Probabilistically Tuned LQ Control for Mechatronic Applications

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Abstract
Mechatronic applications are integral part of production machines and industrial robots. The key task is a design of their suitable control, which should ensure safe control actions in spite of sudden changes of working conditions. The paper presents specific probabilistic interpretation of well-known Linear Quadratic control.This interpretation employs complex information on system behavior and gives physical meaning for fine-tuning of control parameters. The principles of fully probabilistic design with emphasis on on-line tuning are demonstrated on physical model of gearbox mechatronic system representing flexible mechanism occurring in rolling machines.

Keywords: adaptive control, state-space realization, mechatronic applications

Introduction

Mechatronic systems comprise elemental part of production machines and industrial robots. They consist of beams, wheels, joints and drives with power electronics. The systems have to be precisely controlled to provide safe motion and elimination of undesired vibrations causing drive wear and damage.

In this paper, the gearbox mechatronic system is used as a representative system. It represents flexible mechanism (Fig.1) occurring in rolling mill machines [3] and also in geared robot arms [8] of serial industrial robots - manipulators. Considered system consists of electric drive, solid wheels and elastic belts or elastic shafts respectively.

The aim is to tune suitably designed control, which should adapt itself for sudden changes of working conditions (load changes, external signal disturbances etc.) making control process stochastic.

The most general formulation of the control design is based on the minimization of expected value of a suitably chosen loss function. The loss function is defined as a function of system inputs, outputs and desired behavior with respect to feedback control strategies. The control strategy has to be chosen in correspondence to the purpose of control. One of well known powerful strategy is LQ (Linear Quadratic) control employing linear system model and quadratic criterion [2]. Its more general probabilistic interpretation [5] with emphasis on on-line parameter fine-tuning is presented here. The on-line tuning protects drives of controlled system from sharp actions induced by unpredicted change of working conditions.

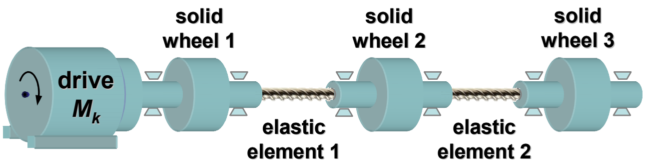

Fig.1 Scheme of gearbox mechatronic system

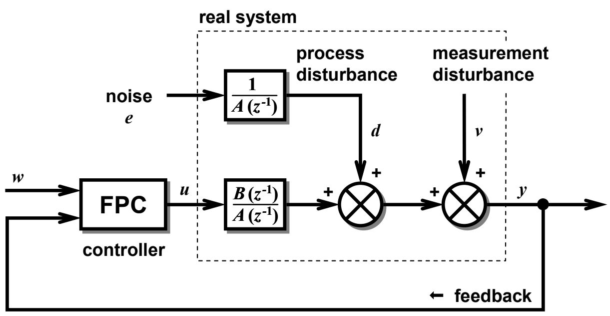

Fig.2 Block diagram of the closed control loop of probabilistic controller and controlled system

The proposed approach considers more complex information on controlled system behavior using probabilistic description of whole closed-loop, block diagram of which is shown in Fig.2. The diagram represents the structure of the closed-loop of considered mechatronic system.

In fully probabilistic approach, all available aspects of the closed-loop including expected and desired inputs and outputs, are defined as probability density functions. Consequently, the probabilistic interpretation may use more of available information contrary to standard design, which may have an insufficient number of representative parameters or interpretations for the information available.

In mechatronic systems (e.g. manipulators - robots [7], [8], the fully probabilistic approach offers to express stochastic inaccuracies of the mechanical elements (e.g. backlashes, friction, wear, elasticity etc.), actuators generating control actions and inaccuracies of measurement sensors and appropriate wiring (signal disturbances). Mechatronic systems represent a chain of different elements, which cause different inaccuracies, combination of which causes stochastic system behavior.

This paper is focused on probabilistic interpretation LQ control design as a promising approach regarding its tuning and application to mentioned mechatronic systems. It starts section, where the basic principles of fully probabilistic control design is briefly outlined. The following sections deal with definition of suitable models describing controlled systems and implementation issues. Then, the princip of on-line fully probabilistic control tuning is explained. At the end, proposed approach is demonstrated on physical model of flexible gearbox mechanism.

Probabilistic design principles

The fully probabilistic control design determines admissible control strategy, which forces the joint distribution of all closed-loop variables as close as possible to the desired (ideal) distribution.linebreak To measure level of proximity of these distributions, the Kullback-Leibler divergence (*KL-divergence*) $D(f \,\|\, {}^{I}f)$ is used as follows [4], [5].

$$D(f \,\|\, {}^{I}f) \equiv E\left\{ \ln \frac{f(X)}{{}^{I}f(X)} \right\} = \int f(X) \ln \frac{f(X)}{{}^{I}f(X)} dX \qquad (1)$$

where the pair of probability density functions (*pdf*s) f and ${}^{I}f$ is considered to be acting on their domains i.e. on a set of all values X^*.

From control point of view, the *KL-divergence* represents the loss function or optimality criterion. By its minimization, the optimal control law is obtained. The following lines outline the mini-linebreak mization process. Due to necessity to consider time for computation of control law, the discrete design within finite time interval is considered.

General assumptions

Let us start from explanation of pair of *pdf*s mentioned in (1), which are evaluated within some specific discrete-time interval. In control design, they represent joint *pdf*s of real and ideal closed-loop behavior:

- joint *pdf* of the real closed-loop behavior

$$f(X) = f_N \equiv f(\mathbf{x}_{k+N}, u_{k+N-1}, \cdots, u_k, \mathbf{x}_k) \qquad (2)$$

- joint *pdf* of the ideal closed-loop behavior

$$f(X) = f_N \equiv f(\mathbf{x}_{k+N}, u_{k+N-1}, \cdots, u_k, \mathbf{x}_k) \qquad (3)$$

These *pdf*s are considered to be defined for values in given time and their parameters to be valid within specific finite horizon N called control horizon. The label N represents the number of discrete time instants j from instant k within the horizon; i.e. $j = k+1, \cdots, k+N$; $u_{(\cdot)}$ are control actions.

Due to practical consequences, the *pdf*s are based on the assumption that succeeding system state $\mathbf{x}_j$ arises from previous system state $\mathbf{x}_{j-1}$ and system input u_{j-1} only. Thus, $\mathbf{x}_j$ is independent of past system states and system inputs. This assumption is formulated as follows:

$$f(\mathbf{x}_j \mid \mathbf{x}_{j-1}, u_{j-1}, \cdots, \mathbf{x}_0, u_0) = f(\mathbf{x}_j \mid \mathbf{x}_{j-1}, u_{j-1}) \qquad (4)$$

$$f(u_j \mid \mathbf{x}_j, u_{j-1}, \cdots, \mathbf{x}_0, u_0) = f(u_j \mid \mathbf{x}_j) \qquad (5)$$

$${}^{I}f(\mathbf{x}_j \mid \mathbf{x}_{j-1}, u_{j-1}, \cdots, \mathbf{x}_0, u_0) = {}^{I}f(\mathbf{x}_j \mid \mathbf{x}_{j-1}, u_{j-1}) \qquad (6)$$

$${}^{I}f(u_j \mid \mathbf{x}_j, u_{j-1}, \cdots, \mathbf{x}_0, u_0) = {}^{I}f(u_j \mid \mathbf{x}_j) \qquad (7)$$

where *pdf* labeled by superscript ^{I} denote user requirements, i.e. user ideals.

Thus, the *pdf*s (2) and (3), or *pdf*s (4) to (7) respectively, describe real and ideal behavior of individual parts of given closed-loop i.e. behavior of the system and controller; e.g. in instant $j = k+1$, the real and ideal system behavior is modeled by *pdf*s $f(\mathbf{x}_{k+1} \mid \mathbf{x}_k, u_k)$ and ${}^{I}f(\mathbf{x}_{k+1} \mid \mathbf{x}_k, u_k)$; and real and ideal controller behavior is modeled by *pdf*s $f(u_k \mid \mathbf{x}_k)$ and ${}^{I}f(u_k \mid \mathbf{x}_k)$, respectively.

The suitable specification of individual *pdf*s will be described in implementation section.

Task specification

The task of fully probabilistic control design is to determine optimal control law - optimal *pdf* ${}^{o}f(u_k \mid \mathbf{x}_k)$ of the *pdf* $f(u_k \mid \mathbf{x}_k)$:

$$\{{}^{o}f(u_{j-1} \mid x_{j-1})\}_{j=k+1}^{k+N} \in \underset{\{f(u_{j-1} \mid x_{j-1})\}_{j=k+1}^{k+N}}{\arg\min} D(f_N \,\|\, {}^{I}f_N) \qquad (8)$$

As indicated in (8), the task of design consists in minimization of *KL-divergence*. The following subsection outlines the minimization procedure, which leads to the optimal *pdf* of controller and the optimal control law respectively.

Outline of minimization procedure

This subsection presents a brief outline of minimization procedure only, detail derivation is described in [5]. Optimal *pdf* of the controller can be obtained using (8).

From control theory point of view, considering the assumptions from subsection of general asumptions, the equation (8) can be interpreted as expression of specific dynamic programming procedure [1].

$$\min_{\{f(u_{j-1} \mid x_{j-1})\}_{j=k+1}^{k+N}} D(f_N \,\|\, {}^{I}f_N) = \min_{\{f(u_{j-1} \mid x_{j-1})\}_{j=k+1}^{k+N}} E\left\{ \sum_{j=k+1}^{k+N} z_j \right\}$$

$$= \min_{\{f(u_k \mid x_k)\}} E\left\{ \sum_{j=k+1}^{k+N} z_j \right\} \cdots = \min_{\{f(u_k \mid x_k)\}} \left\{ E(z_{k+1}) + \cdots \right.$$

$$\left. \min_{\{f(u_{k+N-2} \mid x_{k+N-2})\}} \left\{ E(z_{k+N-1}) + \min_{\{f(u_{k+N-1} \mid x_{k+N-1})\}} \left\{ E(z_{k+N-1}) \right\} \cdots \right\} \right. \qquad (9)$$

where $z_j = \ln \dfrac{f_j(\mathbf{x}_j \mid \mathbf{x}_{j-1}, u_{j-1})}{{}^{I}f_j(\mathbf{x}_j \mid \mathbf{x}_{j-1}, u_{j-1})}$ is j^{th} partial loss. The expression (9) leads to the following *pdf* of optimal control:

$${}^{o}f(u_k \mid \mathbf{x}_{kj}) = \frac{1}{\gamma(\mathbf{x}_k)} {}^{I}f(u_k \mid \mathbf{x}_k)\, e^{-\delta(u_k, \mathbf{x}_k)} \qquad (10)$$

where $\delta(u_k, \mathbf{x}_k)$ and $\gamma(\mathbf{x}_k)$ are suitably formed artificial quantities defined as follows

$$\delta(u_k, \mathbf{x}_k) = f(\mathbf{x}_{k+1} \mid u_k, \mathbf{x}_k) \ln \frac{f_j(\mathbf{x}_{k+1} \mid \mathbf{x}_k, u_k)}{{}^{I}f_j(\mathbf{x}_{k+1} \mid \mathbf{x}_k, u_k)}\, d\,\mathbf{x}_{k+1} \qquad (11)$$

$$\gamma(\mathbf{x}_k) = \int {}^{I}f(u_k \mid \mathbf{x}_k)\, e^{-\delta(u_k, \mathbf{x}_k)}\, du_k \qquad (12)$$

Probabilistic model

As formerly mentioned, the system behavior can be described by probability density function (*pdf*). If the system behavior is normally distributed, then its *pdf* denoted by $f(y)$ is defined as follows

$$N(\mu_y, r_y) = f(y) = \frac{1}{\sqrt{2\pi r_y}} e^{-\frac{(y-\mu_y)^2}{2r_y}} \tag{13}$$

where μ_y represents mean value, i.e. expected value of system output y ($\mu_y = E\{y\}$), $\sigma_y^2 = r_y$ denotes a dispersion (variance; $r_y = E\{(y-\mu_y)^2\}$). In control design, these parameters are considered to be continuous in values and discrete in time. Their continuity follows from the system character. The discreteness in time is given by discrete realization of control, which naturally respects the time for its computation. Internal structure of parameters mentioned above can be specified in more detail either as ARX model or as state-space model. The ARX model [6] with normally distributed noise is defined as:

$$y_k = \sum_{i=1}^{n} b_i u_{k-i} - \sum_{i=1}^{n} a_i y_{k-i} + e_{y_k}, \quad e_{y_k} \sim N(0, r_y) \tag{14}$$

where n is an order and e_{y_k} is a model noise, which has a dispersion r_y. The state-space model is defined as:

$$\mathbf{x}_{k+1} = \underbrace{\mathbf{A}\mathbf{x}_k + \mathbf{B}u_k}_{\mu_{\mathbf{x}}} + e_{\mathbf{x}_k}, \quad e_{\mathbf{x}_k} \sim N(0, \mathbf{R}) \tag{15}$$

$$y_k = \mathbf{C}\mathbf{x}_k + \tilde{e}_{y_k}, \quad \tilde{e}_{y_k} \sim N(0, \tilde{r}_y) \tag{16}$$

Equations (15) and (16) represent general state-space notation, in which the state $\mathbf{x}_k$ may belinebreak available or not; e.g. it has not a physical interpretation and for the control purposes it has to be estimated.

To avoid mentioned drawback, it is possible to use so-called pseudo state-space model [2], which is a direct reinterpretation of ARX model (14). Such reinterpretation means state-space model with non-minimal state, which contains only delayed values of inputs and outputs. An internal structure of the reinterpretation is de-fined as follows:

$$\mathbf{x}_k = \begin{bmatrix} u_{k-1} \\ \vdots \\ u_{k-n+1} \\ y_k \\ \vdots \\ y_{k-n+1} \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} 0 & \cdots & 0 & 0 & \cdots & 0 \\ 1 & \ddots & \vdots & \vdots & & \vdots \\ 0 & \ddots & 0 & 0 & \cdots & 0 \\ b_2 & \cdots & b_n & -a_1 & \cdots & -a_n \\ 0 & \cdots & 0 & 1 & \cdots & 0 \\ 0 & \ddots & 0 & 0 & \ddots & 0 \end{bmatrix}, \tag{17}$$

$$\mathbf{B} = \begin{bmatrix} 1 & \cdots & 0 & b_1 & \cdots & 0 \end{bmatrix}^T, \tag{18}$$

$$\mathbf{C} = \begin{bmatrix} 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \end{bmatrix}^T, \quad \tilde{r}_y = 0 \tag{19}$$

Relation of the pseudo-state space model to ARX model is obvious from the following corollary:

$$y_k = \underbrace{\mathbf{CA}\mathbf{x}_{k-1} + \mathbf{CB}u_{k-1}}_{\mu_y} + \underbrace{\mathbf{C}\mathbf{e}_{\mathbf{x}_k}}_{e_{yk}}, \quad \mathbf{e}_{yk} \sim N(0, r_y), \tag{20}$$

Models (14); or (15) and (16); or (15) to (19) are used as models for implementation of fully probabilistic control design described below.

Implementation of control

Let us start from general expression (10) representing optimal *pdf* (section on principles). To compute real parameters of this *pdf*, individual *pdf*s from assumptions (4), (6) and (7) have to be defined. These emph{pdf}s represent both real and ideal behavior of closed-loop (Fig.2). Assuming model given by (15) to (19), i.e. finite memory and known parameters of appropriate distributions, then emph *pdf*s are defined as follows:

- *pdf* of the real controlled system output

$$N(\mu_y, r_y): f(y_{k+1} \mid u_k, \mathbf{x}_k) = \frac{1}{\sqrt{2\pi\, r_y}} e^{-\frac{1}{2}(y_{k+1}-\mu_y)^T r_y^{-1} (y_{k+1}-\mu_y)} \tag{21}$$

- *pdf* of the ideal controlled system output

$$N({}^I\mu_y, {}^Ir_y): {}^If(y_{k+1} \mid u_k, \mathbf{x}_k) = \\ = \frac{1}{\sqrt{2\pi\, {}^Ir_y}} e^{-\frac{1}{2}(y_{k+1}-{}^I\mu_y)^T\, {}^Ir_y^{-1} (y_{k+1}-{}^I\mu_y)} \tag{22}$$

where ideal ${}^I\mu_y$ is the desired output value w_{k+1};

- *pdf* of the ideal controlled system input

$$N({}^I\mu_u, {}^Ir_u): {}^If(u_k \mid \mathbf{x}_k) = \\ = \frac{1}{\sqrt{2\pi\, {}^Ir_u}} e^{-\frac{1}{2}(u_{k+1}-{}^I\mu_u)^T\, {}^Ir_u^{-1} (u_{k+1}-{}^I\mu_u)} \tag{23}$$

where ${}^I\mu_u$ is assumed to be the previous action u_{k-1} and the dispersion ${}^Ir_u$ can be viewed as a tuning parameter of the controller. For *pdf*s defined like that, the computation of *pdf* (10) leads to the following expressions:

$$\begin{aligned} {}^of(u_k \mid \mathbf{x}_k, u_{k-1}) &= \frac{1}{\sqrt{2\pi\, {}^or_u}} e^{-\frac{1}{2}(u_k - {}^or_u\, b)^T\, {}^or_u^{-1} (u_k - {}^or_u\, b)} \\ &= \frac{1}{\sqrt{2\pi\, {}^or_u}} e^{-\frac{1}{2}\left\{u_k + \mathbf{k}_k\mathbf{x}_k - \sum_{j=k+1}^{k+N+1} k_{wj} w_j - k_u u_{k-1}\right\}^2} \end{aligned} \tag{24}$$

$${}^ou_k = -\mathbf{k}_k\mathbf{x}_k + \sum_{j=k+1}^{k+N+1} k_{wj} w_j + k_u u_{k-1} \tag{25}$$

where ${}^ou_k$ is the optimal control law.

On-line probabilistic tuning

This section focuses on tuning of control parameters. In general, the parameters of the controllers determine the character of the control actions responding on changes of working conditions and user requirements. Usually, the parameters - their values - are selected according to user experiences or according to some simple empirical rule. The values are constant for whole control process or sometimes they are discontinuously reset. It is not suitable for dynamic systems within changeable environment.

Presented probabilistic formulation of LQ control is suitable for on-line tuning or fine-tuning. Partly, it can use local consecutively-changed models (model adaptation) and partly, can use different slightly-changed control parameters (controller adaptation). The former can be characterized as some change of system properties i.e. model parameters and the latter can represent the quality of the description i.e. quality of the model parameters. Thus, good reliable model gives more accurate and brisk controller and vice versa.

The both mentioned ways of adaptation can be covered in the control law (25), which represents standard form of LQ control. The gains $\mathbf{k}_k$, k_{wj} and k_u contain parameters of the system model (e.g. model (20)) and simultaneously control parameters, which are presented by dispersions ${}^{I}r_u$ and ${}^{I}r_y$. These dispersions are very important, because they are determining factors for the gains $\mathbf{k}_k$, k_{wj} and k_u in (24) and (25).

In comparison with non-probabilistic LQ control design, reciprocal values of the dispersions represent input and output penalization factors ($q_u = {}^{I}r_u$, $q_y = {}^{I}r_y$), which together adjust individual terms in quadratic loss-function.

As was already mentioned, their choice is based on experience or on experimental tuning. In fully probabilistic control design, interpretation of these quantities is more straightforward. The equations (22) and (23) imply that ${}^{I}r_u$ and ${}^{I}r_y$ represent noise dispersions for ideal distribution of the system and controller.

The algorithm proposed in this paper is intended for systems (e.g. mechatronic one), where the mathematical model together with the noise can change substantially, possibly due to additional interference, that may occur randomly during the control. Inadequate choice of input and output penalizations or ${}^{I}r_u$ with ${}^{I}r_y$ respectively, can cause serious device failures, e.g. system actuators (drives) might not be able to achieve designed control or may be damaged by them.

Unexpected system noise increase may force the controller to generate inputs out of any reasonable physical range of the device. In such undesired cases, it would usually be acceptable to decrease control quality in order to achieve at least some reasonable control actions. Probabilistic control interpretation of penalization factors as dispersions can achieve indicated strategy via on-line control tuning.

The tuning is based on the idea of changing of dispersion ${}^{I}r_y$ so that its amplitude is propor-tional to the output dispersion r_y or practically to its estimate

$$\hat{r}_{y_i} = e_{y_i} e_{y_i}^T = (y_i - \hat{\mu}_{y_i})(y_i - \hat{\mu}_{y_i})^T \quad (26)$$

calculated from current data y_i and model. The effect is that during periods of increased output noise, output ideal is set to be less strict. It causes the output to be tracked less closely. This allows the input to stay in its reasonable constraints.

However, current output dispersionlinebreak can change very quickly causing big changes in dispersion ${}^{I}r_y$. In order to avoid this, $\hat{r}_{y_i}$ has to be filtrated. As a suitable filter, exponential forgetting is used. It can be defined as follows:

$$\tilde{r}_{y_1} = (1-\lambda)\,\hat{r}_{y_1} \quad (27)$$

$$\tilde{r}_{y_i} = \lambda\,\tilde{r}_{y_{i-1}} + (1-\lambda)\,\hat{r}_{y_i}, \quad i = 2,\cdots,k \quad (28)$$

where λ is a forgetting factor influencing quickness of weight decrease of individual contributions $\hat{r}_{y_i}$. The equations (27) and (28) can form one general expression:

$$\tilde{r}_{y_k} = (1-\lambda)\sum_{i=1}^{k} \lambda^{k-i}\,\hat{r}_{y_i} \quad (29)$$

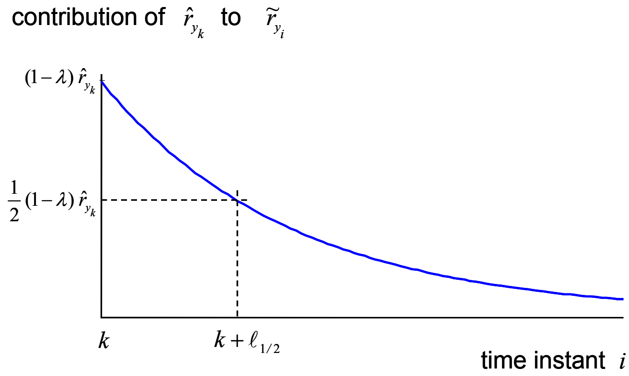

Fig.3 Trend of contribution of $\hat{r}_{yk}$ to $\tilde{r}_{yi}$

In order to find reasonable value for parameter λ the suitable number of time instants ℓ has to be defined in correspondence to the characterlinebreak of control process During these ℓ time instants, the contribution of $\hat{r}_{yk}$ to $\hat{r}_{yi}$ drops to the given level.

Standard choice is to select number of instants (denoted by $\ell_{1/2}$) that cause dropping the contribution of $\hat{r}_{yk}$ to one half of the original value. It implies that $\ell_{1/2}$ satisfies the equation:

$$\lambda^{\ell_{1/2}}(1-\lambda)\,\hat{r}_{yk} = \frac{1}{2}(1-\lambda)\,\hat{r}_{yk} \quad (30)$$

See Fig.3 for illustration of this effect. Producing `half-time' $\ell_{1/2}$ is user-friendly way to find a suitable value for constant λ, because user can easily imagine what is the time needed for a contribution of $\hat{r}_{yk}$ to drop to one half. Consequently, suitable λ can be found like this:

$$\lambda = \left(\frac{1}{2}\right)^{1/\ell_{1/2}} \quad (31)$$

where $\ell_{1/2}$ is provided by the user.

On-line tuned LQ control of gearbox mechatronic system

This section demonstrates the presented fully probabilistic interpretation of LQ control design including the on-line parameter tuning. The aim is to illustrate improvements of control process that follow from consequences of previous section.

As was mentioned in introduction, the gearbox system (see Fig.4(e)), consists of three wheels, which are mutually connected by two elastic belts. Position of the wheel 1 is controlled by servo-motor, and the position of the wheel 3 is measured.

From control design point of view, the mechatronic system is modelled by ARX model (14) of order $n = 6$, which is determined by the fackt, that each solid wheel represents approximatelly 2 orders. Real control of the system is provided by adaptive LQ controller.

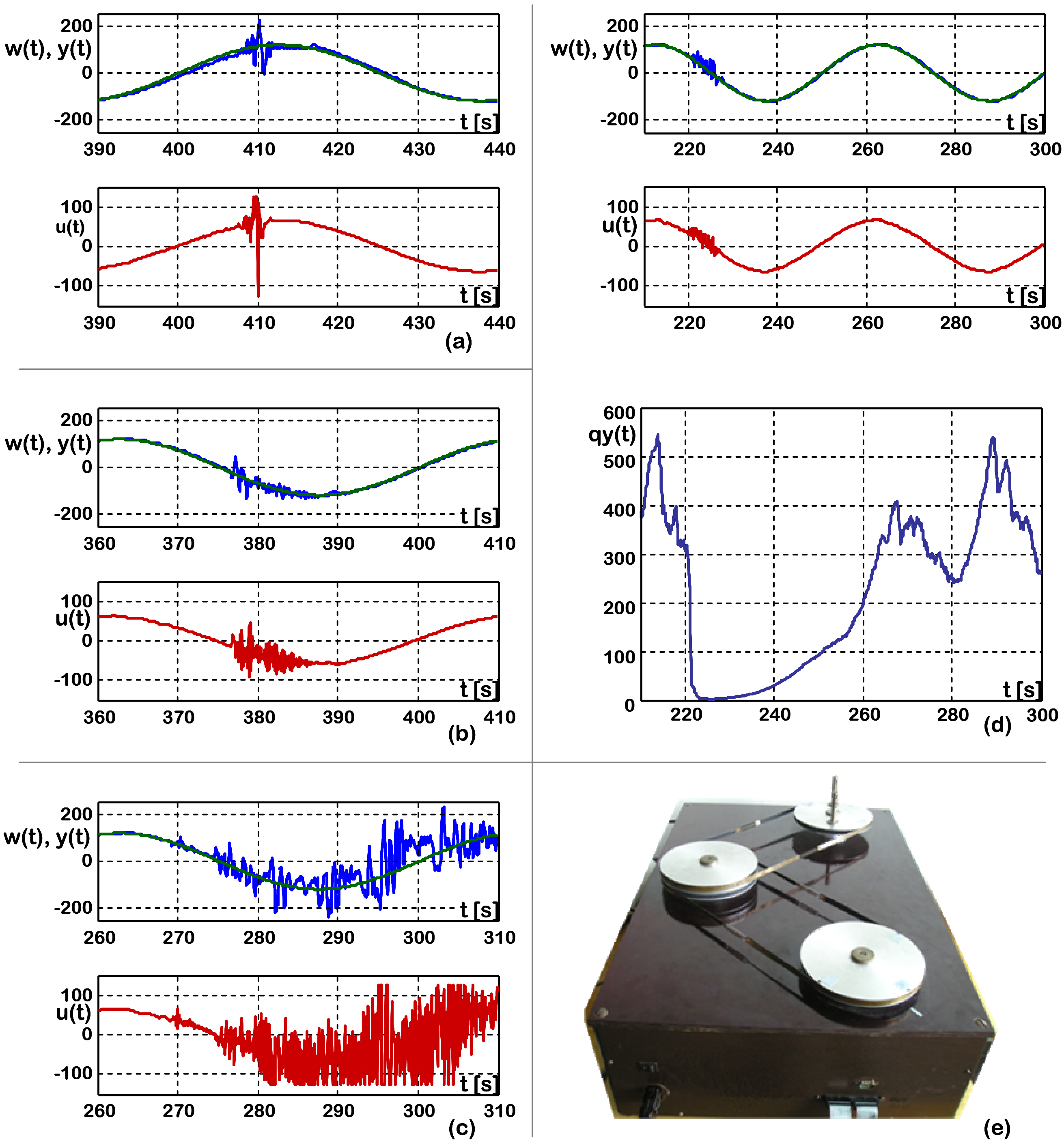

Fig.4 Real experiment: (a), (b) and (c) comparison of standard LQ control $q_y = 1$, $q_y = 100$ and $q_y = 200$ respectively; (d) control generated by probabilistic design with tuning (desired and real system output $w(t)$ and $y(t)$; input $u(t)$; penalization $q_y(t)$; (e) gearbox system

During control process, the discrepancy between model estimated and the real system occurs. This causes sharp changes of control actions, which do not follow from desired profile of system output but just from temporal discrepancy of estimated model from reality. In ideal conditions, this undesirable state damp out shortly.

However, in real conditions, it can cause unpredictable behaviour damaging drives and even it can damage other structural elements of the system. This phenomenon is being suppressed by tuning algorithm proposed in this paper (see Fig.4(d)).

Fig.4 specifically, demonstrates four runs of real control process. The individual sub-figures (a), (b), and (c) show control runs with differentlinebreak but constant output penalization (q_y). In all caseslinebreak of constant q_y, the input magnitude startslinebreak to change rapidly due to sudden disturbance. The process eventually stabilizes, however, in case (c) the controller have not stabilized at all.

With adaptive tuning proposed in this paper (sub-figure (d) of Fig.4) the changes in input are reasonably small, moreover, the output matches desired value much better.

Conclusion

The paper outlines the principles and practical aspects of fully probabilistic interpretation of LQ control. Consequently, the on-line tuning was introduced. This way of design forms sound physical interpretation for tunable controller parameters. The design with tuning was applied and demonstrated on real gearbox mechatronic system occuring frequenty in production machines (e.g. rolling mills) and in industrial robots (geared robotic arms). The representative results are discussed in section dealing with real time control of gearbox mechatronic system.

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