The Efficient Market Hypothesis Testing on the Prague Stock Exchange

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Abstract:

This article attempts to answer the question, to what extent can the Czech Capital Market (Prague Stock Exchange) be considered efficient and tries to explain its volatility. The weak-form efficiency market hypothesis is evaluated. Martingale and random walk models are starting points of our analysis.

The stock returns behaviour on the Czech capital market is not consistent with the weak-form efficiency market hypothesis because high autocorrelation of returns is present is demonstrated. Moreover, contrary to the random walk hypothesis, stock price returns are heteroskedastic. Tthe GARCH (1,1) model is used to analyze the heteroskedasticity. It is observed, that the volatility increased in 1997, especially during political crises. Finally, it is explored by means of the GARCH-M model, the extent of risk averson on the Czech capital market. However, this element is not statistically significant and therefore the hypothesis of risk neutrality cannot be refuted.

Keywords: the weak-form efficiency market hypothesis, volatility, high autocorrelation of returns, the Czech Capital Market

JEL classification numbers:E12, E52

I. Introduction

One of basic questions concerning capital markets is the question of its efficiency. Price system P_t, playing basic allocation task, is an attribute of each capital market. It is usually admitted that capital market is efficient if its price system reflects all information important for its dynamics. More formally, efficient capital market should satisfy following properties:

$$\log \mathcal{P}_{t+1} = \mathcal{E}(\log \mathcal{P}_{t+1} \big| \boldsymbol{\Phi}_t) + \varepsilon_{t+1} \tag{1a}$$

$$r_{t+1} = \mathcal{E}(r_{t+1} \big| \boldsymbol{\Phi}_t) + \varepsilon_{t+1} \tag{1b}$$

where ε_{t+1}, $t \geq 1$ is independent and identically distributed (IID) sequence of random variables with zero mean and constant variance. ϕ_t is an information set given at time t and yield r_t is defined as:

$$r_t = \log(P_t / P_{t-1}) = \log P_t - \log P_{t-1}. \tag{2}$$

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According to the information set ϕ, we distinguish three forms of the Efficient Market Hypothesis (EMH). A capital market is called weak efficient if trading rules based on the information set ϕ^{WE} containing all historical prices does not yield any systematic positive returns. A capital market is called semi-strong efficient if trading rules based on the information set ϕ^{SSE} that besides ϕ^{WE} contains all publicly available information concerning capital market does not yield any systematic positive returns. A capital market is called strong efficient, if trading rules based on the information set ϕ^{SE} that contains all information concerning the capital market does not yield any systematic positive returns. It is obvious, that the following implications hold:

$$\Phi^{WE} \subset \Phi^{SSE} \subset \Phi^{SE} \tag{3}$$

The formulation (1a) and (1b) are too general to be testable. They only express that investors form its expectations rationally. To make the Efficient Market Hypothesis operational, we need to assume some predetermined model of equilibrium (expected) returns. Therefore we will assume further, that the equilibrium return is time invariant and equal to zero[1]. This assumption is quite restrictive. The requirement of capital market efficiency is also compatible with more general assumption that returns change systematically over time but is not possible to forecast future deviations from the expected returns[2]. Pursuant to this assumption, the share price follows a process called martingale. An efficient capital market has therefore the following property:

$$E(P_{t+1} | \Phi_t) = P_t \tag{4}$$

If (4) holds on the information set ϕ^{WE} then a capital market is called ***weak efficient***. If (4) holds on the information set ϕ^{SSE} then a capital market is called ***semi-strong efficient***. If (4) holds on the information set ϕ^{SE} then a capital market is called ***strong efficient***. To test the EMH means to test the hypothesis whether the relation (4) holds on given information set or not.

The equation (4) is also restrictive in the following way. Autocorrelation with past returns does not mean that the EMH is rejected. The fundamental argument of the EMH is that it is unpossible to get on the basis of the information set ϕ_t expected abnormal returns. Zero autocorrelation with past returns is a sufficient condition only but not a necessary one. Nonzero

[1] One obvious possibility is that stock market returns depend on interest rate. However, this exploratory variable was not statistically significant.
[2] Fama (1970) explores this issue, see also LeRoy (1989)

autocorrelation with past returns was recorded even on standard capital markets. Such situation is usually explained by institutional factors and trading rules constraints[3]. Lo & MacKinlay (1988) have suggested another explanation. In their case, the nonzero autocorrelation is caused by different behaviour of small and large companies whose shares trade with different frequencies. Small firms' share prices absorb information with delay as compared to large firms. This situation results in nonzero autocorrelation of index series when it includes both small and large companies.

The Czech capital market is considered as emerging capital market[4]. Therefore, we will test the weak form EMH only. One of possible formulations of the EMH used for weak form tests is that share prices follow a random walk. It means that returns are realizations of IID sequence of random variables[5].

The logarithmic random walk model is given by

$$\log P_t = \log P_{t-1} + \varepsilon_t \tag{5}$$

where

$$E(\varepsilon_t) = 0,\ E(\varepsilon_t \varepsilon_{t-s}) = \sigma^2 \quad for \quad s = 0 \quad and \quad E(\varepsilon_t \varepsilon_{t-s}) = 0 \quad for \quad s \neq 0$$

Because $r_t = \varepsilon_t$, normal distributed returns are independent, with zero mean and variance equal to σ^2. Hence, the best estimate for tomorrow share price is today price.

The random walk formulation of the EMH is not equivalent to the martingale formulation. It is more restrictive. The martingale formulation excludes linear dependence among returns only. On the other hand, the random walk model claims that empirical distribution functions are independent. Especially the assumption of time invariant variance (homoskedasticity) is very strong and contradicts reality, where cool periods alternate with turbulent periods. Copeland (1976), Epps (1976), Akgiray (1989) consider variance of returns as a function of an inflow of new information on a capital market. Thus, rejection of the EMH in the form of equation (5) does not mean rejection of the EMH in its weak form.

Engle (1982) supplied necessary statistical tools for the return heteroskedasticity analysis. Because of heavy reliance on these tools in further analysis, we review the most important models

[3] see Hasbrouck, Ho (1987)
[4] see Vošvrda (1997)
[5] see Fama (1970)

with heteroskedastic residuals. Engel's ARCH (Autoregressive Conditional heteroskedasticity) process is very similar to the equation (5)

$$\log P_t = \log P_{t-1} + \varepsilon_t \tag{6}$$

$$\varepsilon_t \left| \Phi_{t-1} \sim N(0, h_t^2) \right. \tag{6a}$$

$$h_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 \tag{6b}$$

where $\alpha_t \geq 0$ for $i = 0,\ldots, p$.

An order of the ARCH process p and its parameters are estimated by the Maximum Likelihood Estimation method.

Bollerslev (1986) generalized the ARCH (p) process in a form of GARCH (Generalized ARCH) process where heteroskedasticity of returns is given not only by past errors but also by past variance. This model is usually better than ARCH model in describing reality.

GARCH (p,q) model is given by:

$$\log \mathcal{P}_t = \log \mathcal{P}_{t-1} + \varepsilon_t \tag{7}$$

$$\varepsilon_t \left| \Phi_{t-1} \sim N(0, h_t^2) \right. \tag{7a}$$

$$h_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j h_{t-j}^2 \tag{7b}$$

where $\alpha_t \geq 0$ for $i = 0,\ldots, p$,
$\beta_j \geq 0$ for $j = 1,\ldots, q$.

II. The EMH testing on the Czech capital market- methodology

As a first step in our analysis, we test the EMH in the form (5). The weak form EMH will be confirmed, if, in the equation

$$\Delta \log \mathcal{P}_t = \gamma_0 + \sum_{i=1}^{p} \gamma_i \Delta \log \mathcal{P}_{t-i} + \varepsilon_t. \tag{8}$$

The return $\Delta \log P_t$ is integrated of de gree zero, I (0), or alternatively, that the share price $\log P_t$ is integrated of degree one, $I(1)$[6], γ_t equals zero for all i=1,..., p and the sequence of random errors is IID. The degree of integration can be estimated by means of Dickey-Fuller[7] test and the existence of nonzero coefficients by means of autocorrelation function and OLS estimate of the

[6] If the share price is I (1) we cannot test the equation (7) directly, because the estimates would be biased.

equation (8). We will apply the Bera-Jarque[8] test and the Kolmogorov-Smirnov test to test the normality hypothesis.

One reason for the violation of normality assumption can be that the parameters of random error distribution function are not time invariant. We will therefore use the LM[9] test and squared residuals test to question the existence of heteroskedasticity. In case of positive answer we can adjust the simple model of form (8) to allow for models with conditional heteroskedasticity (ARCH, GARCH, TARCH, EGARCH, Component Model, ARCH in Mean Model). The selection of the most suitable model is based on following criteria: Akaike Information Criterion, Schwarz Information Criterion and Log Likelihood Value. In case of our data, the best fitting model is GARCH (1,1) model:

$$\Delta \log \mathcal{P}_t = \gamma_0 + \sum_{k=1}^{r} \gamma_k \Delta \log \mathcal{P}_{t-k} + \varepsilon_t \tag{9}$$

$$\varepsilon_t \left| \Phi_{t-1} \sim N(0, h_t^2) \right. \tag{9a}$$

$$h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}^2 \tag{9b}$$

If $\gamma_k = 0$ for all $k = 0,\ldots,$ r and ε_t is IID, we confirm the weak form EMH formulated as a martingale. If for some k, $\gamma_k \neq 0$ we reject this hypothesis. The normality of residuals will be tested again by means of Bera-Jarque test and Kolmogorov-Smirnov test. If the residuals are not normally distributed, we can say that the GARCH process is not capable of describing of all the determinants of residuals variance or the assumption of normality is itself false. LM test and squared residuals test of the equation (9) errors will be applied again to determine which of these cases is closer to reality.

The sum of coefficients $\alpha_1 + \beta_1$ determines the extent of persistence of variance. If the sum equals 1, than random errors influence all future values of variance with constant weights. Lamoureux a Lastrapes (1990) define the statistics of a half-life of a shock to the variance (*HL*). It means the time period over which the shock diminishes to half its original size. The half-life for GARCH (1,1) process is defined as

$$HL = 1 - \frac{\log 2}{\log(\alpha_1 + \beta_1)} \tag{10}$$

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[7] Dickey and Fuller (1979)
[8] Bera and Jarque (1982)

III The EMH testing on the Czech capital market - empirical results

We used the Prague Stock Exchange Index PX50 in the time period from January 5, 1995 to October 3, 1997 to test the efficiency of the Czech capital market. The PX 50 Index is a weighted index containing 50 the most liquid titles with weights changing according to the market capitalization. The starting period of the Prague Stock Exchange from September 7, 1993 to January 4, 1995 was omitted in the data set because of small information value of the data. Vast majority of the data was influenced by the legal maximum limit of 10% (later 5%) daily change. These data do not convey all the available information.

We define the daily rate of return of index as

$$r_t = \log PX50_t - \log PX50_{t-1} \tag{12}$$

where t denotes days. The following table 1 and graph 1 show the distribution of daily returns (12). Table 2 shows values of the Dickey-Fuller unit root test applied on the level and return of PX50. If the value of DF test exceeds the critical value, we reject the hypothesis of a unit root and thus the hypothesis of nonstationarity of the series. On the basis of estimated values we can reject the hypothesis of nonstationarity of the PX50 returns. We do not reject this hypothesis for levels of the PX50 Index. This conclusion is consistent with the weak form EMH.

Table 1: Descriptive statistics *PX 50*

No. of observations	649
Mean	-5.03117E-05
Median	0.000196804
25% quantile (Q25)	-0.00406422
75% quantile (Q75)	0.004131844
Minimum	-0.042253356
Maximum	0.043133936
Standard error	0.008501343
Variance	7.22728E-05
Skewness	-0.103700101
Kurtosis	3.598496531

Graph 1:

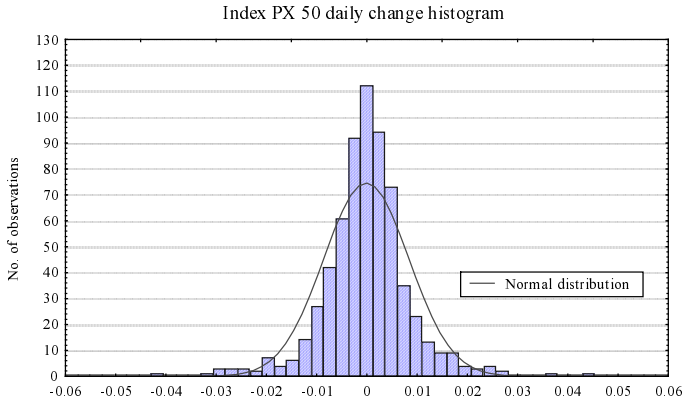


Table 2: *Dickey-Fuller test*

	Log PX50		**ΔLog PX50**	
	no trend	**with trend**	**no trend**	**with trend**
DF	-1.140316	-3.380025	-17.27716	-17.35441

[9]Lagrange Multiplier test, Engle (1982)

ADF(1)	-1.580902	-3.582603	-13.61221	-13.70347
ADF(2)	-1.687368	-3.785215	-12.21594	-12.30862
ADF(3)	-1.596133	-3.649101	-11.13004	-11.21240
ADF(4)	-1.476164	-3.395533	-9.99331	-10.07692
MacKinnon 1% critical value			-3.4430	-3.9769

The rejection of the hypothesis of nonstationarity of stock market returns does not imply that returns are uncorrelated. We have therefore computed the values of the autocorrelation function and partial autocorrelation function for 10 lagged returns.

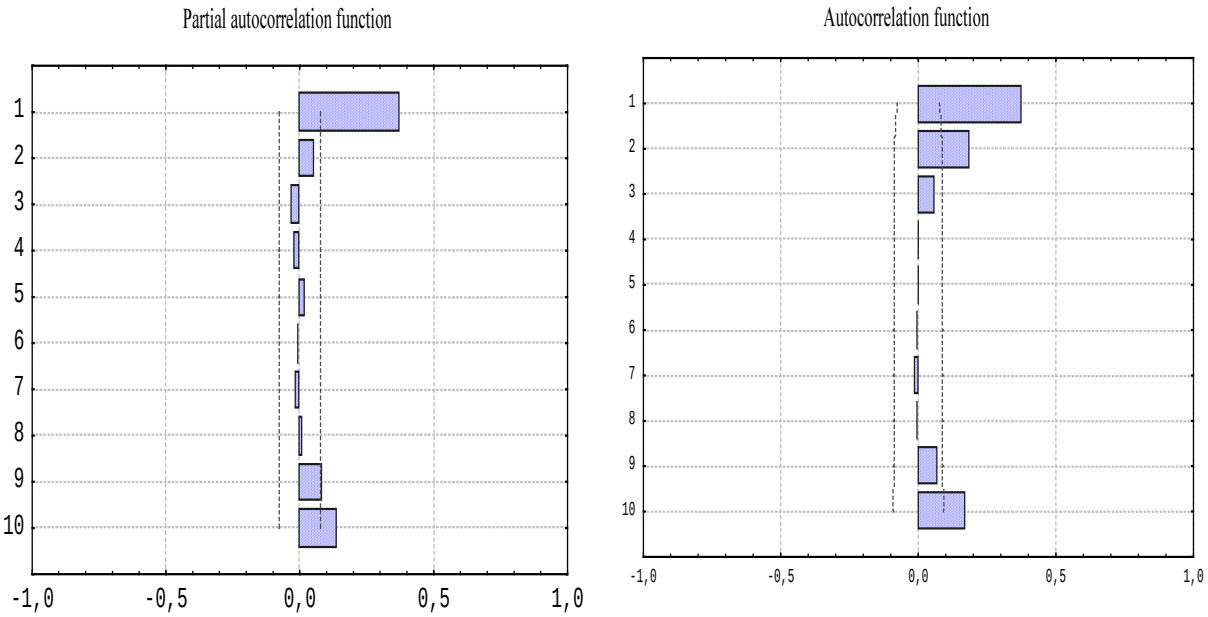
The table 3 shows, together with graphs 2 and 3, that there is statistically significant autocorrelation between $\Delta\log PX50_t$ and $\Delta\log PX50_{t-1}$ and, surprisingly, between $\Delta\log PX50_t$ a $\Delta\log PX50_{t-10}$.

Table 3: ***Autocorrelation analysis***

Lag	**Autocorrelation**	**Partial AC**	**Ljung-Box Q**	**Probability**
t-1	0.370	0.370	89.140	0.000
t-2	0.180	0.049	110.18	0.000
t-3	0.061	-0.024	112.57	0.000
t-4	0.010	-0.015	112.63	0.000
t-5	0.013	0.018	112.74	0.000
t-6	0.005	-0.001	112.76	0.000
t-7	-0.013	-0.020	112.88	0.000
t-8	-0.002	0.009	112.88	0.000
t-9	0.072	0.086	116.27	0.000
t-10	0.173	0.142	136.01	0.000

Although the reasons for weak-form-EMH consistent autocorrelation we have presented in the first part can be valid for the Czech capital market as well, the magnitude of the first order autocorrelation leads us to conclude, that weak form EMH does not apply to the Czech capital market. Fama (1970) presents autocorrelations of 30 American companies. The average is 0.026 and the highest magnitude is 0.123. Although more detailed analysis of whether above-average returns can be earned on the basis of historical prices is definitely needed, the value of first order autocorrelation (0.370) can be only hardly consistent with EMH.

Graph 2 **Graph 3**



On the basis of autocorrelation function we have chosen the basic regression equation (8) with regressors $\Delta\log PX50_{t-1}$ a $\Delta\log PX50_{t-10}$. The summary of the regression can be found in the table 4.

Table 4: ***Regresssion equation (8)***

Variable	Coefficient	Standard error	t-statistics	Probability
constant	-8.79E-06	0.000307	-0.028600	0.9772
ΔLog $PX50_{t-1}$	0.354225	0.036557	9.689741	0.0000
ΔLog $PX50_{t-10}$	0.148222	0.036241	4.089901	0.0000
R-squared		0.155593	No. of observations	648
Adjusted R-squared		0.152933		

Except the constant γ_0, the other regression coefficients are statistically significant.

The tests of normality of residuals reject the hypothesis of normality. The values of the Bera-Jarque test are χ^2 distributed with 2 degrees of freedom. Significance level $\alpha = 0.001$ implies the value of 9.210, but the Bera-Jarque statistics is 704.2. The hypothesis of normality is also rejected by the Kolmogorov-Smirnov test. The results are summarized in table 5:

Table 5: ***Normality tests***

Kolmogorov-Smirnov	K-S d	p-value
Non-normal (skewness, kurtosis)	0.048967	p<0.10
Log-normal	0.096745	p<0.01
Normal	0.098040	p<0.01

Bera-Jarque	704.9089	p<0.00

At this moment, the weak form EMH can be rejected, especially on the basis of high autocorrelation. This conclusion applies both to the martingale and to the random walk specification of weak form EMH. Even if more complicated GARCH models are applied, this conclusion will be unaffected.

None the less further progress in our analysis can be beneficial, because it can reject or accept the existence of heteroskedasticity and bring about more realistic model than equation (8). We have undertaken the Lagrange Multiplier test of equation (8) residuals that confirms the existence of heteroskedastic PX50 returns.

The presence of heteroskedasticity can be also verified by the squared residuals autocorrelation. Strong autocorrelation indicates heteroskedastic residuals. The conclusions of this test stay in accord with previous test and justify using GARCH models for modelling stock market returns (equation (9)). The values of autocorrelation function are presented in the table 7.

Table 6: ***ARCH-LM heteroscedasticity test***

F-statistics	15.12338		**Probability**	0.000000
Variable	**Coefficient**	**Standard error**	**t-statistics**	**Probability**
constant	3.89E-05	7.10E-06	5.482684	0.0000
$\varepsilon^2_{\tau-1}$	0.264586	0.039881	6.634436	0.0000
$\varepsilon^2_{\tau-2}$	0.068327	0.041278	1.655295	0.0984
$\varepsilon^2_{\tau-3}$	0.019105	0.041278	0.462835	0.6436
$\varepsilon^2_{\tau-4}$	0.000421	0.039907	0.010542	0.9916

Table 7: ***Squared residuals autocorrelation*** Graph 4:

Lag	Autocorrelation	Partial AC	Ljung-Box Q	p
t-1	0.204	0.204	26.789	0.000
t-2	0.115	0.076	35.235	0.000
t-3	0.065	0.030	37.992	0.000
t-4	0.034	0.008	38.733	0.000
t-5	0.072	0.059	42.107	0.000
t-6	0.013	-0.017	42.220	0.000
t-7	0.016	0.005	42.390	0.000
t-8	0.028	0.020	42.899	0.000
t-9	0.031	0.020	43.511	0.000
t-10	0.104	0.091	50.599	0.000

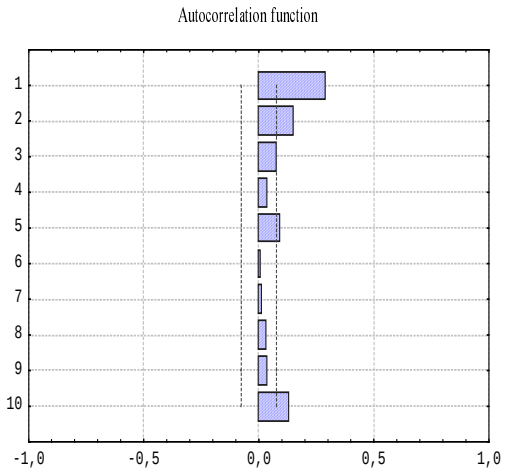


Table 8: ***GARCH(1,1) process***

Variable	Coefficient	Standard error	t-statistics	Probability
ΔLog PX50$_{t-1}$	0.383092	0.041001	9.343426	0.0000
ΔLog PX50$_{t-10}$	0.091729	0.034468	2.661267	0.0080
Variance equation (9b)				
constant	5.80E-06	7.81E-07	7.423986	0.0000
ε_{t-1}	0.159481	0.029438	5.417617	0.0000
h^2_{t-1}	0.751671	0.031917	23.55092	0.0000

R-squared	0.151748	Mean dependent var	-3.71E-05
Adjusted R-squared	0.146388	S.D. dependent var	0.008430
S.E. of regression	0.007789	Akaike info criterion	-9.702363
Sum squared resid	0.038400	Schwarz criterion	-9.667423
Log likelihood	2243.975	F statistic	28.31014
Durbin-Watson stat	2.088778	Prob(F-statistic)	0.000000

Comparison of different models suggested, that GARCH (1,1) model fits the PX50 returns in the best way. The estimated parameters of the GARCH process are shown in table 8.

Normalized residuals were again tested both by Lagrange Multiplier tests and by squared residuals autocorrelation function. The results indicate that after application of GARCH (1,1) model the heteroskedasticity was removed and the residuals are homoskedastic. Both tests are shown in tables 9, 10 and on graph 5.

Let's now focus on the estimated coefficients in the variance equation (9b). In case of the Prague Stock Exchange the value of $\alpha_1+\beta_1$ equals 0.911, which means quite high variance persistence level. The half-life is 8.45. The variance shock thus diminishes to half of its original

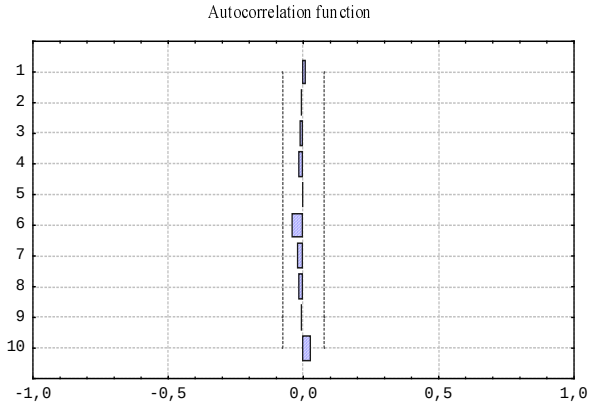
Table 9: ***ARCH-LM heteroscedasticity test***

F-statistics	0.101481		**Probability**	0.981946
Variable	**Coefficient**	**Standard error**	**t-statistics**	**Probability**
constant	1.033542	0.149015	6.935848	0.0000
ε^2_{t-1}	0.009429	0.039866	0.236514	0.8131
ε^2_{t-2}	-0.005918	0.039906	-0.148307	0.8821
ε^2_{t-3}	-0.012671	0.039906	-0.317530	0.7509
ε^2_{t-4}	-0.018750	0.039910	-0.469819	0.6386

Table 10: ***Squared residuals autocorrelation***

Lag	**Autocorrelation**	**Partial AC**	**Ljung-Box Q**	**p**
t-1	0.009	0.009	0.0567	0.812
t-2	-0.007	-0.007	0.0869	0.957
t-3	-0.013	-0.013	0.2032	0.977
t-4	-0.019	-0.019	0.4378	0.979
t-5	-0.003	-0.002	0.4419	0.994
t-6	-0.043	-0.044	1.6561	0.948
t-7	-0.023	-0.023	2.0091	0.959
t-8	-0.016	-0.016	2.1661	0.975
t-9	-0.007	-0.008	2.1975	0.988
t-10	0.031	0.029	2.8342	0.985

Graph 5:



size in slightly more than 8 trading days. Estimated conditional variance h_t^2 is depicted on graph 6. It is apparent from the graph, that the largest swings in volatility on Prague Stock Exchange can be associated with politically unstable situations especially around parliamentary elections in 1996 and in the period when the economic restrictions were announced in May 1997 (the so called Second Economic Package).

It is obvious, that the hypothesis of normality must be rejected again. The residuals are even further from being normally distributed than the equation (8) residuals. The next question to be answered is, whether the GARCH (1,1) residuals are normally distributed (table 11).

IV. Conclusions

We have shown that in the period of 1995-1997 the weak form Efficient Market Hypothesis does not apply to the Prague Stock Exchange. This conclusion is supported especially by the magnitude of autocorrelation between subsequent returns. Due to this result, the confirmation and analysis of heteroskedasticity can be considered as the most important contribution of this article. We used GARCH (1,1) model to eliminate heteroskedasticity.

Normalized residuals obtained from this estimate are indeed homoskedastic, but the hypothesis of normality was rejected.

Graph 6:

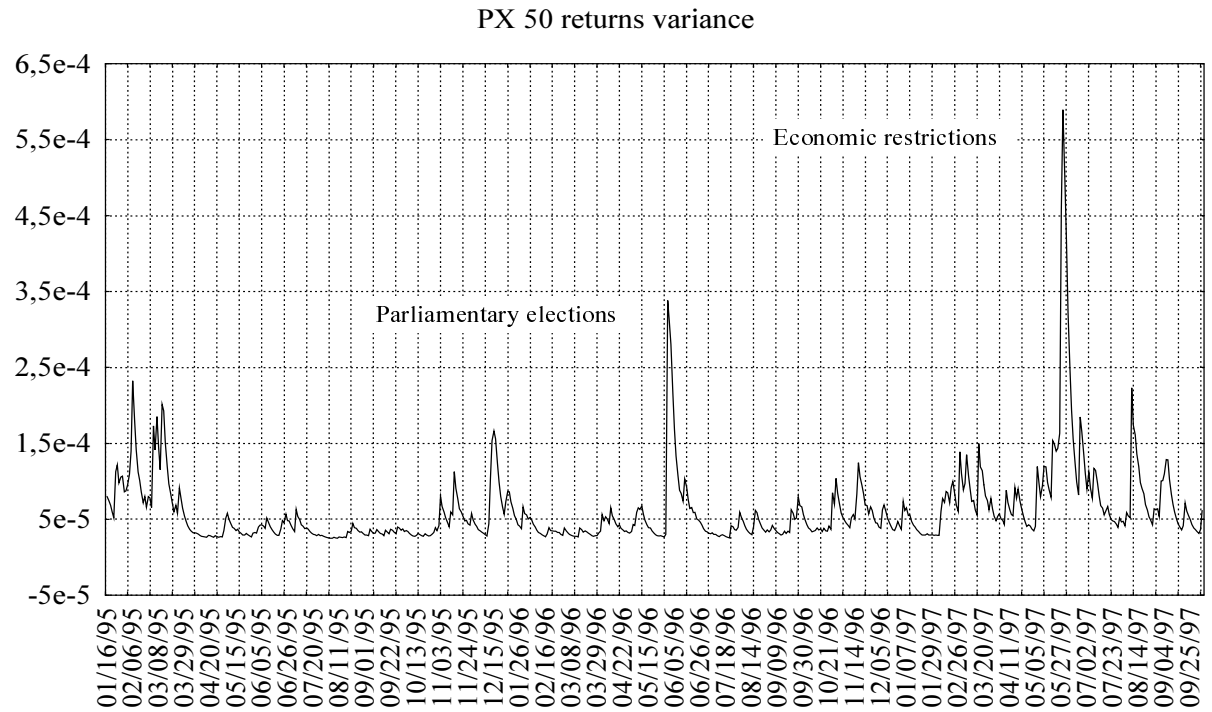


Table 11: ***Normality tests***

Kolmogorov-Smirnov	**K-S d**	**p-value**
Non-normal (skewness, kurtosis)	0.043638	p<0.20
Log-normal	0.142217	p<0.01
Normal	0.087393	p<0.01
Bera-Jarque	1667.7179	p<0.00

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