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General Measure Theory

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A very few decades ago, measure theory appeared to be a classical and well established branch of mathematics with stabilized arrangement of its presentation, and its textbooks were organized in an expected way. This classical measure theory offered reliable tools for the advanced probability theory presented by A. N. Kolmogorov, and the measure itself was defined as a non-negative, additive set function vanishing in the empty set. Nevertheless, the mathematics has developed, its scope has expanded, and qualitatively new concepts have appeared. Some of them display properties partially compatible with the concept of measure, however, this compatibility belongs rather to the methodological roots of the relevant theories than to their formalism. Probably the first monograph dealing with these phenomenon was *Fuzzy Measure Theory* written by Z. Wang and G. J. Klir and published in 1992.

The progress of mathematics, as well as the extensive development of its tools, motivated the authors to innovate the book mentioned above, by means of including the new measure-like concepts and generalizing the classical ideas related to the measure theory concepts. The referred book represents the result of their endeavour.

It is worth stressing that the new monograph widely exceeds the original intention to complete the Fuzzy Measure Theory by simple addition of new but close concepts and related results. The concept of measure is generalized to its essence – namely, the *general measure* is defined as a nonnegative set function vanishing at empty set. Even this concept can be extended by omitting the requirement of non-negativeness to *signed general measures* fulfilling a single condition – the vanishing at empty set. So general set functions cover a wide class of special cases like, e. g., monotone or continuous measures, dealt with in particular chapters and sections.

The self-contained book demanding only a solid background in mathematical analysis, is organized in fifteen main chapters. After two introductory ones, next five chapters are devoted to the general measure concept and its analysis. Especially, to its basic ideas, special areas, and extensions. Also the structural characteristics for set functions and the properties of measurable functions on monotone measure spaces belong to this area of topics.

The next group of five chapters, namely those with numbers 8 to 12, deal with integration and closely related concepts like Sugeno and Choquet integrals, and general integration concept covering both Lebesgue and Sugeno integrals and called pan-integral. Chapter 12 deals with upper and lower integrals with their relation to the integration models.

Finally, the last group of three chapters present attractive topics regarding the applicability of the general measures theory, namely constructing general measures, fuzzification of generalized measures and the Choquet integral, and finally the proper applications of generalized measure theory.

The volume is completed by two appendices (containing glossaries of key concepts and symbols), by the rich list of references (430 representative items), and by the subject and name indices.

Each chapter, except the introductory Chapter 1, is concluded by Notes containing historical, bibliographical and other relevant comments, and almost all chapters (No. 1, 12

and 14 represent the exceptions) are concluded by exercises.

The authors themselves characterize their book as a text for one-semester graduate or upper division course. Of course, it can be used even in this way – the organization of chapters and their sections, the introduction of Notes and Exercises sections, and also the interesting and inspiring comments to the analyzed concepts make such use of the volume very easy. Nevertheless, the characterization is deeply under-evaluating. The volume offers qualitatively new view on the roots of the concept of measure and on its theory. It is consistent with the classical one, but it extends its scope in a prodigious way. Surely, the referred volume can serve as a handbook for specialists in the measure and integral theory, but it represents much more opening new horizons for the theory and potential applications of measure-like set functions.

Not only the organization of the chapters and sections but also the presentation of particular concepts and results, and the depth of explanatory comments illustrate the skill of the authors and their careful analysis of the presented topics. Their understanding for the demands of the subject of their interest allows them, in some cases, to modify slightly even the former terminology, when the logic of the structure of concepts needs such modification. This care for formal cleanliness results in an excellently written book. The above characteristics mean that the referred monograph can be recommended to everybody who wishes to be up-to-date in modern measure theory and to be able to follow its future development.

Milan Mareš