

Markov Bases of Conditional Independence Models for Permutations

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Abstract: The L-decomposable and the bi-decomposable models are two families of distributions on the set S_n of all permutations of the first n positive integers. Both of these models are characterized by collections of conditional independence relations. We first compute a Markov basis for the L-decomposable model, then give partial results about the Markov basis of the bi-decomposable model. Using these Markov bases, we show that not all bi-decomposable distributions can be approximated arbitrarily well by strictly positive bi-decomposable distributions.

Keywords: conditional independence; Markov basis; closure of exponential family; permutation; L-decomposable;

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