Wavelets and Estimation of the Persistence in the Heterogeneous Agents Model

Lukas Vacha and Miloslav Vosvrda

Institute of Information Theory and Automation, Academy of Sciences of the Czech Republic, Prague, vachal@utia.cas.cz, vosvrda@utia.cas.cz

1 Introduction

Modern finance is undergoing an important change in the perception of economic agents, i.e., from a representative, rational agent approach towards a behavioral, agent-based approach with markets boundedly rational, where heterogeneous agents apply rule of thumb strategies. The traditional approach, rested on simple analytically tractable models with a representative, perfectly rational agent and mathematics as the main tool of analysis. The new behavioral approach fits much better with agent-based simulation models and computational and numerical methods have become an important tool of analysis, [5]. The new behavioral, heterogeneous agents approach challenges the traditional representative rational agent framework. Heterogeneity in expectations can lead to market instability and complicated price dynamics. Prices are driven by endogenous market forces. Typically, in the heterogeneous agents model (HAM), two types of agents are distinguished: fundamentalists and chartists. Fundamentalists base their expectations about future asset prices and their trading strategies on market fundamentals and economic factors, such as dividends, earnings, macroeconomic growth, unemployment rates, etc. Chartists or technical analysts try to extrapolate observed price patterns, such as trends, and exploit these patterns in their investment decisions. One such model was developed by Brock and Hommes in 1998, [3]. In our early work we focused on a simple HAM with two or four types of beliefs, [15], [16], [17], [18]. These beliefs were fixed for all our simulations. In our previous papers, [16], [18], we introduced a memory and some learning schemes to the model of Brock and Hommes. In this paper we use the core of the Brock and Hommes model introducing further extensions, such as stochastic formation of beliefs and parameters including the memory length. Another extension is the application of the Worst Out Algorithm (WOA). In [14] we showed how memory length distribution in the agents' performance measure affects the persistence of the generated price time series. Our motivation is to trace the memory length in the price time series with different replacement ratios of the improved WOA. When used in the HAM, the WOA should increase the persistence of returns. Wavelet analysis is a convenient tool that can be used to detect these events and especially to identify activities at various scales of the price time series. Wavelets are more useful for frequency detection in price time series than Fourier analysis because they are better in identifying price changes in the mood of the financial market over time. The wavelet analysis uses the time-scale domain instead of the time-frequency domain. A typical stylized fact for financial markets is the existence of clusters of both high positive returns and low negative returns in the realizations of the price time series. We can retrospectively analyze which part of the set of trading strategies was used on the financial market and we can estimate their statistical properties.

The first part of the paper concerns a heterogeneous agent model, which is an extension of the Brock and Hommes model, [3]. The second part briefly introduces wavelets. Part three deals with the implementation of the WOA into HAM. The last part of the paper investigates qualitative changes in the financial market structure represented by the HAM under the presence of sentiment persistence and sentiment change. Sentiment change is simulated by changing the mean of the trend g_h for the new strategies that enter the market via the WOA.

2 A Heterogeneous Agent Model

Capital markets are perceived as systems of interacting agents who immediately process new information. Agents adapt their predictions by choosing from a limited number of beliefs (predictors or trading strategies). Each belief is appreciated by a performance measure. Agents on the capital market use this performance measure to make a rational choice which depends on the heterogeneity in agent information and subsequent decisions of the agent either as a fundamentalist or as a chartist, [7], [8].

The model presents a form of evolutionary dynamics, called Adaptive Belief System, in a simple present discounted value (PDV) pricing model. The first part of this model was elaborated by Brock and Hommes, [3].

Consider an asset pricing model with one risky asset and one risk free asset. Let p_t denote the price (ex dividend) per share of the risky asset at time t, and let $\{\mathbf{y}_t\}$ be a stochastic dividend process of the risky asset. The supply of the risk free asset is perfectly elastic at the gross risk free interest rate R, which is equal to $1 + r$, where r is the interest rate. For the dynamics of the wealth we can write

$$\mathbf{W}_{t+1} = RW_t + (\mathbf{p}_{t+1} + \mathbf{y}_{t+1} - Rp_t)\, z_t, \tag{1}$$

where z_t denotes the number of shares of the asset purchased at time t. Random variables at time t are in bold. E_t and V_t are the conditional expectation and conditional variance operators based on the set of publicly available information consisting of past prices and dividends, i.e., on the information set $\mathcal{F}_t = \{p_t, p_{t-1}, \ldots; y_t, y_{t-1}, \ldots\}$. Let $E_{h,t}$ and $V_{h,t}$ denote beliefs (or forecast) of type h investor about the conditional expectation and conditional variance. Investors are supposed to be a myopic mean-variance maximizers so that the demand $z_{h,t}$ for risky asset is obtained by a solving of the following criterion

$$\max_{z_{h,t}} \left\{ E_{h,t}\left[\mathbf{W}_{t+1}\right] - \frac{a}{2} V_{h,t}\left[\mathbf{W}_{t+1}\right] \right\}, \tag{2}$$

where the risk aversion coefficient, $a > 0$, is assumed to be the same for all traders. Thus the demand $z_{h,t}$ of type h for risky asset has the following form

$$E_{h,t}\left[\mathbf{p}_{t+1} + \mathbf{y}_{t+1} - Rp_t\right] - a\sigma^2 \mathbf{z}_{h,t} = 0, \tag{3}$$

$$z_{h,t} = \frac{E_{h,t}\left[\mathbf{p}_{t+1} + \mathbf{y}_{t+1} - Rp_t\right]}{a\sigma^2}, \tag{4}$$

assuming that the conditional variance of excess returns is a constant for all investor types

$$V_{h,t}\left(\mathbf{p}_{t+1} + \mathbf{y}_{t+1} - Rp_t\right) = \sigma_h^2 = \sigma^2. \tag{5}$$

Let z_t^s be the supply of outside risky shares. Let $n_{h,t}$ be a fraction of type h investor at time t. The equilibrium of the demand and supply is

$$\sum_{h=1}^{H} n_{h,t} \left\{ \frac{E_{h,t}\left[\mathbf{p}_{t+1} + \mathbf{y}_{t+1} - Rp_t\right]}{a\sigma^2} \right\} = z_t^s, \tag{6}$$

where H is the number of different investor types.

For the special case of zero supply, i.e., $z_t^s = 0$, for all t, we get a benchmark notion of the rational expectations fundamental solution p_t^*. We can thus write

$$Rp_t^* = E_t\left[p_{t+1}^* + \mathbf{y}_{t+1}\right]. \tag{7}$$

In the case when the dividend process $\{\mathbf{y}_t\}$ is an independent, identically-distributed (IID) process, $E_t\{\mathbf{y}_{t+1}\} = \bar{y}$, which is a constant. In the special case when $\{\mathbf{y}_t\}$ is IID we have a standard notion of fundamental. Let $p_t^* = \bar{p}$, where $\bar{p}$ solves

$$R\bar{p} = \bar{p} + \bar{y}. \tag{8}$$

Equation (7) has infinitely many solutions, but only one constant solution $\bar{p} = \bar{y}/\left(R-1\right)$ of (8) that satisfies the transversality condition

$$\lim_{t\to\infty} \frac{E_t\left[p_t\right]}{R^t} = 0. \tag{9}$$

For our purpose, it is convenient to work with a deviation x_t from the benchmark fundamental price p_t^*, i.e., $x_t = p_t - p_t^*$.

In the case of zero supply of outside shares , i.e., $z_t^s = 0$, the market equilibrium is as follows

$$Rp_t = \sum_{h=1}^{H} n_{h,t}\left\{E_{h,t}\left[\mathbf{p}_{t+1} + \mathbf{y}_{t+1}\right]\right\}. \tag{10}$$

Now, let us make the following assumptions:

A1)

$$E_{h,t}\left[\mathbf{y}_{t+1}\right] = E_t\left[\mathbf{y}_{t+1}\right], \qquad h = 1, ..., H, \tag{11}$$

A2) all forecasts $E_{h,t}\left[\mathbf{p}_{t+1}\right]$ have the following form

$$E_{h,t}\left[\mathbf{p}_{t+1}\right] = E_t\left[p_t^*\right] + f_h\left(x_{t-1}, ..., x_{t-m_h}\right). \tag{12}$$

Each forecast $f_h\left(x_{t-1}, ..., x_{t-m_h}\right)$ represents a model of the capital market, for which trader type h believes that prices deviate from the fundamental price. The number of lags m_h denotes the size of the information set of trader type h, which is the memory length, i.e., the investment horizon of trader type h.

Let us concentrate on the evolutionary dynamics of the fractions $n_{h,t}$ of different h-investor types, i.e.

$$Rx_t = \sum_{h=1}^{H} n_{h,t-1} f_h\left(x_{t-1}, ..., x_{t-m_h}\right) \equiv \sum_{h=1}^{H} n_{h,t-1} f_{h,t}^{m_h}, \tag{13}$$

where $n_{h,t-1}$ denotes the fraction of investor type h at the beginning of period t, before the equilibrium price x_t has been observed. The realized excess return in period t over period $t+1$ denoted, $\mathbf{Z}_{t+1}$ is computed as

$$\mathbf{Z}_{t+1} = \mathbf{p}_{t+1} + \mathbf{y}_{t+1} - Rp_t. \tag{14}$$

Now we need a performance measure generated by forecasts $f_{h,t}^{m_h}$. Let the performance measure $\pi_{h,t}$ be defined by

$$\pi_{h,t} = E_t\left[\frac{\mathbf{Z}_{t+1}\rho_{h,t}}{a\sigma^2}\right], \tag{15}$$

where

$$\rho_{h,t} = E_{h,t}\left[\mathbf{Z}_{t+1}\right] = f_{h,t}^{m_h} - Rx_t. \tag{16}$$

So the π-performance is given by the realized performance for the h-investor. Let the updated fractions $n_{h,t}$ be given by the discrete choice probability

$$n_{h,t} = \exp\left(\beta \frac{1}{m_h} \sum_{k=1}^{m_h} \pi_{h,t-k}\right) / Y_t, \tag{17}$$

where

$$Y_t = \sum_{j=1}^{H} \exp\left(\beta \frac{1}{m_h} \sum_{k=1}^{m_h} \pi_{h,t-k}\right). \tag{18}$$

When $m_h = 1$ is the same for all types, we get the Brock and Hommes model, [3]. If $b_h = 0$, the investor is called a pure trend chaser if $g_h > 0$ and a contrarian if $g_h < 0$. If $g_h = 0$, the investor is called an upward (downward) biased if $b_h > 0$ ($b_h < 0$). In the special case $g_h = b_h = 0$, the investor is called fundamentalist, i.e., the investor believes that prices return to their fundamental values. A fundamentalist's strategy is based on all past prices and dividends in his information set, but he does not know the fractions $n_{h,t}$ of the other belief types.

2.1 Stochastic Beliefs Formation

To demonstrate the dynamics of the investor types we simulate the trend g_h and the bias b_h of type h trader as realizations from the normal distributions, $N(0, 0.16)$ and $N(0, 0.09)$. The memory length m_h of the trading strategy $f_{h,t}$ is a realization from the uniform distribution, $U(1, 100)$.

3 Wavelets

The traditional approach to the application of spectral techniques to economic and financial data, such as the Fourier transform, has focused on the identification of frequency components. The key problem with the application of the Fourier transform to financial data is that the highlighted spectrum of a signal is global rather than being localized. Conversely, the wavelet transform offers localized frequency decomposition. It also provides information what frequency components are present and where they are occurring. As a result wavelets have significant advantages over a basic Fourier analysis when the object under study is non-stationary and non-homogeneous [1].

A very important attribute of wavelets is that they are zero phase, not very smooth, and that they have very few vanishing moments, which makes then suitable for detecting regime shifts and discontinuities. Wavelet analysis provides an alternative and preferable solution, especially in financial market time series, since it allows the degree of localization to be automatically and appropriately adapted, see [4], [1], [12].

3.1 Multiresolution Analysis

The main feature of wavelet analysis is the possibility to decompose a time series into its constituent multiresolution components. The multiresolution analysis (MRA) of a time series $x(t)$ is given by the following expression

$$x\left(t\right) = S_J + D_J + D_{J-1} + \ldots + D_j + \ldots + D_1, \tag{19}$$

where

$$S_J = \sum_k s_{J,k} \phi_{J,k}\left(t\right), \tag{20}$$

$$D_j = \sum_k d_{j,k} \psi_{j,k}\left(t\right). \tag{21}$$

S_J denotes the smooth and $D_J, D_{J-1}, \ldots, D_j, \ldots, D_1$ are the details of the signal. The sequence of terms S_J and $D_J, D_{J-1}, \ldots, D_j, \ldots, D_1$ represents a set of signal components that provide representations of the signal at the different resolution levels $j = 1, 2, \ldots, J$. For a more detailed treatment see [1] and [12]. For the MRA we use the Daubechies (4) wavelets.

3.2 Wavelet Variance Analysis

The wavelet variance (or energy) decomposes the variance of stochastic processes on scale basis and hence is important in financial time series processing. The wavelet variance is a succinct alternative to the power spectrum based on the Fourier transform and is often easier to interpret than the frequency-based spectrum [12]. Such scale decomposition in time helps us track the evolution of the energy contribution at various scales, which is related to traders' investment horizons.

We also use wavelet variance decomposition to get values of the aggregate energy at scales for all simulation periods. We depict these values on a pie graph, see Figure 4.

4 Learning with the WOA

This section implements wavelets and the WOA into HAM modeling. As mentioned above, wavelet analysis is a very convenient tool for activity detection at various scales of the price time series. It is also more suitable for frequency detection in price time series than Fourier analysis because price changes in the financial market are better detected in time.

4.1 The Worst Out Algorithm

In the heterogeneous agents model presented in the previous section it is possible to set different degrees of agents' heterogeneity, such as trend, bias and memory. Due to these features the model more closely replicates the events of a real financial market, see [14]. The WOA adds behavioral aspect to the model and we obtain outcomes that are closer to stylized facts.

The WOA periodically replaces the trading strategies that have the lowest performance level of the strategies presented on the market by new ones. Without loss of generality, this algorithm is constructed so that it evaluates and ranks the performance of all fifteen strategies in the market after every 40 iterations in descending order and the last strategy and/or several strategies are replaced by a new strategy and/or several strategies. The new strategies that enter the financial market have the same stochastic parameters as the initial set of strategies. The use of the WOA in simulations shows that the implementation of the WOA can significantly change price time series parameters and modify the behavior of investors on the financial market.

4.2 Simulations

We have used the WOA in an effort to gain a better understanding of the evolution of financial market dynamics. We compare eight cases that differ in the number of replaced trading strategies, i.e., the replacement ratio. Our motivation is to trace the memory structure in the price time series with different replacement ratios of the updated version of the WOA. The WOA should increase the persistence of the returns.

The algorithm replaces zero, one, two, three, four, five, six, and eight strategies with the lowest performance by 0, 1, 2, 3, 4, 5, 6, and 8 new strategies, respectively (i.e. 0WOA, 1WOA, 2WOA, 3WOA, 4WOA, 5WOA, 6WOA, and 8WOA, respectively). The set of all strategies used for the simulation is composed of fifteen different trading strategies with specific parameters. The replacement ratio of the market strategies ranges from 0% to 53.3%. The higher the replacement ratio in the simulation, the more dramatic changes in the mood on the financial market are observed.

As mentioned above, simulations are performed with fifteen trading strategies or beliefs and the intensity of choice, β, is set to 40.

4.3 Persistence

The persistence level of the simulated financial time series is represented here by the Hurst exponent. For the 0WOA case (no replacement of strategies), the initial set of strategies remains unchanged throughout all simulations, i.e., there is no learning effect due to replacement, and we can expect the values of the Hurst exponent to be close to the EMH case, i.e., $H = 0.5$. When the WOA is implemented, we observe a strong learning effect that leads to long-memory (persistent) behavior of price returns.

The highest level of persistence is in the 2WOA case (13.3% replacement rate) when the market has enough time to learn. When the number of replaced strategies is higher, the learning effect is weakened by the randomly chosen new strategies that appear on the financial market. With higher replacement ratio, the value of the Hurst exponent declines as the learning is "diluted" by new strategies that randomly enter the financial market. This is caused by a large number of incoming strategies (randomly generated) that makes the market portfolio of strategies richer on random events and restrains the market learning effect. This phenomenon takes place from the 5WOA case to the 8WOA case (33 % - 53 % replacement rate), see Figure 1.

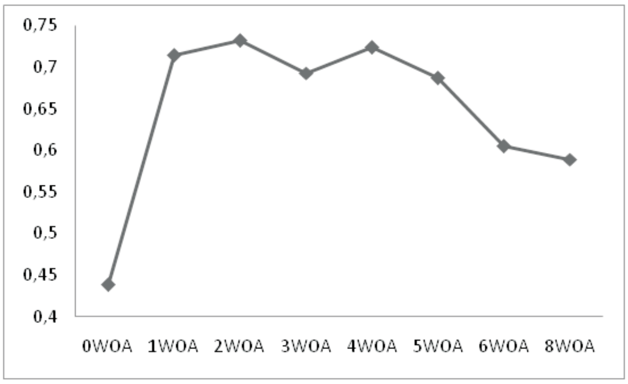

Fig. 1. The value of the Hurst exponent with different replacement rate of the WOA.

4.4 Energy Decomposition at Scales

The following Figures 2 and 3 depict scale energy decomposition, i.e., decomposition of the wavelet variance. For this decomposition we use a moving variance (window length 255) of the MRA details $D_1, ..., D_6$. Examining the cases 1WOA and 8WOA, we investigate how the energy changes at scales during the simulation. In Figure 2, the 1WOA case, we see a dramatic change of financial market structure during simulations. For example, there is a high increment of the energy at scales D_1 and D_2 at time 6000. This increment lasts for about 200 iterations. Unlike the 1WOA case, there are no such changes in Figure 3, the 8WOA case. In comparison to the 1WOA case, the overall energy level at scales D_1, D_2, D_3 is higher. This is mainly due to the higher replacement ratio of the WOA.

Comparing the aggregate energy values at scales for low and high replacement ratios, we observe significant differences. Figure 4 depicts the low replacement ratio (6.7%, 1WOA) and the high replacement ratio (33%, 5WOA). The main difference is at scales D_2 and D_3, where a higher activity at lower frequencies or higher scale in the 1WOA case is observed. This implies that strategies with longer investment horizons are preferred when there is a smaller fluctuation of traders on the financial market.

5 Sentiment Change on the Market

With the objective of deepening the behavioral aspects of the HAM, in this subsection we investigate the sentiment of investors present on

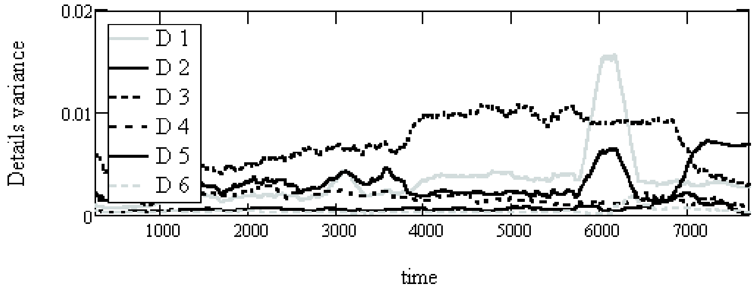

Fig. 2. The 1WOA case. Moving sum of squared wavelet detail vectors (window length 255) of the Daublet (4) DWT multiresolution analysis

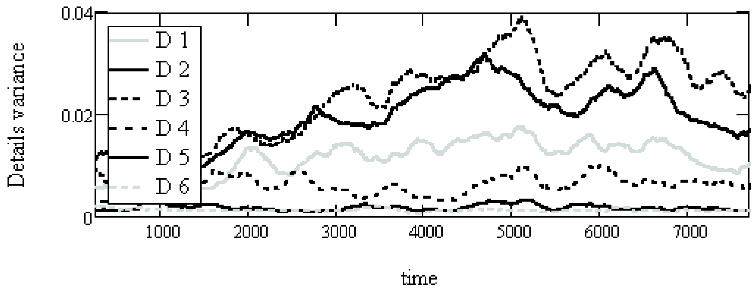

Fig. 3. The 8WOA case. Moving sum of squared wavelet detail vectors (window length 255) of the Daublet (4) DWT multiresolution analysis

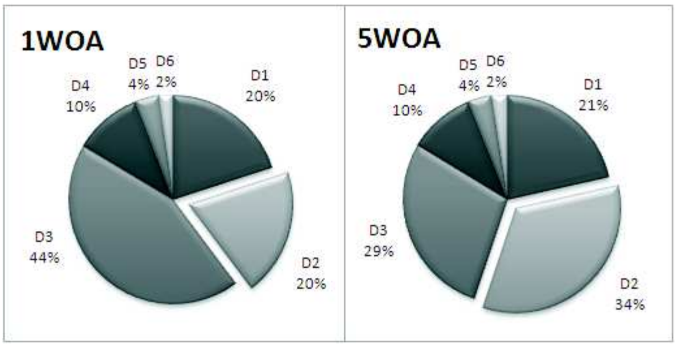

Fig. 4. Aggreagate energy at scales of the simulated time series with the 1WOA and 5WOA

the financial market. For this purpose, we introduce the change of the sentiment. We define the change of the sentiment as a shift of the trend g_h of a newly incoming investor strategy h on the financial market.

Our hypothesis is that when there are changes of the sentiment on the financial market, represented by shifts of the expected value of the trend g_h, the Hurst exponent of the returns will decrease due to the break up of the traders' structure that was created by the WOA. For the model defined above the sentiment change simulates more information and different mood phases entering the financial market.

In the first case, called the no sentiment change case, the trend g_h is generated by a normal distribution with zero mean and variance equal to 0.16 throughout the entire simulation. The second case, i.e., the sentiment change case, examines consequences of the sentiment change, specifically how the persistence of the generated time series is influenced. In the sentiment change case the expected value of g_h varies. For the first 2000 iterations the trend g_h of the newly incoming strategies on the financial market is generated from the distribution $N(0, 0.16)$. Next, for the interval $2000-4000$ iterations, the trend g_h is generated from the distribution $N(+0.1, 0.16)$. For another 2000 steps (the interval $4000-6000$) the trend g_h is generated from the distribution $N(-0.1, 0.16)$. For the interval $6000-8000$ the trend g_h is generated from the distribution $N(+0.1, 0.16)$, and finally for the interval $8000-10000$ the trend g_h is generated from the distribution $N(0, 0.16)$. This procedure defines the sentiment structure on the financial market.

In the sentiment case, the simulations are also performed with fifteen trading strategies (beliefs). We use 4WOA, i.e., we replace four strategies, so the replacement ratio is 27%. This ratio may seem quite high, however, such setup is good for the simulation of the sentiment changes. Other parameters have the same values as in the preceding simulation.

The values of the Hurst exponent for both cases, the change of the sentiment case and the case without sentiment change, are in Table 1 and the descriptive statistics of returns are in Table 2. The first row contains the value of the Hurst exponent for the whole interval (all 10000 iterations). Next rows present the values of the intervals during simulation. In general, the case with a sentiment change has lower Hurst exponent and more closely replicates real financial market data. The higher Hurst exponent for the no sentiment change case is a consequence of a learning mechanism on the financial market even though it is disturbed by the WOA. It is clear that in terms of financial market efficiency the WOA has a lower impact than the change of the sentiment or forecasts of investors.

Table 1. Values of the Hurst exponent for the sentiment and no sentiment change case

Interval	H.exponent Sentiment	H.exponent No Sentiment
0 – 10K	0.620	0.656
0 – 2K	0.698	0.794
2 – 4K	0.719	0.783
4 – 6K	0.676	0.748
6 – 8K	0.619	0.799
8 – 10K	0.547	0.744

Table 2. Descriptive statistics for the sentiment and no sentiment change case

Descriptive stat.	Sentiment change	No Sentiment change
Mean	0.032	0.011
Variance	0.059	0.0009
Kurtosis	3.584	1.081
Skewness	0.945	0.875

5.1 Wavelet Analysis of the Sentiment Change

The overall activity in the no sentiment change case is low in comparison to the sentiment change case, see Figures 5 and 6. The sentiment change causes the activity or energy to shift to higher frequencies, i.e., lower scales, for example compare Figures 5 and 6.

6 Conclusions

The concept of beliefs replacement represented by the WOA dramatically modifies the financial market dynamics. In the first part of the paper we demonstrate that the behavior of the HAM changes considerably when we implement the modified WOA. The implementation of this type of the WOA, in which we can change more beliefs (trading

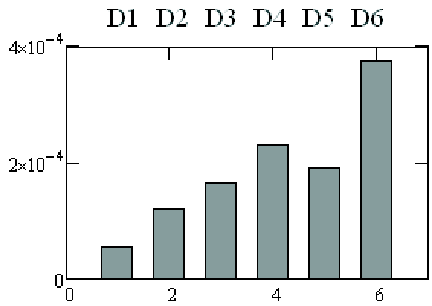

Fig. 5. The no sentiment change case aggregate activity (energy) of investors at six scales. D1 is a low scale component (high frequency), D6 is the highest scale component (low frequency) of a given time series.

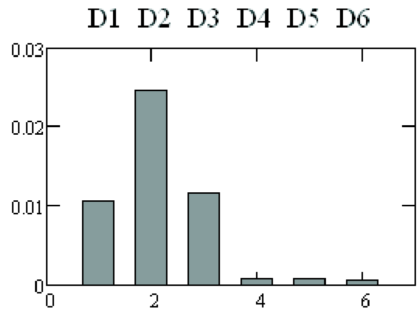

Fig. 6. The sentiment change case aggregate activity (energy) of investors at six scales. D1 is a low scale component (high frequency), D6 is the highest scale component (low frequency) of a given time series.

strategies), considerably increases the level of persistence of the price time series. However when we increase the number of replaced strategies beyond some point (4WOA), the value of the Hurst exponent declines as the learning is "diluted" by new strategies that randomly enter the financial market.

The application of the sentiment change on financial markets provides a new possibility for the incorporation of the behavioral approach into the theoretical financial market model. In general, the case with a sentiment change has a lower value of the Hurst exponent and its value is closer to the level of the Hurst exponent estimated from real financial market data. The higher level of the Hurst exponent for the no sentiment change case is a consequence of a learning mechanism on the financial market even though it is disturbed by the WOA. In terms of financial market efficiency, the WOA has a lower impact than a change of the sentiment or forecasts of investors.

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