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Stability Estimating in Optimal Sequential Hypotheses Testing

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Abstract: We study the stability of the classical optimal sequential probability ratio test based on independent identically distributed observations $X_1, X_2, \ldots$ when testing two simple hypotheses about their common density f: $f = f_0$ versus $f = f_1$. As a functional to be minimized, it is used a weighted sum of the average (under f_0) sample number and the two types error probabilities. We prove that the problem is reduced to stopping time optimization for a ratio process generated by $X_1, X_2, \ldots$ with the density f_0. For τ_* being the corresponding optimal stopping time we consider a situation when this rule is applied for testing between f_0 and an alternative $\tilde{f}_1$, where $\tilde{f}_1$ is some approximation to f_1. An inequality is obtained which gives an upper bound for the expected cost excess, when τ_* is used instead of the rule $\tilde{\tau}_*$ optimal for the pair $(f_0, \tilde{f}_1)$. The inequality found also estimates the difference between the minimal expected costs for optimal tests corresponding to the pairs (f_0, f_1) and $(f_0, \tilde{f}_1)$.

Keywords: sequential hypotheses test; simple hypothesis; optimal stopping; sequential probability ratio test; likelihood ratio statistic; stability inequality;

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References

[1] Y. S. Chow, H. Robbins, and D. Siegmund: Great Expectations: The Theory of Optimal Stopping. Houghton Mifflin Company, Boston 1971.

[2] E. I. Gordienko and F. S. Salem: Estimates of stability of Markov control processes with unbounded costs. Kybernetika 36 (2000), 195–210.

[3] E. I. Gordienko and A. A. Yushkevich: Stability estimates in the problem of average optimal switching of a Markov chain. Math. Methods Oper. Res. 57 (2003), 345–365.

[4] P. J. Huber: A robust version of the probability ratio test. Ann. Math. Statist. 36 (1965), 1753–1758.

[5] A. Kharin: On robustifying of the sequential probability ratio test for a discrete model under "contaminations". Austrian J. Statist. 3 (2002), 4, 267–277.

[6] A. Kharin: Robust sequential testing of hypotheses on discrete probability distributions. Austrian J. Statist. 34 (2005), 2, 153–162.

[7] G. Lorden: Structure of sequential tests minimizing an expected sample size. Z. Wahrsch. Verw. Gebiete 51 (1980), 291–302.

[8] V. Mackevičius: Passage to the limit in problems of optimal stopping of Markov processes (in Russian). Litovsk. Mat. Sb. (Russian) 13 (1973), 1, 115–128, 236.

[9] R. Montes-de-Oca, A. Sakhanenko, and F. Salem-Silva: Estimates for perturbations of general discounted Markov control chains. Appl. Math. 30 (2003), 287–304.

[10] A. Novikov: Optimal sequential tests for two simple hypotheses. Sequential Analysis 28 (2009), No. 2.

[11] A. Novikov: Optimal sequential tests for two simple hypotheses based on independent observations. Internat. J. Pure Appl. Math. 45 (2008), 2, 291–314.

[12] V. V. Petrov: Sums of Independent Random Variables. Springer, New York 1975.

[13] P. X. Quang: Robust sequential testing. Ann. Statist. 13 (1985), 638–649.

[14] A. N. Shiryayev: Statistical Sequential Analysis. Nauka, Moscow 1969. (In Russian.)

[15] A. Wald and J. Wolfowitz: Optimum character of the sequential probability ratio test. Ann. Math. Statist. 19 (1948), 326–339.

[16] J. Whitehead: The Design and Analysis of Sequential Clinical Trials. Wiley, New York 1997.