigates several approaches to time parameterization of user demands intended for mechatronic systems. Presented techniques ensure continuous and smooth curves including their derivatives. It is important especially at highly dynamic systems, where sharp and discontinuous curve profiles can cause failure of the controlled system.

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