



# Fully probabilistic control design

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## Abstract

Stochastic control design chooses the controller that makes the closed-loop behavior as close as possible to the desired one. The fully probabilistic design describes both the closed loop and its desired behavior in probabilistic terms and uses Kullback–Leibler divergence as their proximity measure. This approach: (i) unifies stochastic control design methodology; (ii) provides explicit minimizer.

The paper completes the previous solutions of various particular cases by formulating and solving the fully probabilistic control design in the general, discrete-time, state-space setting.

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*Keywords:* Stochastic control design; Fully probabilistic design; State-space models

## 1. Introduction

The standard stochastic control problem is formulated as minimization of an expected loss function with respect to feedback control strategies, e.g. [1,2]. This minimization can be interpreted as an attempt to influence selected characteristics of the joint distribution of variables occurring in the optimized closed loop. This interpretation provides an alternative formulation: *the joint distribution of closed-loop variables should be forced to be as close as possible to their desired distribution*. The above formulation of the control design problem, called *fully probabilistic design* (FPD), was proposed in [7]. It has been extended to systems modeled by finite probabilistic mixtures [8].

The approach has the following special features.

- Explicit minimizer is available in the pair of operations (minimization, expectation) that are applied sequentially when optimizing via stochastic dynamic programming [7].
- Employing the Kullback–Leibler divergence together with case-specific desired distribution makes the resulting loss function to take into account both the deterministic and stochastic properties of the controlled system.
- The use of the multi-modal desired distribution provides a well-justified and feasible multiple-objective control design [10,4].

The listed features indicate a significant application potential of the FPD and justify attempts to formulate it as generally as possible. Up to now, the published versions of the FPD considered only controlled problems with the observable variables. Relaxation of this restriction is the main contribution of the current paper, which provides the fully probabilistic control design coping with stochastic state-space models.

Section 2 prepares the necessary notions and notations. Section 3 contains the main result, i.e., the FPD in the state-space setting. Concluding remarks close the paper.

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## 2. Preliminaries

The following notations are used throughout the paper:  $\equiv$  is an equality by definition;  $X^*$  denotes a set of  $X$ -values;  $\overset{\circ}{X}$  means cardinality of a finite set  $X^*$ ;  $f(\cdot|\cdot)$  stands for a probability density function (pdf) that is assumed to exist;  $t$  labels discrete-time moments,  $t \in t^* \equiv \{1, \dots, \overset{\circ}{t}\}$ ;  $\overset{\circ}{t} < \infty$  denotes a given control horizon;  $d_t = (y_t, u_t)$  is the data record at time  $t$  consisting of an observed system output  $y_t$  and of an optional system input  $u_t$ ;  $x_t$  denotes an unobserved system state;  $d, x$  are assumed to be finite-dimensional;  $X(t)$  stands for the sequence  $(X_1, \dots, X_t)$ ,  $X(t) \in \{d(t), y(t), u(t), x(t)\}$ .

The following adopted simplifications are also used.

- Names of arguments distinguish respective pdfs. No formal distinction is made between a random variable, its realization and an argument of a pdf.
- Integrals used are always definite and multivariate ones. The integration domain coincides with the support of the pdf in its argument.

The FPD exploits the *Kullback–Leibler (KL) divergence*  $\mathcal{D}(f\|\tilde{f})$  [12] that measures the proximity of a pair of pdfs  $f, \tilde{f}$  acting on a set  $X^*$ . It is defined as follows:

$$\mathcal{D}(f\|\tilde{f}) \equiv \int f(X) \ln \left( \frac{f(X)}{\tilde{f}(X)} \right) dX. \quad (1)$$

The KL divergence has the following key property

$$\mathcal{D}(f\|\tilde{f}) \geq 0, \quad \mathcal{D}(f\|\tilde{f}) = 0 \text{ iff } f = \tilde{f} \text{ almost everywhere on } X^*. \quad (2)$$

The joint pdf  $f(d(\overset{\circ}{t}), x(\overset{\circ}{t})|x_0, d(0))f(x_0|d(0)) = f(d(\overset{\circ}{t}), x(\overset{\circ}{t})|x_0)f(x_0)$  of all random variables involved is known to be the most complete probabilistic description of the controlled closed loop. In it,  $x_0$  is an initial uncertain state and  $d(0)$  denotes the prior information available before the choice of the first input. Habitually,  $d(0)$  is considered only implicitly.

The chain rule for pdfs [14] implies the following decomposition of the joint pdf representing the complete probabilistic description of the closed-loop behavior

$$\begin{aligned} & f(d(\overset{\circ}{t}), x(\overset{\circ}{t})|x_0) \\ &= \prod_{t \in t^*} f(y_t|u_t, x(t), d(t-1))f(x_t|u_t, x(t-1), d(t-1))f(u_t|x(t-1), d(t-1)). \end{aligned} \quad (3)$$

The chosen order of conditioning distinguishes the *observation model*  $f(y_t|u_t, x(t), d(t-1))$ , the *state evolution model*  $f(x_t|u_t, x(t-1), d(t-1))$  and the general *randomized control law*  $f(u_t|x(t-1), d(t-1))$ . The collection of control laws over  $t \in t^*$  forms *control strategy*.

The following assumptions are adopted on particular factors of the closed-loop decomposition (3).

**Assumptions.** *Distribution of the system state*  $x_t$  is determined by the current system input  $u_t$  and the previous system state  $x_{t-1}$  only, i.e.,

$$f(x_t|u_t, x(t-1), d(t-1)) = f(x_t|u_t, x_{t-1}).$$

*Distribution of the observed system output*  $y_t$  is determined by the current system input  $u_t$  and the current system state  $x_t$  only, i.e.,

$$f(y_t|u_t, x(t), d(t-1)) = f(y_t|u_t, x_t).$$

*Admissible control strategies*, generating the system input  $u_t$  from the observed data history  $d(t-1)$  and ignoring the unobserved states  $x(t-1)$ , are considered, i.e.,  $f(u_t|x(t-1), d(t-1)) = f(u_t|d(t-1))$ .

With these Assumptions, the closed-loop description (3) reduces to

$$f(d(\overset{\circ}{t}), x(\overset{\circ}{t})|x_0) = \prod_{t \in t^*} f(y_t|u_t, x_t)f(x_t|u_t, x_{t-1})f(u_t|d(t-1)). \quad (4)$$

1 Note that the omission of  $d(t-1)$  in the first two items of the Assumptions is unnecessary but it makes the explanations more transparent and closer to a usual understanding of the state-space models.

### 3. Problem formulation and solution

5 The control aim and constraints are quantified by the so-called *ideal pdf* that represents the desired joint distribution of the considered closed-loop variables. It is constructed in the way analogous to (4) with user-specified factors distinguished by the superscript  $I$

$$7 \quad {}^I f(d(t), x(t)|x_0) {}^I f(x_0) = \prod_{t \in t^*} {}^I f(y_t|u_t, x_t) {}^I f(x_t|u_t, x_{t-1}) {}^I f(u_t|d(t-1)) f(x_0),$$

9 where ideal pdfs  ${}^I f(y_t|u_t, x_t)$ ,  ${}^I f(x_t|u_t, x_{t-1})$  and  ${}^I f(u_t|d(t-1))$  describe the desired observation model, state evolution model and control law, respectively.

11 The prior pdf on initial states  $x_0^*$  cannot be influenced by the optimized control strategy so that it is left to its fate, i.e.,  ${}^I f(x_0) = f(x_0)$ .

To formulate the FPD concisely and to simplify some formal manipulations, the following shorthand notation is used

$$13 \quad f_t \equiv f(d(t), x(t)|x_0) f(x_0), \quad {}^I f_t \equiv {}^I f(d(t), x(t)|x_0) f(x_0). \quad (5)$$

Under the adopted Assumptions, Section 2, the FPD is formulated as follows.

$$15 \quad \boxed{\text{Find admissible control strategy minimizing the KL divergence } \mathcal{D} \left( f_t^\circ \parallel {}^I f_t^\circ \right)}$$

Solution of the FPD requires the solution of stochastic filtering problem in the closed control loop.

17 **Proposition 1** (*Stochastic filtering in closed control loop*). *Let the prior pdf  $f(x_0)$  be given and the Assumptions hold. Then, the pdf  $f(x_t|d(t))$ , determining the state estimate, and the pdf  $f(x_t|u_t, d(t-1))$ , determining the state prediction, evolve according to the coupled equations*

$$19 \quad \text{Time updating} \quad f(x_t|u_t, d(t-1)) = \int f(x_t|u_t, x_{t-1}) f(x_{t-1}|d(t-1)) dx_{t-1},$$

$$\text{Data updating} \quad f(x_t|d(t)) = \frac{f(y_t|u_t, x_t) f(x_t|u_t, d(t-1))}{\underbrace{\int f(y_t|u_t, x_t) f(x_t|u_t, d(t-1)) dx_t}_{f(y_t|u_t, d(t-1))}}$$

21 *The stochastic filtering does not depend on the used admissible control strategy  $\{f(u_t|d(t-1))\}_{t \in t^*}$  but on the generated inputs only.*

23 **Proof.** The pdf  $f(x_0|d(0)) \equiv f(x_0)$  is given. Let us assume that we have already got  $f(x_{t-1}|d(t-1))$  for a generic  $t \in t^*$ . Then the time updating is implied by the following identities

$$\begin{aligned} f(x_t|u_t, d(t-1)) &\stackrel{\text{marginalization}}{\equiv} \int f(x_t, x(t-1)|u_t, d(t-1)) dx(t-1) \\ &\stackrel{\text{chain rule}}{\equiv} \int f(x_t|u_t, x(t-1), d(t-1)) f(x(t-1)|u_t, d(t-1)) dx(t-1) \\ &\stackrel{\text{Assumptions}}{\equiv} \int f(x_t|u_t, x_{t-1}) f(x(t-1)|d(t-1)) dx(t-1) \\ &\stackrel{\text{marginalization}}{\equiv} \int f(x_t|u_t, x_{t-1}) f(x_{t-1}|d(t-1)) dx_{t-1}. \end{aligned}$$

25

1 The data updating completes the recursion and it is implied by the identities

$$\begin{aligned} f(x_t|d(t)) &\stackrel{\text{conditioning}}{=} \frac{f(x_t, y_t, u_t|d(t-1))}{f(y_t, u_t|d(t-1))} \\ &\stackrel{\text{chain rule}}{=} \frac{f(y_t|u_t, x_t, d(t-1))f(x_t|u_t, d(t-1))f(u_t|d(t-1))}{f(y_t|u_t, d(t-1))f(u_t|d(t-1))} \\ &\stackrel{\text{Assumptions}}{=} \frac{f(y_t|u_t, x_t)f(x_t|u_t, d(t-1))}{\int f(y_t|u_t, x_t)f(x_t|u_t, d(t-1)) dx_t}. \quad \square \end{aligned}$$

3 **Proposition 2** (Solution of the FPD). Let both the joint pdf  $f(x(t), d(t)|x_0)$  and its ideal counterpart  ${}^I f(x(t), d(t)|x_0)$  meet the Assumptions.

5 Then, the optimal admissible control strategy minimizing  $\mathcal{D}(f_t^\circ || f_t^\circ)$  is randomized one given by the pdfs

$${}^o f(u_t|d(t-1)) = {}^I f(u_t|d(t-1)) \frac{\exp[-\omega(u_t, d(t-1))]}{\gamma(d(t-1))}, \quad t \in t^*, \quad (6)$$

$$7 \quad \gamma(d(t-1)) \equiv \int {}^I f(u_t|d(t-1)) \exp[-\omega(u_t, d(t-1))] du_t.$$

Starting with  $\gamma(d(t)) \equiv 1$ , the functions  $\omega(u_t, d(t-1))$  are generated recursively for  $t = t^*, t^* - 1, \dots, 1$  in the backward manner, as follows:

$$11 \quad \Omega(u_t, d(t-1)) \equiv \int \Omega(u_t, d(t-1), x_{t-1}) f(x_{t-1}|d(t-1)) dx_{t-1}$$

with  $f(x_t|d(t))$  updated according to Proposition 1 and

$$\begin{aligned} &\Omega(u_t, d(t-1), x_{t-1}) \\ &\equiv \int f(y_t|u_t, x_t) f(x_t|u_t, x_{t-1}) \ln \left( \frac{f(y_t|u_t, x_t) f(x_t|u_t, x_{t-1})}{\gamma(d(t)) {}^I f(y_t|u_t, x_t) {}^I f(x_t|u_t, x_{t-1})} \right) dy_t dx_t. \end{aligned}$$

13 **Proof.** The result is implied by the following sequence of equalities in which we use the definition of the KL divergence (1), its basic property (2), Fubini theorem on multiple integration [15], marginalization and normalization of pdfs and the chain rule for them [14] together with conditional independencies expressed by the adopted Assumptions. The definition  $\gamma(d(t)) \equiv 1$  and the shorthand notation (5) are used. Moreover, the notation of differentials is simplified by taking  $dx(t) \equiv dx(t)dx_0$ .

17 The KL divergence can be viewed as an expectation of the additive loss function with the  $t$ th partial loss equal to

$$\ln \left( \frac{f(y_t|u_t, x_t) f(x_t|u_t, x_{t-1}) f(u_t|d(t-1))}{{}^I f(y_t|u_t, x_t) {}^I f(x_t|u_t, x_{t-1}) {}^I f(u_t|d(t-1))} \right).$$

19 This observation, together with the definition  $\gamma(d(t)) = 1$ , implies the following identity

$$\begin{aligned} \mathcal{D}_t^\circ &\equiv \min_{\{f(u_t|d(t-1))\}_{t=1}^{\circ}} \mathcal{D}(f_t^\circ || f_t^\circ) = \min_{\{f(u_t|d(t-1))\}_{t=1}^{\circ}} \left\{ \mathcal{D}(f_{t-1}^\circ || f_{t-1}^\circ) \right. \\ &+ \min_{f(u_t^\circ|d(t-1))} \int f_{t-1}^\circ \left[ \int f(y_t^\circ|u_t^\circ, x_t^\circ) f(x_t^\circ|u_t^\circ, x_{t-1}^\circ) f(u_t^\circ|d(t-1)) \right. \\ &\times \ln \left( \frac{f(y_t^\circ|u_t^\circ, x_t^\circ) f(x_t^\circ|u_t^\circ, x_{t-1}^\circ) f(u_t^\circ|d(t-1))}{\gamma(d(t)) {}^I f(y_t^\circ|u_t^\circ, x_t^\circ) {}^I f(x_t^\circ|u_t^\circ, x_{t-1}^\circ) {}^I f(u_t^\circ|d(t-1))} \right) dy_t^\circ dx_t^\circ du_t^\circ \left. \right] dx(t-1) dd(t-1) \left. \right\}. \quad (7) \end{aligned}$$

- 1 Let us deal with the second term in the expression (7), which depends on the last member  $f(u_{\circ}|d(\overset{\circ}{t}-1))$  of the opti-  
 2 mized admissible strategy. In this term, the factor  $A_{\circ}$ , defined as the expression in the square brackets, can be rearranged  
 3 as follows

$$\begin{aligned}
 A_{\circ} &\equiv \int f(y_{\circ}|u_{\circ}, x_{\circ})(x_{\circ}|u_{\circ}, x_{\circ_{t-1}})f(u_{\circ}|d(\overset{\circ}{t}-1)) \\
 &\quad \times \ln \left( \frac{f(y_{\circ}|u_{\circ}, x_{\circ})f(x_{\circ}|u_{\circ}, x_{\circ_{t-1}})f(u_{\circ}|d(\overset{\circ}{t}-1))}{\gamma(d(\overset{\circ}{t}))^I f(y_{\circ}|u_{\circ}, x_{\circ})^I f(x_{\circ}|u_{\circ}, x_{\circ_{t-1}})^I f(u_{\circ}|d(\overset{\circ}{t}-1))} \right) dy_{\circ} dx_{\circ} du_{\circ} \\
 &= \int f(u_{\circ}|d(\overset{\circ}{t}-1)) \left[ \ln \left( \frac{f(u_{\circ}|d(\overset{\circ}{t}-1))}{{}^I f(u_{\circ}|d(\overset{\circ}{t}-1))} \right) \right. \\
 &\quad \left. + \underbrace{\int f(y_{\circ}|u_{\circ}, x_{\circ})f(x_{\circ}|u_{\circ}, x_{\circ_{t-1}}) \ln \left( \frac{f(y_{\circ}|u_{\circ}, x_{\circ})f(x_{\circ}|u_{\circ}, x_{\circ_{t-1}})}{\gamma(d(\overset{\circ}{t}))^I f(y_{\circ}|u_{\circ}, x_{\circ})^I f(x_{\circ}|u_{\circ}, x_{\circ_{t-1}})} \right) dy_{\circ} dx_{\circ}}_{\Omega(u_{\circ}, d(\overset{\circ}{t}-1), x_{\circ_{t-1}})} \right] du_{\circ}.
 \end{aligned}$$

- 5 This rearrangement uses Fubini theorem, non-presence of  $y_{\circ}, x_{\circ}$  in the optimized pdf  $f(u_{\circ}|d(\overset{\circ}{t}-1))$  and normalization  
 $\int f(y_{\circ}|u_{\circ}, x_{\circ})(x_{\circ}|u_{\circ}, x_{\circ_{t-1}}) dy_{\circ} dx_{\circ} = 1$ .  
 7 With the adopted notation (5) and the introduced function  $\Omega(u_{\circ}, d(\overset{\circ}{t}-1), x_{\circ_{t-1}})$ , the term  $B_{\circ} \equiv \int f_{\circ_{t-1}}[A_{\circ}] dx(\overset{\circ}{t}-1) dd(\overset{\circ}{t}-1)$   
 in (7) influenced by  $f(u_{\circ}|d(\overset{\circ}{t}-1))$  becomes

$$\begin{aligned}
 B_{\circ} &\equiv \int f(d(\overset{\circ}{t}-1)) \left\{ \int f(u_{\circ}|d(\overset{\circ}{t}-1)) \left[ \ln \left( \frac{f(u_{\circ}|d(\overset{\circ}{t}-1))}{{}^I f(u_{\circ}|d(\overset{\circ}{t}-1))} \right) \right. \right. \\
 &\quad \left. \left. + \underbrace{\int f(x(\overset{\circ}{t}-1)|d(\overset{\circ}{t}-1))\Omega(u_{\circ}, d(\overset{\circ}{t}-1), x_{\circ_{t-1}}) dx(\overset{\circ}{t}-1)}_{\omega(u_{\circ}, d(\overset{\circ}{t}-1))} \right] du_{\circ} \right\} dd(\overset{\circ}{t}-1). \quad (8)
 \end{aligned}$$

- 9 To get (8), the chain rule  $f(d(\overset{\circ}{t}-1), x(\overset{\circ}{t}-1)|x_0)f(x_0) = f(x(\overset{\circ}{t}-1), x_0|d(\overset{\circ}{t}-1))f(d(\overset{\circ}{t}-1))$ , Fubini theorem, non-occurrence of  
 11  $x(\overset{\circ}{t}-1)$  and  $x_0$  in the control law  $f(u_{\circ}|d(\overset{\circ}{t}-1))$ , and the normalization  $\int f(x(\overset{\circ}{t}-1), x_0|d(\overset{\circ}{t}-1)) dx(\overset{\circ}{t}-1) = 1$  are sequentially  
 exploited.

- 13 As the function  $\Omega(u_{\circ}, d(\overset{\circ}{t}-1), x_{\circ_{t-1}})$  does not depend on  $x(\overset{\circ}{t}-2)$  and  $x_0$ , these superfluous variables can be integrated  
 15 out from the joint pdf. Hence, the function  $\omega(\cdot)$  defined in (8) can be evaluated using the pdf determining the state estimate  
 only

$$\omega(u_{\circ}, d(\overset{\circ}{t}-1)) = \int f(x_{\circ_{t-1}}|d(\overset{\circ}{t}-1))\Omega(u_{\circ}, d(\overset{\circ}{t}-1), x_{\circ_{t-1}}) dx_{\circ_{t-1}}.$$

1 Proposition 1 has shown that the filtering result  $f(x_{t-1}^\circ | d(t-1))$  does not depend on the optimized admissible strategy. The  
 2 optimized  $f(u_t^\circ | d(t-1))$  enters only the functional in the compound brackets in (8) as follows

$$\begin{aligned}
 & \int f(u_t^\circ | d(t-1)) \ln \left( \frac{f(u_t^\circ | d(t-1))}{\frac{\int f(u_t^\circ | d(t-1)) \exp[-\omega(u_t^\circ, d(t-1))] \gamma(d(t-1))}{\gamma(d(t-1))}} \right) du_t^\circ \\
 & \quad - \underbrace{\ln \left( \int \int f(u_t^\circ | d(t-1)) \exp[-\omega(u_t^\circ, d(t-1))] du_t^\circ \right)}_{-\ln(\gamma(d(t-1)))}. \tag{9}
 \end{aligned}$$

3 By adding and subtracting  $\ln(\gamma(d(t-1)))$  in (9), the first term in (9) has become a conditional version of the KL divergence.  
 4 The basic property (2) of the KL divergence and independence of  $\ln(\gamma(d(t-1)))$  on the optimized  $f(u_t^\circ | d(t-1))$  imply that  
 5 this expression is minimized by the claimed pdf (6)

$$6 \quad {}^o f(u_t^\circ | d(t-1)) = \frac{\int f(u_t^\circ | d(t-1)) \exp[-\omega(u_t^\circ, d(t-1))] \gamma(d(t-1))}{\gamma(d(t-1))}.$$

7 The expression (8) evaluated for this pdf provides the reached minimum

$$8 \quad - \int f(d(t-1)) \ln(\gamma(d(t-1))) dd(t-1) = - \int f_{t-1}^\circ \ln(\gamma(d(t-1))) dx(t-1) dd(t-1).$$

9 Inserting this minimum into (7) shows that  $-\ln(\gamma(d(t-1)))$  enters  $\mathcal{D}(f_{t-1}^\circ \| f_{t-1}^\circ)$  at the same place as  $-\ln(\gamma(d(t))) = 0$   
 10 enters  $\mathcal{D}(f_t^\circ \| f_t^\circ)$ . Thus, the whole procedure can be repeated for the decreasing horizons  $t-1, t-2, \dots, 1$ .  $\square$

#### 4. Concluding remarks

11 This paper completes the development of the fully probabilistic control design by covering the control design for the stochastic  
 12 state-space model. Advantageous properties of the FPD, see [7,8,10,4] and the brief outline in Introduction, apply to the treated  
 13 case, too. For instance, the formula (6) implies that the support of the ideal control law  $\int f(u_t | d(t-1))$  includes the support of  
 14 the designed control law  ${}^o f(u_t | d(t-1))$ , i.e. the ideal pdf  $\int f(u_t | d(t-1))$  may set hard constraints on the range of inputs or  
 15 its changes.

16 Besides, the unified probabilistic treatment opens a novel way of distributed control by adopting technology developed in  
 17 connection with graphical models [5,11] for creating the compound probabilistic models.

18 In a sense, the initial phase of development of the FPD theory is concluded here by extending the FPD to the state-space  
 19 models that allow for a richer and more realistic modeling of the controlled system than the input–output models. A further  
 20 development will include the following directions.

- 21 • *Applications of the FPD to specific models.*

22 For instance, the complete linear-quadratic-Gaussian design with the Gaussian state-space model is obtained if all pdfs  
 23 considered within the problem are Gaussian. This allows the rich analytical results available, for instance, general notions  
 24 like controllability and other “abilities”, [13] to be widely exploited by the FPD.

25 On the other hand, the general theory of the FPD contributes to a deeper understanding of the “traditional” notions. For  
 26 example, the interpretation of penalization matrices in quadratic loss function as precision matrices of the ideal pdf helps  
 27 to choose their values, cf. [7].

28 Algorithmic novelty is expected from the applications to the linear dynamic models with the uniform noise. A sort of  
 29 dynamic linear programming will arise.

- 30 • *Computational and algorithmic aspects.*

31 In our opinion, a further development should predominantly concentrate on the algorithmic aspects. The computational  
 32 complexity barrier that is ever present in any stochastic design is the primary problem. The fact that the minimizer is found

1 explicitly reduces this problem to simpler, but still very hard, multivariate integration and an approximate interpolation of  
the involved multivariate functions. The recently proposed stationary version of FPD [9] is expected to help further on.  
3 The practical solvability of the underlying evaluation problems is indicated by an existence of the solutions for the case of  
probabilistic mixtures, see the cited references and the vast amount work done in this respect world-wide, e.g. [3,6].

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