Completion and Sharing of Probabilistic Information

[1]Kárný Miroslav, [2]Guy Tatiana Valentine , [3]Bodini Antonella , [4]Rugerri Fabrizio

Bayesian paradigm [11, 2] provides a justified prescriptive framework for decision making under uncertainty. This theory covers an extreme width of research and application domains: classical statistical tasks, signal processing, pattern recognition, stochastic and adaptive control, micro and macro economics, societal decision making, cooperation etc.

The necessity of a complete probabilistic description of system's behavior – joint distribution of all variables considered – significantly impedes wide use of this elegant framework. In the majority of real-world tasks, the fragmental information pieces processed *do not* completely specify the required joint distribution.

The maximum entropy principle [3] is frequently used to create a sort of extension of these information pieces. The resulting joint distribution respects the available information pieces and has the highest entropy. An axiomatic justification of this intuitively plausible choice has been proposed for processing generalised moments [9]. The present contribution unifies the result with the important result [1], relating the Kerridge inaccuracy [8] to a Bayesian approximation of distributions and to a formal justification of the so-called fully probabilistic design of decision-making strategies [6, 4]. This unification significantly improves a merging of fragmental information pieces [7, 10, 5]. The improvement removes some redundant constraints of previous solutions and interprets the classical Bayesian estimation as its special case.

Application of the proposed methodology to a merging of fragmentally expressed decision-making aims provides a new way of constructing the probability density function expressing *multiple* decision-making aims and constraints. The methodology is applicable, irrespectively, whether the aims and constraints concern one or several decision makers.

Acknowledgment The research has been supported by MŠMT ČR 1M0572, GAČR 102/08/0567 and Czech-Italian Cooperation project.

References

[1] J. M. Bernardo. Expected information as expected utility. *The Annals of Statistics*, 7,686–690, 1979.

[2] M.H. DeGroot. *Optimal Statistical Decisions*. McGraw-Hill, New York, 1970.

[3] E.T. Janes (Ed. G.L.Bretthorst). *Probability theory: the logic of science*. Cambridge University Press, Cambridge, UK, 2003.

[4] M. Kárný. Bayesian paradigm and fully probabilistic design. In *Preprints of the 17th IFAC World Congress*. IFAC, 2008.

[5] M. Kárný. Knowledge elicitation via extension of fragmental knowledge pieces. In *Proceedings of the European Control Conference 2009*. European Union Control Association, 2009.

[6] M. Kárný and T. V. Guy. Fully probabilistic control design. *Systems & Control Letters*, 55(4):259–265, 2006.

[7] M. Kárný, T.V. Guy, A. Bodini, and F. Ruggeri. Cooperation via sharing of probabilistic elements. *IJCIStudies*, II(2), 2009. accepted.

[8] D.F. Kerridge. Inaccuracy and inference. *Journal of Royal Statistical Society*, B 23:284–294, 1961.

[9] J. Shore and R. Johnson. Axiomatic derivation of the principle of maximum entropy and the principle of minimum cross-entropy. *IEEE Transactions on Information Theory*, 26(1):26–37, 1980.

[10] V. Šmídl and M. Kárný. Log-normal merging for distributed system identification. In *Proceedings of the 15th IFAC Symposium on System Identification*. IFAC, 2009.

[11] A. Wald. *Statistical Decision Functions*. John Wiley, New York, London, 1950.

[1]Institute of Information Theory and Automation of the ASCR, Department of Adaptive Systems, *school@utia.cas.cz*
[2]Institute of Information Theory and Automation of the ASCR, Department of Adaptive Systems, *guy@utia.cas.cz*
[3]Istituto di Matematica Applicata e Tecnologie Informatiche, Milano Department, *anto@mi.imati.cnr.it*
[4]Istituto di Matematica Applicata e Tecnologie Informatiche, Milano Department, *fabrizio@mi.imati.cnr.it*