Multiobjective Stochastic Programming via Multistage Problems

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Abstract

Economic activities developing over time are very often influenced simultaneously by a random factor (modelled mostly by a stochastic process) and a “decision” parameter that has to be chosen according to economic possibilities. Moreover, it is necessary often to evaluate the economic activities simultaneously by a few “utility” functions. Evidently, the mentioned economic situations can lead to mathematical models corresponding to multistage multiobjective stochastic programming problems. Usually, the multiobjective (one–stage) problems and multistage (one–objective) problems have been investigated separately. The aim of this contribution will be to try to analyze a relationship between these two approaches.

Key words
Multistage stochastic programming problems, multiobjective stochastic programming, efficient points, weight approach

1 Introduction

Multistage stochastic programming problems belong to optimization problems depending on a probability measure. Usually, the operator of mathematical expectation appears in the objective function and constraints set can depend on the probability measure also. The multistage stochastic programming problems correspond to applications that can be considered with respect to some finite "discrete" (say $(0, M)$; $M \geq 1$) time interval and simultaneously there exists a possibility to decompose them with respect to the individual time points. A decision, at every individual time point say k, can depend only on the random elements realizations and the decisions to the time point $k-1$ (we say that it must be nonanticipative). To define the multistage stochastic programming problems we employ an approach in which the multistage stochastic programming problem is introduced as a finite system of parametric (one–stage) optimization problems with an inner type of dependence (for more details see e.g. [1]). The multistage stochastic programming problem can be then introduced in the following form.

Find

$$\varphi_{\mathcal{F}}(M) = \inf \{\mathsf{E}_{F^{\xi^0}} g^0_{\mathcal{F}}(x^0, \xi^0)|\ x^0 \in \mathcal{K}^0\}, \tag{1}$$

where the function $g^0_{\mathcal{F}}(x^0, z^0)$ is defined recursively

$$\begin{aligned} g^k_{\mathcal{F}}(\bar{x}^k, \bar{z}^k) \quad &= \quad \inf\{\mathsf{E}_{F^{\xi^{k+1}|\bar{\xi}^k=\bar{z}^k}}\ g^{k+1}_{\mathcal{F}}(\bar{x}^{k+1}, \bar{\xi}^{k+1})\,|x^{k+1} \in \mathcal{K}^{k+1}_{\mathcal{F}}(\bar{x}^k, \bar{z}^k)\}, \\ k = 0,\, 1,\, \ldots,\, M-1, & \\ g^M_{\mathcal{F}}(\bar{x}^M, \bar{z}^M) \quad &:= \quad g^M_0(\bar{x}^M, \bar{z}^M), \quad \mathcal{K}_0 := X^0. \end{aligned} \tag{2}$$

$\xi^j := \xi^j(\omega)$, $j = 0, 1, \ldots, M$ denotes an s–dimensional random vector defined on a probability space $(\Omega, \mathcal{S}, P)$; $F^{\xi^j}(z^j)$, $z^j \in R^s$, $j = 0, 1 \ldots, M$ the distribution function of the ξ^j and $F^{\xi^k|\bar{\xi}^{k-1}}(z^k|\bar{z}^{k-1})$, $z^k \in R^s$, $\bar{z}^{k-1} \in R^{(k-1)s}$, $k = 1, \ldots, M$ the conditional distribution function (ξ^k conditioned by $\bar{\xi}^{k-1}$); $P_{F^{\xi^j}}$, $P_{F^{\xi^{k+1}|\bar{\xi}^k}}$, $j = 0, 1, \ldots, M$, $k = 0, 1, \ldots, M-1$ the corresponding probability measures; $Z^j := Z_{F^{\xi^j}} \subset R^s$, $j = 0, 1, \ldots, M$ the support of the probability measure $P_{F^{\xi^j}}$. Furthermore, $g_0^M(\bar{x}^M, \bar{z}^M)$ denotes a continuous function defined on $R^{n(M+1)} \times R^{s(M+1)}$; $X^k \subset R^n$, $k = 0, 1, \ldots, M$ is a nonempty compact set; the symbol $\mathcal{K}_{\mathcal{F}}^{k+1}(\bar{x}^k, \bar{z}^k) := \mathcal{K}_{F^{\xi^{k+1}|\bar{\xi}^k}}^{k+1}(\bar{x}^k, \bar{z}^k)\,(\mathcal{K}_{F^{\xi^{k+1}|\bar{\xi}^k}}^{k+1}(\bar{x}^k, \subset X^k)$, $k = 0, 1, \ldots, M-1$ denotes a multifunction mapping $R^{n(k+1)} \times R^{s(k+1)}$ into the space of subsets of R^n. $\bar{\xi}^k(:= \bar{\xi}^k(\omega)) = [\xi^0, \ldots, \xi^k]$; $\bar{z}^k = [z^0, \ldots, z^k]$, $z^j \in R^s$; $\bar{x}^k = [x^0, \ldots, x^k]$, $x^j \in R^n$; $\bar{X}^k = X^0 \times X^1 \ldots \times X^k$; $\bar{Z}^k := \bar{Z}_{\mathcal{F}}^k = Z_{F^{\xi^0}} \times Z_{F^{\xi^1}} \ldots \times Z_{F^{\xi^k}}$, $j = 0, 1, \ldots, k$, $k = 0, 1, \ldots, M$. Symbols $\mathsf{E}_{F^{\xi^0}}$, $\mathsf{E}_{F^{\xi^{k+1}|\bar{\xi}^k = \bar{z}^k}}$, $k = 0, 1, \ldots, M-1$ denote the operators of mathematical expectation corresponding to F^{ξ^0}, $F^{\xi^{k+1}|\bar{\xi}^k = \bar{z}^k}$, $k = 0, \ldots, M-1$.

The problem (1) is a "classical" one–stage, one–objective stochastic problem, the problems (2) are (generally) parametric one–stage, one–objective stochastic optimization problems. Let us assume a special case when the function $g_0^M(\bar{x}^M, \bar{z}^M)$ fulfils the following assumption.

i.1 there exist continuous functions $\bar{g}^j(x^j, z^j)$, $j = 0, 1, \ldots, M$ defined on $R^n \times R^s$ such that

$$g_0^M(\bar{x}^M, \bar{z}^M) = \sum_{j=0}^{M} \bar{g}^j(x^j, z^j). \tag{3}$$

Evidently, under i.1, the function $\bar{g}^j(x^j, z^j)$ corresponds to an evaluation of the economic activity at the time point $j \in \{0, 1, \ldots, M\}$. However, it happens rather often that it is reasonable to evaluate this economic activity simultaneously by several "utility" functions, say $\bar{g}_i^j(x, , z)$, $i = 1, \ldots, l$. Including this reality we can see that the "underlying" common objective function $g_0^M(\bar{x}^M, \bar{z}^M)$ (in (1) and (2)) has to be replaced by the following multiobjective criterion function.

$$g_{0,i}^M(\bar{x}^M, \bar{z}^M) = \sum_{j=0}^{M} \bar{g}_i^j(x^j, z^j), \quad i = 1, \ldots, l. \tag{4}$$

Consequently, assuming the same inner time dependence as it was assumed in the problem (1) and (2), we obtain formally the following multistage, multiobjective problem.

Find
$$\varphi_{\mathcal{F}}(M, i) = \inf \mathsf{E}_{F^{\xi^0}} g_{\mathcal{F}}^{0,i}(x^0, \xi^0), \ i = 1, \ldots, l \quad \text{subject to} \quad x^0 \in \mathcal{K}^0, \tag{5}$$

where the function $g_{\mathcal{F}}^{0,i}(x^0, z^0)$, $i = 1, \ldots, l$ are defined recursively

$$\begin{aligned} g_{\mathcal{F}}^{k,i}(\bar{x}^k, \bar{z}^k) \quad &= \quad \inf \mathsf{E}_{F^{\xi^{k+1}|\bar{\xi}^k = \bar{z}^k}}\ g_{\mathcal{F}}^{k+1,i}(\bar{x}^{k+1}, \bar{\xi}^{k+1}),\ i = 1, \ldots, l \\ &\qquad \text{subject to} \quad x^{k+1} \in \mathcal{K}_{\mathcal{F}}^{k+1}(\bar{x}^k, \bar{z}^k), \quad k = 0, 1, \ldots, M-1, \\ g_{\mathcal{F}}^{M,i}(\bar{x}^M, \bar{z}^M) \quad &:= \quad g_{0,i}^M(\bar{x}^M, \bar{z}^M), \quad i = 1, \ldots, l, \quad \mathcal{K}_0 := X^0. \end{aligned} \tag{6}$$

The problem (5) is formally a problem of one–stage multiobjective optimization theory. The problems (6) are one–stage multiobjective parametric optimization problems. It is known that there doesn't exists (mostly) an optimal solution simultaneously with respect to all criteria. Consequently, the optimal solution has to be mostly replaced by a set of efficient points. Consequently, $g_{\mathcal{F}}^{k,i}$, $i = 1, \ldots, l$, $k = 0, \ldots, M-1$ have to be replaced by multifunctions $\mathcal{G}_{\mathcal{F}}^{k,i}$, $i = 1, \ldots, l$, $k = 0, \ldots, M-1$ corresponding to the function values in efficient points (for definition of the efficient points see [2] or the following section).

2 Some Definitions and Auxiliary Assertion

2.1 One–Stage Deterministic Multiobjective Problems

A multiobjective deterministic optimization problem can be introduced as the problem.

Find

$$\min f_i(x), \ \ i = 1, \ldots, l \quad \text{subject to } x \in \mathcal{K}. \tag{7}$$

f_i, $i = 1, \ldots, l$ are functions defined on R^n, $\mathcal{K} \subset R^n$ is a nonempty set.

Definition 1. [4] The vector x^* is an efficient solution of the problem (7) if and only if $x^* \in \mathcal{K}$ and if there exists no $x \in \mathcal{K}$ such that $f_i(x) \le f_i(x^*)$ for $i = 1, \ldots, l$ and such that for at least one i_0 one has $f_{i_0}(x) < f_{i_0}(x^*)$. We denote the set of efficient points of the problem (7) by $\mathcal{K}_E$.

Definition 2. [4] The vector x^* is a properly efficient solution of the multiobjective optimization problem (7) if and only if it is efficient and if there exists a scalar $M > 0$ such that for each i and each $x \in \mathcal{K}$ satisfying $f_i(x) < f_i(x^*)$ there exists at least one j such that $f_j(x^*) < f_j(x)$ and

$$\frac{f_i(x^*) - f_i(x)}{f_j(x) - f_j(x^*)} \le M. \tag{8}$$

We denote the set of properly efficient points of problem (7) by $\mathcal{K}_{PE}$.

Definition 3. [2] The vector x^* is called weakly efficient solution of the problem (7) if and only if $x^* \in \mathcal{K}$ and if there exists no $x \in \mathcal{K}$ such that $f_i(x) < f_i(x^*)$ for every $i = 1, \ldots, l$. We denote the set of weakly efficient points of the problem (7) by $\mathcal{K}_{wE}$.

Furthermore, let us define the following parametric optimization problem.

Find

$$\begin{aligned} &\min_{x \in \mathcal{K}} f^{\lambda}(x), \quad \lambda \in \Lambda, \quad \text{where} \quad f^{\lambda}(x) = \sum_{i=1}^{l} \lambda_i f_i(x) \quad \text{and} \\ &\Lambda = \{\lambda \in R^l : \lambda = (\lambda_1, \ldots, \lambda_l), \ \lambda_i \in \langle 0, 1\rangle, \ i = 1, \ldots, l, \ \sum_{i=1}^{l} \lambda_i = 1\}. \end{aligned} \tag{9}$$

Proposition 1. [2] If

1. $\hat{x} \in \mathcal{K}$ is a solution of (9) for $\lambda \in \Lambda$, then $\hat{x} \in \mathcal{K}_{wE}$,
2. $\mathcal{K}$ is a convex set, f_k, $k = 1, \ldots, l$ are convex functions, then

$$\hat{x} \in \mathcal{K}_{wE} \iff \text{there exists } \ \lambda \in \Lambda \ \text{ such that } \ \hat{x} \ \text{ is optimal in } \ (9).$$

Proposition 2. [4] If

1. $\hat{x}$ is optimal in the problem (9) for some fixed $\lambda = (\lambda_1, \ldots, \lambda_l) \in \Lambda$ with $\lambda_i > 0$, $i = 1, \ldots, l$, then $\hat{x} \in \mathcal{K}_{PE}$,
2. $\mathcal{K}$ is a convex set and f_i, $i = 1, \ldots, l$ are convex functions on $\mathcal{K}$, then

$$\hat{x} \in \mathcal{K}_{PE} \iff \text{there exists } \ \lambda \in \Lambda \ \text{ with } \ \lambda_i > 0, i = 1, \ldots, l \ \text{ such that } \ \hat{x} \ \text{ is optimal in (9).}$$

If we denote by the symbols $f(\mathcal{K}_E)$, $f(\mathcal{K}_{PE}) \subset R^l$ the image of $\mathcal{K}_E$, $\mathcal{K}_{PE} \subset R^n$ obtained by the vector function $f = (f_1, \ldots, f_l)$, then the following implication has been recalled in [4].

$$\begin{array}{ll} \mathcal{K} \quad \text{closed and convex}, \quad f_i,\, i = 1, \ldots, l \quad \text{continuous and convex on} \quad \mathcal{K} \\ \Longrightarrow \quad f(\mathcal{K}_{PE}) \subset f(\mathcal{K}_E) \subset \bar{f}(\mathcal{K}_{PE}), \quad \text{where} \quad \bar{f}(\mathcal{K}_{PE}) \quad \text{denotes a closure of} \quad f(\mathcal{K}_{PE}). \end{array} \tag{10}$$

Lemma 1. [10] Let $\mathcal{K} \subset R^n$ be a nonempty set, f_i, $i = 1, \ldots, l$ be functions defined on R^n. Let, moreover, the function f^λ be defined by the relation (9). If $f_i, i = 1, \ldots, l$ are bounded functions on $\mathcal{K}$, $(|f_i(x)| \le \bar{M},\, x \in \mathcal{K},\, i = 1, \ldots, l,\, \bar{M} > 0)$, then for every $x \in \mathcal{K}$, f^λ is a Lipschitz function on Λ with a Lipschitz constant not greater then $l\bar{M}$.

2.2 Deterministic Parametric Optimization

Definition 4. [5] Let $h(x)$ be a real–valued function defined on a convex set $\mathcal{K} \subset R^n$. $h(x)$ is a strongly convex function with a parameter $\rho > 0$ if

$$h(\lambda x^1 + (1-\lambda)x^2) \le \lambda h(x^1) + (1-\lambda)h(x^2) - \lambda(1-\lambda)\rho\|x^1 - x^2\|^2 \quad \text{for every} \quad x^1,\, x^2 \in \mathcal{K},\, \lambda \in \langle 0,\, 1\rangle.$$

Lemma 2. [6] Let $\mathcal{K} \subset R^n$ be a nonempty, compact, convex set. Let, moreover, $h(x)$ be a strongly convex with a parameter $\rho > 0$, continuous, real–valued function defined on $\mathcal{K}$. If x^0 is defined by the relation $x^0 = \arg\min_{x \in \mathcal{K}} h(x)$, then

$$\|x - x^0\|^2 \le \frac{2}{\rho}|h(x) - h(x^0)| \quad \text{for every } x \in \mathcal{K}.$$

Lemma 3. [10] Let $\mathcal{K} \subset R^n$ be a nonempty convex set, $\varepsilon \in (0,\, 1)$. Let, moreover, f_i, $i = 1, \ldots, l$ be convex functions on $\mathcal{K}$. If

1. f_1 is a strongly convex (with a parameter $\rho > 0$) function on $\mathcal{K}$, then f^λ defined by (9) is for $\lambda = (\lambda_1, \ldots, \lambda_l) \in \Lambda$ with $\lambda_1 \in (\varepsilon,\, 1)$ a strongly convex function on $\mathcal{K}$,
2. f_i, $i = 1, \ldots, l$ are strongly convex function on $\mathcal{K}$ with a parameter ρ, then f^λ defined by (9) is a strongly convex function on $\mathcal{K}$.

Lemma 4. [7] Let $\mathcal{K} \in R^n$, $Y \in R^m$, $n,\, m \ge 1$ be nonempty convex sets. Let, furthermore, $\bar{\mathcal{K}}(y)$ be a multifunction mapping Y into the space of nonempty closed subsets of $\mathcal{K}$, $h(x,\, y)$ function defined on $X \times Y$ such that

$$\bar{\varphi}(y) \;=\; \inf\{h(x,\, y) | x \in \bar{\mathcal{K}}(y)\} > -\infty \quad \text{for every} \quad y \in Y.$$

If

1. $h(x,\, y)$ is a convex function on $X \times Y$ and simultaneously
$$\lambda(\bar{\mathcal{K}}(y(1)) + (1-\lambda)(\bar{\mathcal{K}}(y(2)) \;\subset\; \bar{\mathcal{K}}(\lambda y(1) + (1-\lambda)y(2)) \quad \text{for every} \quad y(1),\, y(2) \in Y,$$
then $\bar{\varphi}(y)$ is a convex function on Y.
2. $h(x,\, y)$ is a Lipschitz function on $X \times Y$ with the Lipschitz constant L and simultaneously
$$\Delta[\bar{\mathcal{K}}(y(1)),\, \bar{\mathcal{K}}(y(2))] \le \bar{C}\|y(1) - y(2)\| \quad \text{for every} \quad y(1),\, y(2) \in Y \text{and a} \, \bar{C} \ge 0$$
then $\bar{\varphi}(y)$ is a Lipschitz function on Y with the Lipschitz constant not greater then $L(\bar{C}+1)$. (the symbol $\Delta[\cdot,\, \cdot]$ denotes the Hausdorff distance, for the definition see e.g. [12].)

3 Problem Analysis

In this section, we return to the problem introduced by (6) and (7). In particular, the aim of this section will be to try to characterize points obtained by "weight" approach and consequently to approximate efficient points sets corresponding to the individual problems (6), (7). To this end we assume.

A.1 $\bar{g}_i^j(x^j,\, z^j),\ i = 1,\, \ldots,\, l, \quad j = 0,\, \ldots,\, M$ are for every $\bar{z}^M \in \bar{Z}^M$ convex functions of $\bar{x}^M \in \bar{X}^M$,

A.2 $\Delta[\mathcal{K}_{\mathcal{F}}^{k+1}(\bar{x}^k(1),\, \bar{z}^k),\, \bar{x}^k(2),\, \bar{z}^k)] \le C\|\bar{x}^k(1) - \bar{x}^k(2)\|$ for every $\bar{x}^k(1),\, \bar{x}^k(2) \in \bar{X}^k,\, \bar{z}^k \in \bar{Z}^k$ and some $C > 0$,

A.3 $\lambda\mathcal{K}_{\mathcal{F}}^{k+1}(\bar{x}^k(1),\, \bar{z}^k) + (1-\lambda)\mathcal{K}_{\mathcal{F}}^{k+1}(\bar{x}^k(2),\, \bar{z}^k) \subset \mathcal{K}_{\mathcal{F}}^{k+1}(\lambda\bar{x}^k(1) + (1-\lambda)\bar{x}^k(2),\, \bar{z}^k)$ for every $\bar{x}^k(1),\, \bar{x}^k(2) \in \bar{X}^k,\, \bar{z}^k \in \bar{Z}^k$,

A.4 $\mathcal{G}_{\mathcal{F}}^{k,\,i}(\bar{x}^k,\, \bar{z}^k) \in (-\infty,\, +\infty)$ for every $\bar{x}^k \in \bar{X}^k,\, \bar{z}^k \in \bar{Z}^k$.

The cases under which the assumption A.2, A.3 and A.4 are fulfilled can be found e.g. in [8] or [11].

Evidently, according to the assertions of Proposition 1, Proposition 2 and the relation (10) it is reasonable to set a weight approach to the multiobjective function (5). We obtain by this one–objective multistage parametric problem of the type (1), (2) in which

$$g_0^M(\bar{x}^M,\, \bar{z}^M) \quad \text{is replaced by} \quad g_{\mathcal{F}}^{M,\,\lambda}(\bar{x}^M,\, \bar{z}^M) = \sum_{j=0}^{M}\sum_{i=1}^{l}\lambda_i\bar{g}_i^j(x^j,\, z^j), \quad \lambda = (\lambda_1,\, \ldots,\, \lambda_l) \in \Lambda.$$

Obviously, for given $\lambda \in \Lambda$ the values of the component λ_i for every $i \in \{1,\, \ldots,\, l\}$ corresponds to the relevance of the component $\sum\limits_{j=1}^{M}\bar{g}_i^j(x^j,\, z^j)$ in the multiobjective criterion $\sum\limits_{j=1}^{M}\bar{g}_1^j(x^j,\, z^j),\, \ldots,$ $\sum\limits_{j=1}^{M}\bar{g}_l^j(x^j,\, z^j)$ (for more details see e.g. [3] or [4]). Consequently, we obtain (by this approach) the following multistage stochastic parametric programming problem.

Find

$$\varphi_{\mathcal{F}}^{\lambda}(M) = \inf\{\mathsf{E}_{F^{\xi^0}}\, g_{\mathcal{F}}^{0,\,\lambda}(x^0,\, \xi^0)|\ x^0 \in \mathcal{K}^0\}, \tag{11}$$

where the function $g_{\mathcal{F}}^{0,\,\lambda}(x^0,\, z^0)$ is defined recursively for $k = 0,\, 1,\, \ldots,\, M-1$ by

$$g_{\mathcal{F}}^{k,\,\lambda}(\bar{x}^k,\, \bar{z}^k) = \inf\{\mathsf{E}_{F^{\xi^{k+1}|\bar{\xi}^k=\bar{z}^k}}\,[\sum_{i=1}^{l}\lambda_i\bar{g}_i^k(x^k,\, z^k) + g_{\mathcal{F}}^{k+1,\,\lambda}(\bar{x}^{k+1},\, \bar{\xi}^{k+1})]\,|x^{k+1} \in \mathcal{K}_{\mathcal{F}}^{k+1}(\bar{x}^k,\, \bar{z}^k)\}. \tag{12}$$

According to Lemma 3, if for every $k \in \{0,\, \ldots,\, M\}$ at least one $\bar{g}_i^k(x^k,\, z^k),\ i \in \{1,\, \ldots,\, l\}$ is a strongly convex function of $x^k \in X^k$, then for $\lambda \in (0,\, \varepsilon)$ (ε arbitrary small) there exists only one solution of every individual problem (12). Employing, the assertions of Lemma 1 and Lemma 4 we can see that the optimal function of $\varphi_{\mathcal{F}}^{\lambda}(M)$ is (under general assumptions) a Lipschitz function of $\lambda \in \langle 0,\, 1\rangle$ and x. Completed this consideration by scenario approach based on some stability results (for details see e.g. [11]), according to Lemma 2 and Lemma 4 we can obtain a relatively "good" approximation of criteria value functions and approximation of efficient points sets also.

Remark. Evidently, the introduced approach doesn't introduce completely efficient points in the multiobjective, multistage problems. However, we obtain a "reasonable" solution corresponding to the evaluation of individual criteria corresponding to the relevance of "underlying" economic problem. Maybe that some others results can be obtain employing the definition of multistage problems as a problem in some abstract mathematical space. However this consideration is over the possibilities of this contribution.

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References

[1] J. Dupačová: Multistage Stochastic Programs: the State–of–the–Art and Selected Bliography. Kybernetika *31* (1995),2, 151–174.

[2] M. Ehrgott: Multicriteria Optimization. Springer, Berlin 2005.

[3] A. M. Geoffrion: Solving Biccriterion Mathematical Programs. Operations Research *15* (1967), 1, 39–54.

[4] A. M. Geoffrion: Proper Efficiency and the Theory of Vector Maximization. J. Math. Anal. Appl. *22*, (1968), 3, 618–630.

[5] J.-B. Hiriart–Urruty, C. Lamaréchal: Fundamentals of Convex Analysis. Springer, Berlin 2000.

[6] V. Kaňková: Stability in Stochastic Programming–the Case of Unknown Location Parameter. Kybernetika *29* (1993), 1, 97–112.

[7] V. Kaňková: Convexity, Lipschitz Property and Differentiability in Two–Stage Stochastic Nonlinear Programming Problems. Proceedings of the 3rd International Conference on Approximation and Optimization in the Carribean, Puebla, Mexico 1995. Aportaciones Matematicas, Serie Comunicaciones *24* (1999), 135–149.

[8] V. Kaňková: A Note on Multistage Stochastic Programming. In: Proc. 11th Joint Czech–Gerany–Slovak Conference: Mathematical methods in Economy and Industry. University of Technology , Liberec 1998, 45–52.

[9] V. Kaňková: Stochastic Programming Approach to Multiobjective Optimization Problems with Random Elements I. Research Report UTIA 2000, No. 1990.

[10] V. Kaňková: A Note on the Relationship between Strongly Convex Functions and Multiobjective Stochastic Programming Problems. In: Operations Research Proceedings 2004 (Heiin Fleuren, Dick den Hertog and peter Kort, eds.), Springer, Berlin 2005, 305–312.

[11] V. Kaňková: Multistage Stochastic Programs vis Autoregressive Sequences and Individual Probability Constraints. Kybernetika *44* (2008), 2, 151–170 (to appear).

[12] G. Salinetti and R. J.-B. Wets: On the Convergence of Sequences of Convex Sets in Finite Dimension. SIAM Review *21* (1979), 18–33.