Problem of two Managers via Stochastic Programming Problem with Linear Recourse

Vlasta Kaňková

Department of Econometrics
Institute of Information Theory and Automation, AS ČR
Pod Vodárenskou věží 4, 182 08 Praha 8
kankova @utia.cas.cz

Abstract

[1] Stochastic programming problems with linear recourse correspond to many economic problems. It is generally known that these problems are a composition of two (outer and inner) optimization problems. A solution of the outer problem depends on an "underlying" probability measure while a solution of the inner problem depends on the solution of the outer problem and on the random element realization. Evidently, a position and optimal behaviour of two managers can be (in many cases) described by this type of the model in which the optimal behaviour of the main manager is determined by the outer problem while the optimal behaviour of the second manager is described by the inner problem. We focus on an investigation of properties of the inner problem.

Keywords

Stochastic programming problems with linear recourse, stability, empirical estimates, Lipschitz function, strongly convex function, multiobjective optimization problem, efficient points

JEL classification: C 44
AMS classification: 90 C 15

1 Introduction

Let $\xi := \xi(\omega)\,(m \times 1)$ be a random vector defined on a probability space $(\Omega, \mathcal{S}, P)$; $q\,(n_1 \times 1)$, $W(m \times n_1)$, $m \le N_1$, $T(m \times n)$, $m \le n$ be a deterministic vector and matrices. We denote by F^{ξ}, $P_{F^{\xi}}$ the distribution function and the probability measure corresponding to the random vector ξ; $Z_{F^{\xi}}$ the support of $P_{F^{\xi}}$. Let, moreover, $g_0(x, z)$ be a function defined on $R^n \times R^m$; $C \subset R^n$ be a nonempty, closed convex set. Symbols x, y denote n–dimensional decision vector and n_1–dimensional decision vector depending on the decision x and the realization of ξ. (R^n denotes the n–dimensional Euclidean space.)

Stochastic programming problems with linear recourse (in a rather general setting) can be introduced as the following problem:

Find

$$\varphi(F^{\xi}) = \min_{x \in C} \mathsf{E}_{F^{\xi}} \{ g_0(x, \xi) + \min_{\{y \in R^{n_1}:\, Wy = \xi - Tx,\, y \ge 0\}} q^{'} y \}. \tag{1}$$

[1]This research was supported by the grant agency of the Czech Republic under Grants 402/07/1113, 402/05/0115 and 402/06/0990.

(E_{F^ξ} denotes the operator of mathematical expectation corresponding to F^ξ; q' denotes a transposition of the vector q.)

It is known, from the stochastic optimization theory, that usually an "underlying" problem with a random element is in the form:

Find
$$\min_{\{x \in R^n : x \in C, Tx = \xi\}} g_0(x, \xi) \tag{2}$$

and that, moreover, the problem (2) corresponds to the situation when

- a solution x has to be determined without knowledge of the ξ realization,
- it is possible to correct the solution x (after ξ realization) by a new problem:

  Find
  $$\min_{\{y \in R^{n_1} : Wy = \xi - Tx, y \geq 0\}} q' y, \tag{3}$$

- it is reasonable to determine x w.r.t. the mathematical expectation of the objective function
  $$g_0(x, \xi) + \min_{\{y \in R^{n_1} : Wy = \xi - Tx, y \geq 0\}} q' y.$$

A sense of the problem (3) has been originally to compensate the unfulfilled constraints with random element in the problem (2). The problem (1) is then considered se an outer problem and the problem (3) as an inner problem. This approach, to the problem (2), is acceptable for many applications. Let us recall some of them: Financial problems (investment problem, portfolio revision problem, see e.g. [3]), farmer's problem (see e.g. [1]), melt control problem (see c.g. [3]), power–station planning (see e.g. [6], [22]), aircraft allocation problem, transportation problem (see e.g. [16]), location problem (see e.g. [24]), production planning (see e.g. [9], [11]). Evidently, most of this applications are connected with a loss or a profit by some economic activities. Some other applications can be found in [14].

A great attention has been paid (in the stochastic programming literature) to investigate the properties of the problem (1). However, if we employ these results we can obtain the properties of the inner problem (3) and, consequently, a valuable results for some other economic activities (see e.g. [12], [13]). In more details, employing the well–known stability results of the "classical" approach to the problems with recourse (see e.g. [15], [21] and [23]), new stability results for the inner problem can be obtained (for details see ([12], [13]).

In the contribution, first, we shall recall the above mentioned results concerning the inner problem. Furthermore, we shall introduce en economic problem of two managers for which just these new results are very important. At the end, we shall try to employ specifications of this type of the problems to introduce some new approach.

2 Some Definitions and Auxiliary Assertions

First, we introduce a system of the assumptions:

A.1 W is a complete recourse matrix (for every $z \in R^m$ there exists $y \geq 0$ such that $Wy = z$),

A.2 there exists $u \in R^m$ such that $u'W \leq q$,

A.2' there exists a vector $\bar{u} \in R^m$ such that $\bar{u}'W < q$ componentwise,

A.3 there exists $\int\limits_{R^m} \|z\| dP_{F^\xi} < +\infty$ ($\| \cdot \|$ denotes the Euclidean norm in R^m), A.4 the probability measure P_{F^ξ} is absolutely continuous with respect to the Lebesgue measure on R^m,

A.4' A.4 is fulfilled and, moreover, there exists a convex open set $V \subset R^m$, constants $r > 0$, $\rho > 0$ such that a probability density f^ξ corresponding to P_{F^ξ} fulfils the following relation

$$f^\xi(t^{'}) \ge r \quad \text{for all} \quad t^{'} \in R^m \quad \text{with} \quad \mathrm{dist}(t^{'}, V) \le \rho,$$

A.5 a. for every $x \in C$ there exists a finite $\int\limits_{R^m} g_0(x, \xi) dP_{F^\xi}$,

b. for every $z \in Z_{F^\xi}$, $g_0(x, z)$ is a strongly convex function on C,

c. $g_0(x, z)$ is a uniformly continuous function on $C \times Z_{F^\xi}$,

A.6 Z_{F^ξ} is a convex set, $f^\xi(t^{'}) \ge r$ for all $t^{'} \in Z_{F^\xi}$.

(For the definition of a strongly convex function see e.g. [7].)

Remark 1. Evidently, if the assumptions A.4 and A.6 are fulfilled, then the assumption A.4' is fulfilled for every open convex $V \subset Z_{F^\xi}$ such that $V(\varepsilon) \subset Z_{F^\xi}$ for some $\varepsilon > 0$. ($V(\varepsilon)$ denotes an ε–neighbourhood of V.)

Furthermore, we recall the definition of the Wasserstein metric $d_{W_1}(\cdot, \cdot)$ (for more details see e.g. [18]). To this end let $\mathcal{P}(R^m)$ denote the set of all (Borel) probability measures on R^m, $m \ge 1$. If $\mathcal{M}_1(R^m) = \{\nu \in \mathcal{P}(R^m) : \int\limits_{R^m} \|z\| \nu(dz) < \infty\}$ and $\mathcal{D}(\nu, \mu)$ denotes the set of measures in $\mathcal{P}(R^m \times R^m)$ whose marginal measures are ν, μ, then

$$d_{W_1}(\nu, \mu) = \inf\{ \int\limits_{R^m \times R^m} \|z - \bar{z}\| \kappa(dz \times d\bar{z}) : \kappa \in \mathcal{D}(\nu, \mu)\}, \quad \nu, \mu \in \mathcal{M}_1(R^m).$$

If we denote

$$\begin{aligned}
Q(x, \xi) &= \min_{\{y \in R^{n_1} : Wy = \xi - Tx, y \ge 0\}} q^{'} y, \qquad Q^1_{F^\xi}(x) = \mathsf{E}_{F^\xi} Q(x, \xi), \\
\mathcal{Y}(x, \xi) &= \{y \in R^{n_1} : Q(x, \xi) = q^{'} y;\, Wy = \xi - Tx,\, y \ge 0\}, \\
Q_{F^\xi}(x) &= \mathsf{E}_{F^\xi}\{g_0(x, \xi) + \min_{\{y \in R^{n_1} : Wy = \xi - Tx, y \ge 0\}} q^{'} y\}, \\
\mathcal{X}(F^\xi) &= \{x \in C : Q_{F^\xi}(x) = \varphi(F^\xi)\},
\end{aligned} \tag{4}$$

then evidently, $Q(x, \xi)$, $\mathcal{Y}(x, \xi)$ (under the assumptions A.1 and A.2) depend on the vector x only through the value Tx. Consequently, there exists a function $Q_T(t, z)$ defined for $t \in R^m$, $t = Tx$, $x \in C$ such that

$$Q_T(t, z) = Q(x, z) \quad \text{for} \quad x \in C, \quad t = Tx, \quad z \in R^m. \tag{5}$$

A multiobjective deterministic optimization problem can be introduced as the problem:

Find

$$\max f_i(v), \; i = 1, \ldots, l \quad \text{subject to } v \in \mathcal{K}. \tag{6}$$

f_i, $i = 1, \ldots, l$ are functions defined on R^{n_2}, $\mathcal{K} \subset R^{n_2}$ is a nonempty set.

Definition 1. [5] The vector v^* is an efficient solution of the problem (6) if and only if there exists no $v \in \mathcal{K}$ such that $f_i(v) \geq f_i(v^*)$ for $i = 1, \ldots, l$ and such that for at least one i_0 one has $f_{i_0}(v) > f_{i_0}(v^*)$.

Let us consider a special case when

i.1 there exists $d_i \in R^{n_2}$, $i = 1, \ldots, l$ such that $f_i(v) = d_i^{'} v$,

i.2 $\mathcal{K} = \{v \in R^{n_2} | \, Av = h, \, v \geq 0\}$, where $A(m \times n_2)$, $h(m \times 1)$ are a deterministic matrix and a deterministic vector.

We recall the Theorem of Issermann 1974 (for more details see e.g. [4]).

Theorem 1. Let the assumptions i.1 and i.2 are fulfilled. A feasible $v^0 \in \mathcal{K}$ be an efficient solution of the problem (6) if and only if there exists a $\lambda = (\lambda_1, \ldots, \lambda_l) \in R^l$, $\lambda > 0$, $i = 1, \ldots, l$ such that

$$\sum_{i=1}^{l} \lambda_i d_i^{'} v^0 \geq \sum_{i=1}^{l} \lambda_i d_i^{'} v \quad \text{for every} \quad v \in \mathcal{K}.$$

Remark 2. The definition of the multiobjective problem has been recalled for the maximization. However, there exists a well–known relations between maximization and minimization problems and, moreover, the problems of two managers shall be introduced as problem of maximization. Consequently, we have not transformed this definition to minimization.

3 Stability Results of Inner Problem

In this section we recall results dealing with stability and statistical estimates in the case of the inner problem. First, we shall deal with a "deterministic" case. In particular, we shall deal with the case when the distribution function G^{ξ} is a deterministic approximation of F^{ξ}. Such situation can happen, for example, when the second manager has some information about theoretical F^{ξ}; however, an actual distribution function is a little contaminated or if it is known only that F^{ξ} belongs to some family of distribution functions.

Theorem 2. [13] Let the assumptions A.1, A.2', A.3 and A.4' be fulfilled, C be a polyhedral set, $\mathcal{X}(F^{\xi})$ be a nonempty bounded set. If

1. there exists an n–dimensional deterministic vector c such that $g_0(x, z) = c^{'} x$,
2. $x(F^{\xi}) \in \mathcal{X}(F^{\xi}), \quad T\mathcal{X}(F^{\xi}) \subset V \quad$ for some V fulfilling A.4',

then there exist constants $L_Q^1 > 0$, $d_Q^1 > 0$ such that for $P_{G^{\xi}} \in \mathcal{M}_1(R^m)$, $d_{W_1}(P_{F^{\xi}}, P_{G^{\xi}}) \leq d_Q^1$ and $x(G^{\xi}) \in \mathcal{X}(G^{\xi})$ fulfilling the relation

$$|\, Q(x(F^{\xi}), \xi) - Q(x(G^{\xi}), \xi) \,| \leq L_Q^1 [d_{W_1}(P_{F^{\xi}}, P_{G^{\xi}})]^{\frac{1}{2}}, \quad \xi \in R^m. \tag{7}$$

Remark 3. If we replace the assumption 2 of Theorem 2 by the assumption

3. $T\mathcal{X}(F^{\xi}) \subset V \quad$ for some V fulfilling A.4',

then there exist constants $L_Q^1 > 0$, $d_Q^1 > 0$ such that for $P_{G^{\xi}} \in \mathcal{M}_1(R^m)$, $d_{W_1}(P_{F^{\xi}}, P_{G^{\xi}}) \leq d_Q^1$ and $x(G^{\xi}) \in \mathcal{X}(G^{\xi})$ there exists $x(F^{\xi}) \in \mathcal{X}(F^{\xi})$ fulfilling the relation (7).

However, it happen rather often that the decision of the inner in (1) has to been taken on the basis of an experience, while the decision of the outer problem can be determined on the basis knowledge of F^{ξ}. In particular, there exist some data $z^1, \ldots, z^N$ that can be supposed to be realizations of independent identically distributed random elements $\xi^1, \ldots, \xi^N$, $N = 1, 2, \ldots$ corresponding to the distribution function F^{ξ}. If we denote by the symbol F^{ξ}_N an empirical distribution function determined by $\xi^1, \ldots, \xi^N$, then we can introduce two following assertions.

Corollary 1. Let the assumptions of Theorem 2 be fulfilled. If $\xi, \xi^1, \ldots, \xi^N, \ldots$ are stochastically independent m–dimensional random vectors, corresponding to the distribution function F^{ξ}; F^{ξ}_N is an empirical distribution function determined by $\{\xi^k\}_{k=1}^N$, $N = 1, 2, \ldots$, then there exist $x(F^{\xi}_N) \in \mathcal{X}(F^{\xi}_N)$, $N = 1, \ldots$ such that

$$P\{Q(x(F^{\xi}_N), \xi) \longrightarrow_{(N \longrightarrow \infty)} Q(x(F^{\xi}), \xi)\} = 1.$$

Proof. First, it follows from the assumptions of Theorem 2 that there exist constants $L^1_Q > 0$, $d^1_Q > 0$ such that for $P_{G^{\xi}} \in \mathcal{M}_1(R^m)$, $d_{W_1}(P_{F^{\xi}}, P_{G^{\xi}}) \leq d^1_Q$ there exists $x(G^{\xi}) \in \mathcal{X}(G^{\xi})$ fulfilling the relation

$$|\, Q(x(F^{\xi}), \xi) - Q(x(G^{\xi}), \xi) \,| \leq L^1_Q [d_{W_1}(P_{F^{\xi}}, P_{G^{\xi}})]^{\frac{1}{2}}, \quad \xi \in R^m. \tag{8}$$

Replacing G^{ξ} by F^{ξ}_N we can employ the assertion proven in [17] (see also [20]) according that

$$d_{W_1}(P_{F^{\xi}}, P_{F^{\xi}_N}) \longrightarrow_{(N \longrightarrow \infty)} 0 \quad a.s.$$

However, it follows from the last two relations that for every $z \in Z_{F^{\xi}}$

$$P\{Q(x(F^{\xi}_N), z) \longrightarrow_{(N \longrightarrow \infty)} Q(x(F^{\xi}), z)\} = 1.$$

Employing furthermore the properties of the conditional probability measure we can see that the assertion of Corollary is valid.

Theorem 3. [13] Let $Z_{F^{\xi}}$ be a compact set, C be convex and compact set. If

1. the assumptions A.1, A.2, A.3, A.4 and A.5 are fulfilled,
2. $\xi, \xi^1, \ldots, \xi^N, \ldots$ are stochastically independent, corresponding to the distribution function F^{ξ}; F^{ξ}_N is an empirical distribution function determined by $\{\xi^k\}_{k=1}^N$, $N = 1, 2, \ldots$,

then

$$P\{Q(x(F^{\xi}_N), \xi) \longrightarrow_{(N \longrightarrow \infty)} Q(x(F^{\xi}), \xi)\} = 1.$$

4 Mathematical Analysis

It is known from the theory of linear programming that the assumption A.1 and A.2 guarantee the fulfilling of the relation

$$Q(x, \xi) \in (-\infty, +\infty) \quad \text{for} \quad x \in C, \ \xi \in Z_{F^{\xi}} \tag{9}$$

and that the existence of a finite $\mathsf{E}_{F^{\xi}} Q(x, \xi)$ is guaranteed by the assumptions A.1, A.2 and by the existence of finite $\mathsf{E}_{F^{\xi}} \xi$ (for more details see e.g. [8]).

A fulfilling of the assumption A.2 depends only on the (given) deterministic vector q and the (given) deterministic matrix W, however, not on $\xi \in Z_{F^{\xi}}$, $x \in C$. Consequently, A.2 can be verified by some deterministic technique. A condition, under which, A.1 is fulfilled is introduced in [8] or [9].

Lemma 1. [9] $W(m \times n_1,\, m \leq n_1)$ matrix is a complete recourse matrix if and only if

- it has $rank(W) = m$ and,
- assuming without loss of generality that the first m columns $W_1, W_2, \ldots, W_m$ are linearly independent, the linear constraints

$$\begin{array}{lcl} Wy & = & 0 \\ y_i & \geq & 1, \quad i = 1, \ldots, m, \\ y & \geq & 0 \end{array}$$

  has a feasible solution.

Let us recall (according to the definition; see e.g. [8]) that W is a complete recourse matrix, if the set
$$\{y \in R^{n_1} : \; Wy = z,\, y \geq 0\}$$
is nonempty for every $z \in R^m$. However it is easy to see that sometimes there can exist a set $\bar{Z} \subset R^m$, $\bar{Z} \neq R^m$ such that
$$\{y \in R^{n_1} : \, Wy = z,\, y \geq 0\}$$
is nonempty only for $z \in \bar{Z}$ and that simultaneously the set $\bar{Z}$ covers $\{z \in R^m : \, z = \xi - Tx,\, \xi \in Z_{F^{\xi}},\, x \in C\}$.

5 Problem of Two Managers

To introduce the "Problem of Two Managers", first, let us consider an example of a "classical" production planning problem in which quantity b of raw materials is random (say $b := \xi$). In the case of a deterministic technology matrix T and a deterministic cost vector c we obtain (under some additional assumptions) the "underlying" linear programming problem with a random element ξ in the form:

Find
$$\max\{c^{'}x \,|\, x \in C,\, Tx \leq \xi, \; x \geq 0\},$$
where $C \subset R^n$ is usually a polyhedral set, corresponding to the constraints not depending on the random element.

Furthermore, let us complete the above mentioned "classical" example by a special situation under which the unutilized raw materials can be employed for a next production and, moreover, this second production can be organized by a relatively independent manager (maybe in another locality). Evidently, the aim of the second manager is to maximize profit from this additional production. We can suppose that the additional problem can be written in the form:

Find
$$\max_{\{y \in R^{n_1} : \, Wy \leq \xi - Tx,\, y \geq 0\}} q^{'}y.$$
Supposing that the main manager has also a profit from the inner problem, his or her decision is determined by the problem:

Find
$$\max_{x \in C} \mathsf{E}_{F^b}\{c^{'}x + K^* \max_{\{y \in R^{n_1} : \, Wy \leq \xi - Tx,\, y \geq 0\}} q^{'}y\} \quad \text{for some} \quad K^* \in \langle 0,\, 1\rangle. \tag{10}$$
The matrices T, W and the vector q are supposed to be (in this special case) deterministic of suitable types; $C \subset R^n$ is a nonempty (maybe polyhedral) set.

Of course, the aim of the additional manager is to obtain a maximal profit. In the case, when the second manager knows exactly the probability measure $P_{F^{\xi}}$ and, moreover, when he can suppose that the decision of the main manager $x(F^{\xi})$ corresponds to the problem (10), then the optimal decision of the second manager is determined by the problem:

Find

$$\max_{\{y \in R^{n_1}: \, Wy \le \xi - Tx(F^{\xi}), \, y \ge 0\}} q^{'} y. \tag{11}$$

Since the additional manager knows mostly F^{ξ} only approximately, he or her is interesting in the stability (considered w.r.t. probability measure space) of the optimal value of the last problem. In particular, the second manager needs to know profit changes if F^{ξ} will be replaced by a "near" G^{ξ}. Of course, to this end the additional manger can employ the results recalled in the section 3.

We have supposed that the constraints set of the inner problem is in the form

$$\{y \in R^{n_1} : \, Wy \le \xi - Tx, \, y \ge 0\}, \tag{12}$$

where the matrix W is a technology matrix corresponding to the to new production (consequently, the elements of W can be supposed to be nonnegative). Evidently, in this case, the last set given by the relation (12) is nonempty a.s. whenever $\xi - Tx \ge 0$ for every $\xi \in Z_{F^{\xi}}$, $x \in C$. However, it is surely reasonable to assume that the second manager has possibility to win some materials from an additional source. Of course, these possibilities are limited. Consequently, it is (generally) reasonable to assume that the constraints set corresponding to the inner problem is in the form:

$$\{\bar{y} \in R^{n_2} : \, \bar{W}\bar{y} \le h(\xi - Tx), \, y \ge 0\}, \tag{13}$$

or equivalently the constraints set (in a rather general case) of the inner problem can be defined by a system of inequalities:

$$\begin{array}{lll}
w_{1,\,1}\bar{y}_1 + \ldots + w_{1,\,n_1}\bar{y}_{n_1} & +w_{1,\,n_1+1}\bar{y}_{n_1+1} + \ldots + w_{1,\,n_2}\bar{y}_{n_2} & \le \xi_1 - t_1 x_1, \\
\vdots & & \\
w_{m,\,1}\bar{y}_m + \ldots + w_{m,\,n_1}\bar{y}_{n_1} + & +w_{m,\,n_1+1}\bar{y}_{n_1+1} + \ldots + w_{m,\,n_2}\bar{y}_{n_2} & \le \xi_m - t_m x_1, \\
 & w_{m+1,\,n_1+1}\bar{y}_{n_1+1} + \ldots + w_{m+1,\,n_2}\bar{y}_{n_2} & \le \bar{h}_{m+1} \\
\vdots & & \\
 & +w_{m_1,\,n_1+1}\bar{y}_{n_1+1} + \ldots + w_{m_1,\,n_2}\bar{y}_{n_2} & \le \bar{h}_{m_1}, \\
 & \bar{y}_1, \, \ldots, \bar{y}_{n_1}, \, \ldots, \, \bar{y}_{n_2} & \ge 0,
\end{array} \tag{14}$$

where the first n_1 components of $\bar{y}$ corresponds to y; $h(\xi - Tx) = (h_1(\xi - Tx), \, \ldots, \, h_{m_1}(\xi - Tx))$; $h_1(\xi - Tx) = \xi_1 - t_1 x, \, \ldots, \, h_m(\xi - Tx) = \xi_m - t_m x, \, h_{m+1}(\xi - Tx) = \bar{h}_{m+1}, \, \ldots, \, \bar{h}_{m_1}$, $\bar{h}_{m+1}, \, \ldots, \, \bar{h}_{m_1}$ corresponds to the limits for additional activities of the second manger, $\xi = (\xi_1, \, \ldots, \, \xi_m)$, $t_i, = 1, \ldots, m$ are rows of the matrix T. The matrix $\bar{W}$ is completed matrix W just corresponding to the additional activities of the second manger.

According to this new situation the problems (10), (11) have to be replaced by the problem:

Find

$$\max_{x \in C} \mathsf{E}_{F^{\xi}}\{c^{'}x + K^{*} \max_{\{\bar{y} \in R^{n_2}: \, \bar{W}\bar{y} \le h(\xi - Tx), \, \bar{y} \ge 0\}} q^{'}\bar{y}\} \quad \text{for some} \quad K^{*} \in \langle 0, \, 1\rangle; \, n_2 \ge n_1 \tag{15}$$

with the solution denoted by $\bar{x}(F^{\xi})$, and with the problem:

Find

$$\max_{\{\bar{y} \in R^{n_2} \,|\, \bar{W}\bar{y} \leq h(\xi - T\bar{x}(F^{\xi})),\, \bar{y} \geq 0\}} q^{'}\bar{y}. \tag{16}$$

Evidently, to be two last problems solvable it is necessary to be the set

$$\{\bar{y} \in R^{n_2} |\, \bar{W}\bar{y} \,\leq\, h(b - T\bar{x}(F^{\xi})),\, \bar{y} \geq 0\}$$

nonempty for $\xi \in Z_{F^{\xi}}$, $x \in C$. Before discussing this problem generally, let us introduce a simple example.

Example We consider the case in which the original constraints set corresponding to the inner problem are given by a system of inequalities:

$$\begin{array}{lcl} 2y_1 + 3y_2 & \leq & \xi_1 - t_1 x, \\ y_1 + y_2 & \leq & \xi_2 - t_2 x, \quad y_1,\, y_2 \geq 0, \end{array}$$

where $y = (y_1,\, y_2)$, $\xi = (\xi_1,\, \xi_2)$, $t_1,\, t_2$ are rows of the matrix T. Evidently, in this case $m = 2$, $n_1 = 2$ and the constraints set is nonempty (for every $x \in C$) a.s. if and only if $\xi - Tx \geq 0$ a.s. for every $x \in C$. However, if we can assume that (in this special case) the constraints set (14) is given by a system of inequalities

$$\begin{array}{lcl} 2\bar{y}_1 + 3\bar{y}_2 - \bar{y}_3 & \leq & \xi_1 - t_1 x, \\ \bar{y}_1 + \bar{y}_2 - \bar{y}_4 & \leq & \xi_2 - t_2 x, \\ \bar{y}_3 + \bar{y}_4 & \leq & \bar{h}_3, \qquad \bar{y}_1,\, \bar{y}_2,\, \bar{y}_3,\, \bar{y}_4 \geq 0, \end{array}$$

then a set of solutions of the last system is (for every $x \in C$) nonempty a.s. if

$$t_1 x - \xi_1 + t_2 x - \xi_2 \leq \bar{h}_3 \quad a.s. \quad \text{for every} \quad x \in C.$$

Evidently, $\bar{y}_3$, $\bar{y}_4$ corresponds to the amounts that the second manager wins from some additional source and $\bar{h}_3$ corresponds to the upper bound that can be utilized for these additional source.

6 Discussion

To apply the assertions of section 3 to the problems of two managers, first, the constrains inequality

$$\bar{W}\bar{y} \,\leq\, h(\xi - Tx), \quad \bar{y} \geq 0,$$

must be transformed to a system of equations. Of course, it is known from the theory of linear programming, that such transformation is simple. However, it follows from a simple example given in section 5 that the constraints set (13) can be a.s. nonempty also in the case when $\bar{W}$ does not correspond to a complete recourse matrix. It means, that to be the constraints set (13) nonempty it is not necessary to be the assumption A.1 fulfilled. Let us try, furthermore, to analyze the constraints set of the inner problem in the form of inequalities. To this end, first, let us define the assumption A.1' by

A.1' for every $\xi \in Z_{F^{\xi}}$, $x \in C$ there exists a solution of the system (14).

Evidently, the assumption A.1 can be replaced by A.1'. Furthermore, let us define the multiobjective problem:

Find

$$\max\, t_i x - \xi_i, \quad i = 1, \ldots, m \quad \text{subject to} \quad \xi = (\xi_1, \ldots, \xi_m) \in Z_{F^\xi},\, x \in C. \tag{17}$$

If we denote by the symbol $\mathcal{Z}(Z_{F^\xi}, C) \subset R^m \times R^n$ the set of efficient points $(\bar{\xi}, \bar{x})$ of the problem (17). Then evidently, the following assertion can be introduced.

Proposition 1. If for every $(\bar{\xi}, \bar{x}) \in \mathcal{Z}(Z_{F^\xi}, C)$ the system of inequalities (14) is feasible, then the assumption A.1' is fulfilled for every $\xi \in Z_{F^\xi}$, $x \in C$.

Proof The proof of lemma 2 is very simple. It follows immediately from the results presented e.g. in [19].

Evidently, in the case when the the Z_{F^ξ} is given by a system of linear inequalities as well as the set C is also given by a system of linear inequalities, then the set $\mathcal{Z}(Z_{F^\xi}, C)$ can be obtain by a method of parametric linear programming. Moreover, for our purpose it is sufficient to look for a solution of this parametric linear programming that correspond to some basis solution. A modified simplex algorithm to determine them is introduced e.g. in [4].

References

[1] J-R. Birge and F. Louveuac: Introduction to Stochastic Programming. Springer, Berlin 1997.

[2] Caballero, R., Cerdá, E., Muňoz, M. M., Rey, L., Stancu–Minasian, I. M. (2001): Efficient Solution Concepts and their Relations in Stochastic Multiobjective Programming. J. Opt. Theory and Appl. **110**, 1, 53–74.

[3] J. Dupačová, J. Hurt and J. Štěpán: Stochastic Modelling in Economics and Dinance. Kluwer, London 2002.

[4] M. Ehrgott: Multicriteria Optimization. Springer, Berlin 2005.

[5] A. M. Geoffrion: Proper Efficiency and the Theory of Vector Maximization. J. Math. Anal. Appl., **22** (1968), 3, 618–630.

[6] JL. Higle and S. Sen: Stochastic Decomposition. A Statistical Method for Large Scale Stochastic Linear Programming. Kluwer, London 1996.

[7] J-B. Hiriart-Urruty and C. Lemaréchal: Fundamentals of Convex Analysis. Springer, Berlin 2001.

[8] P. Kall: Stochastic Linear Programming. Springer, Berlin 1976.

[9] P. Kall and S. W. Wallace: Stochastic Programming. Willey, Chichester 1994.

[10] V. Kaňková: Optimal Solution of Stochastic Optimization Problem with Unknown Parameters. In: Trans. 7th Prague Conf. 1974, Academia, Prague 1977, 349–353.

[11] V. Kaňková: Differentiability of the Optimalized Function in a Two–Stage Stochastic Nonlinear Programming Problems (in Czech). Ekononicko–matematický obzor *14* (1978), 5, 329–349.

[12] V. Kaňková: On Stability of Stochastic Programming Problems with Linear Recourse. In: Mathematical Methods in Economics, Hradec Králové 2005(H. Skalská, ed.), Faculty of Informatics and Management University of Hradec Králové, Hradec Králové 2005. 188–195.

[13] V. Kaňková: Stochastic Programming Problems with Linear Rcourse; Applications to Problems of Two Managers. In: Proceedings of Mathematical Methods in Economics 2006, (L. Likáš, ed.0, TYPOS–Digital Print. spol. s.r.o.. Pilsen 2006, 283–288.

[14] A. J. King: Stochastic Programming Problems. Examples from the Literarure. In: Numerical Techniques for Stochastic Optimization (Yu. Ermoliev and E. J B Wets, eds.), Springer, Berlin 1988, 543–567.

[15] J. Mayer: Stochastic Linear Programming Algorithms (A Comparison Based on a Model Management System). Gordon and Breach Science Publishers, Amsterdam 1998.

[16] W B. Powel and H. Topaloglu: Stochastic Programming and Logistics. In: Stochastic Programming. Handbooks in Operations Research and Management Science (A. Ruszczynski and A. Shapiro, eds.), Vol 10, Elsevier, Amsterdam 2003.

[17] S. T. Rachev: The Monge–Kantoribich Mass Transference Problem and Its Stochastic Applications. Theory Prob. Appl. *29* (1984), 647–676.

[18] S. T. Rachev: Probability Metrics and the Stability of Stochastic Models. Wiley, Chichester 1991.

[19] J. Rohn: Systems of Interval Linear Equations and Inequalities (Rectangular Case). Technical Report No. 875 (2002), Institute of Computer Science AS CR.

[20] W. Römisch and R. Schulz: Stability Analysis for Stochastic Programs. Annals of Operations Research *30* (1991), 241–266.

[21] W. Römisch and R. Schulz: Stability of Solutions for Stochastic Programs with Complete Recourse. Math. Oper. Res. *18* (1993), 590–609.

[22] W. Römisch and R. Schulz: A Simple Recourse Model for Power Dispatch under Uncertain Demand. Annals of Operations Research *59* (1995), 135–164.

[23] R. Schulz: Strong Convexity in Stochastic Programs with Complete Recourse. J. Comput. Math. *56* (1994), 3–22.

[24] S W. Wallace: A Two–Stage Stochastic Facility Location Problem with Time–Dependent Supply. In: Numerical Techniques for Stochastic Optimization (Yu. Ermoliev and E. J B Wets, eds.), Springer, Berlin 1988, 515–542.