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On the independence of rotation moment invariants

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Abstract

The problem of the independence and completeness of rotation moment invariants is addressed in this paper. First, a general method for constructing invariants of arbitrary orders by means of complex moments is described. As a major contribution of the paper, it is shown that for any set of invariants there exists a relatively small basis by means of which all other invariants can be generated. The method how to construct such a basis and how to prove its independence and completeness is presented. Some practical impacts of the new results are mentioned at the end of the paper.

Keywords: Moment invariants; Complex moments; Independence; Completeness

1. Introduction

Moment invariants have become a classical tool for object recognition during the last 30 years. They were firstly introduced to the pattern recognition community by Hu [1], who employed the results of the theory of algebraic invariants [2,3] and derived his seven famous invariants to the rotation of 2-D objects:

$$\phi_1 = \mu_{20} + \mu_{02},$$

$$\phi_2 = (\mu_{20} - \mu_{02})^2 + 4\mu_{11}^2,$$

$$\phi_3 = (\mu_{30} - 3\mu_{12})^2 + (3\mu_{21} - \mu_{03})^2,$$

$$\phi_4 = (\mu_{30} + \mu_{12})^2 + (\mu_{21} + \mu_{03})^2,$$

$$\begin{aligned}\phi_5 = {} & (\mu_{30} - 3\mu_{12})(\mu_{30} + \mu_{12})((\mu_{30} + \mu_{12})^2 \\ & - 3(\mu_{21} + \mu_{03})^2) + (3\mu_{21} - \mu_{03})(\mu_{21} + \mu_{03}) \\ & \times (3(\mu_{30} + \mu_{12})^2 - (\mu_{21} + \mu_{03})^2),\end{aligned}$$

$$\begin{aligned}\phi_6 = {} & (\mu_{20} - \mu_{02})((\mu_{30} + \mu_{12})^2 - (\mu_{21} + \mu_{03})^2) \\ & + 4\mu_{11}(\mu_{30} + \mu_{12})(\mu_{21} + \mu_{03}),\end{aligned}$$

$$\begin{aligned}\phi_7 = {} & (3\mu_{21} - \mu_{03})(\mu_{30} + \mu_{12})((\mu_{30} + \mu_{12})^2 \\ & - 3(\mu_{21} + \mu_{03})^2) - (\mu_{30} - 3\mu_{12})(\mu_{21} + \mu_{03}) \\ & \times (3(\mu_{30} + \mu_{12})^2 - (\mu_{21} + \mu_{03})^2),\end{aligned} \quad (1)$$

where

$$\mu_{pq} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x - x_c)^p (y - y_c)^q f(x, y)\, \mathrm{d}x\, \mathrm{d}y \quad (2)$$

is the central moment of the object $f(x, y)$ and (x_c, y_c) are the coordinates of the object centroid. Hu also showed how to achieve the invariance to scaling and demonstrated the discriminative power of these features in the case of recognition of printed capital characters.

Since then, numerous works have been devoted to the various improvements and generalizations of Hu's invariants and also to its use in many application areas. Dudani [4] and Belkasim [5] described their application to aircraft silhouette recognition, Wong and Hall [6], Goshtasby [7] and Flusser and Suk [8] employed moment invariants in template matching and registration of satellite images, Mukundan [9,10] applied them to estimate the position and the attitude of the object in 3-D space, Sluzek [11] proposed to use the local moment invariants in industrial quality inspection and many

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authors used moment invariants for character recognition [5,12–15]. Maitra [16] and Hupkens [17] made them invariant also to contrast changes, Wang [18] proposed illumination invariants particularly suitable for texture classification, Van Gool [19] achieved photometric invariance and Flusser et al. [20,21] described moment invariants to linear filtering. Several papers studied recognitive and reconstruction aspects, noise tolerance and other numerical properties of various kinds of moment invariants and compared their performance experimentally [5,22–27]. Moment invariants were shown to be also a useful tool for geometric normalization of an image [28,29]. Large amount of effort has been spent to find the effective algorithms for moment calculation (see [30] for a survey). Recently, Flusser and Suk [31] and Reiss [32] have corrected some mistakes in Hu's theory and have derived the invariants to affine transform.

In the contrast to a large number of application-oriented works, only few attempts to derive invariants from moments of orders higher than three have been done. Li [33] and Wong [34] presented the systems of invariants up to the orders nine and five, respectively. Unfortunately, none paid attention to the mutual dependence/independence of the invariants. The invariant sets presented in their papers are algebraicly dependent.

There is also a group of papers [14,35,36] that use *Zernike moments* to construct rotation invariants. Their motivation comes from the fact that Zernike polynomials are orthogonal on a unit circle. Thus, Zernike moments do not contain any redundant information and are more convenient for image reconstruction. However, Teague [35] showed that Zernike invariants of second and third orders are equivalent to the Hu's ones when expressing them in terms of geometric moments. He presented the invariants up to eight order in explicit form but no general rule as to the way to derive them is given. Wallin [36] described an algorithm for a formation of moment invariants of any order. Since Teague [35] as well as Wallin [36] were particularly interested in the reconstruction abilities of the invariants, they did not pay much attention to the question of independence. However, the independence of the features is a fundamental issue in all the pattern recognition problems, especially in the case of a high-dimensional feature space.

The benefit of this paper is twofold. First, a general scheme how to derive moment invariants of any order is presented. Secondly, we show that there exist relatively small set (basis) of the invariants by means of which all other invariants can be expressed and we give an algorithm for its construction. As a consequence of this, we show that most of the previously published sets of rotation moment invariants including Hu's system (1) are dependent. This is really a surprising result giving a new look at Hu's invariants and possibly yielding a new interpretation of some previous experimental work.

2. A general scheme for deriving invariants

There are various approaches to the theoretical derivation of moment-based rotation invariants. Hu [1] employed the theory of algebraic invariants, Li [33] used the Fourier–Mellin transform, Teague [35] and Wallin [36] proposed to use Zernike moments and Wong [34] used complex monomials which also originate from the theory of algebraic invariants. In this paper, we present a new scheme, which is based on the *complex moments*. The idea to use the complex moments for deriving invariants was already described by Mostafa and Psaltis [24] but they concentrated themselves to the evaluation of the invariants rather than to constructing higher-order systems. In comparison with the previous approaches, this one is more transparent and allows to study mutual dependence/independence of the invariants easily. It should be noted that all the above approaches differ from each other formally by the mathematical tools and the notation used but the general idea behind them is common and the results are similar or even equivalent.

Complex moment c_{pq} of the order $(p+q)$ of an integrable image funciton $f(x, y)$ is defined as

$$c_{pq} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x+\mathrm{i}y)^p (x-\mathrm{i}y)^q f(x,y)\,\mathrm{d}x\,\mathrm{d}y, \tag{3}$$

where i denotes the imaginary unit. Each complex moment can be expressed in terms of geometric moments m_{pq} as

$$c_{pq} = \sum_{k=0}^{p}\sum_{j=0}^{q} \binom{p}{k}\binom{q}{j} (-1)^{q-j}\,\mathrm{i}^{p+q-k-j}\, m_{k+j,p+q-k-j}. \tag{4}$$

In polar coordinates, Eq. (3) becomes the form

$$c_{pq} = \int_{0}^{\infty}\int_{0}^{2\pi} r^{p+q+1} \mathrm{e}^{\mathrm{i}(p-q)\theta} f(r,\theta)\,\mathrm{d}r\,\mathrm{d}\theta. \tag{5}$$

It follows immediately from Eq. (5) that $c_{pq} = c_{qp}^*$ (the asterix denotes a complex conjugate). The following lemma describes an important rotation property of the complex moments.

Lemma 1. *Let f' be a rotated version (around the origin) of f, i.e. $f'(r,\theta) = f(r, \theta+\alpha)$ where α is the angle of rotation. Let us denote the complex moments of f' as c'_{pq}. Then*

$$c'_{pq} = e^{-\mathrm{i}(p-q)\alpha} c_{pq}. \tag{6}$$

Using Eq. (5), the proof of Lemma 1 is straightforward. It can be seen immediately that $|c_{pq}|$ is invariant to rotation for any p and q. However, the moment magnitudes do not generate a complete set of invariants. In the following theorem, we propose a better approach to the construction of rotation invariants.

Theorem 1. *Let $n \geqslant 1$ and let k_i, p_i and q_i; $i = 1, \ldots n$, be non-negative integers such that*

$$\sum_{i=1}^{n} k_i(p_i - q_i) = 0.$$

Then

$$I = \prod_{i=1}^{n} c_{p_i q_i}^{k_i} \tag{7}$$

is invariant to rotation.

The proof of Theorem 1 follows immediately from Lemma 1.

According to Theorem 1, some simple examples of rotation invariants are c_{11}, $c_{20}c_{02}$, $c_{20}c_{12}^2$, etc. As a rule, most invariants (7) are complex. If we want to have real-valued features, we only take the real and imaginary parts of each of them. To achieve also translation invariance (or, equivalently, invariance to the rotation around an arbitrary point), we only use the central coordinates in the definition of the complex moments (3).

It can be seen that the Hu's invariants (1) are nothing else than particular representatives of the general form (7):

$$\phi_1 = c_{11},$$

$$\phi_2 = c_{20}c_{02},$$

$$\phi_3 = c_{30}c_{03},$$

$$\phi_4 = c_{21}c_{12},$$

$$\phi_5 = \mathrm{Re}(c_{30}c_{12}^3),$$

$$\phi_6 = \mathrm{Re}(c_{20}c_{12}^2),$$

$$\phi_7 = \mathrm{Im}(c_{30}c_{12}^3). \tag{8}$$

3. Independence and completeness of the sets of invariants

In this section, our attention will be paid to the construction of a basis of the invariants. Theorem 1 allows us to construct an infinite number of the invariants for any order of moments, but only a few of them are mutually independent. By the term *basis* we intuitively understand the smallest set by means of which all other invariants can be expressed. The knowledge of the basis is a crucial point in all the pattern recognition problems because it provides the same discriminative power as the set of all invariants at minimum computational costs.

To formalize this approach, we introduce the following definitions first.

Definition 1. Let $k \geqslant 1$, let $\mathscr{I} = \{I_1, \ldots, I_k\}$ be a set of rotation invariants of the type (7) and let J be an invariant of the same type. Invariant J is said to be *dependent on* $\mathscr{I}$ if and only if there exists a function F of k variables containing only the operations multiplication, involution with an integer (positive or negative) exponent and complex conjugation, such that

$$J = F(I_1, \ldots, I_k).$$

Definition 2. Let $k > 1$ and let $\mathscr{I} = \{I_1, \ldots, I_k\}$ be a set of rotation invariants of the type (7). The set $\mathscr{I}$ is said to be *dependent* if and only if there exists $k_0 \leqslant k$ such that I_{k_0} depends on $\mathscr{I} - \{I_{k_0}\}$. The set $\mathscr{I}$ is said to be *independent* otherwise.

According to this definition, $\{c_{20}c_{02}, c_{20}^2c_{02}^2\}$, $\{c_{21}^2c_{02}, c_{21}c_{12}, c_{21}^3c_{02}c_{12}\}$ and $\{c_{20}c_{12}^2, c_{02}c_{21}^2\}$ are the examples of the dependent invariant sets.

Definition 3. Let $\mathscr{I}$ be a set of rotation invariants of the type (7) and let $\mathscr{B}$ be its subset. $\mathscr{B}$ is a basis of $\mathscr{I}$ if and only if

- $\mathscr{B}$ is independent,
- Any element of $\mathscr{I} - \mathscr{B}$ depends on $\mathscr{B}$ (this property is called *completeness*).

Now we can formulate the central theorem of this paper that tells us how to construct an invariant basis above a given set of moments.

Theorem 2. *Let $\mathscr{M}$ be a set of the complex moments of any orders (not necessarily of all moments), let $\mathscr{M}^*$ be a set of their complex conjugates and let $c_{p_0q_0} \in \mathscr{M} \cup \mathscr{M}^*$ such that $p_0 - q_0 = 1$ and $c_{p_0q_0} \neq 0$. Let $\mathscr{I}$ be a set of all rotation invariants created from the elements of $\mathscr{M} \cup \mathscr{M}^*$ according to (7). Let $\mathscr{B}$ be constructed as follows:*

$$(\forall p, q | p \geqslant q \wedge c_{pq} \in \mathscr{M} \cup \mathscr{M}^*)(\Phi(p, q) \equiv c_{pq}c_{q_0p_0}^{p-q} \in \mathscr{B}).$$

Then $\mathscr{B}$ is a basis of $\mathscr{I}$.

For the proof of Theorem 2 see Appendix A.

4. Some consequences of Theorem 2

In this section, we highlight some consequences of Theorem 2 that are of practical importance.

4.1. On the dependence of the previously published invariants

We show that the previously published systems of rotation invariants including the famous Hu's one are dependent. This fact has not been reported in the literature yet. Using Eq. (8) and assuming $c_{21} \neq 0$, it is easy to prove that

$$\phi_3 = c_{30}c_{03} = \frac{c_{03}c_{21}^3c_{30}c_{12}^3}{(c_{21}c_{12})^3} = \frac{\phi_5^2 + \phi_7^2}{\phi_4^3}.$$

Moreover, the Hu's system is incomplete. There are two third-order invariants – $c_{20}c_{12}^2$ and $c_{30}^2c_{02}^3$ – that are independent of $\{\phi_1, \ldots, \phi_7\}$.

Li [33] published a set of invariants from moments up to the ninth order. Unfortunately, his system includes the Hu's one as its subset and therefore it also cannot be a basis.

Wong [34] presented a set of 16 invariants from moments up to the third order and a set of "more than 49" invariants from moments up to the fourth order. It follows immediately from Theorem 2 that a basis of the third-order invariants has only six elements and a basis of the fourth-order invariants has 11 elements (these numbers relate to the real-valued invariants). Thus, most of Wong's invariants are dependent.

The systems of invariants published by Teague [35] and Wallin [36] are also dependent. We can, however, obtain an independent system just by removing all skew invariants. The proof of completeness is given in Ref. [36] but this term is in that paper defined as a possibility to recover all the moments up to the given order from the set of invariants.

4.2. *An explicit construction of the third-order and fourth-order bases*

In this section, we present the bases of low-order invariants that have been constructed according to Theorem 2 and that we recommend to use for 2-D object recognition.

- Second and third orders:

$$\psi_1 = c_{11} = \phi_1,$$

$$\psi_2 = c_{21}c_{12} = \phi_4,$$

$$\psi_3 = \mathrm{Re}(c_{20}c_{12}^2) = \phi_6,$$

$$\begin{aligned}\psi_4 &= \mathrm{Im}(c_{20}c_{12}^2)\\ &= \mu_{11}((\mu_{30} + \mu_{12})^2 - (\mu_{03} + \mu_{21})^2)\\ &\quad - (\mu_{20} - \mu_{02})(\mu_{30} + \mu_{12})(\mu_{03} + \mu_{21}),\end{aligned}$$

$$\psi_5 = \mathrm{Re}(c_{30}c_{12}^3) = \phi_5,$$

$$\psi_6 = \mathrm{Im}(c_{30}c_{12}^3) = \phi_7.$$

- Fouth order:

$$\psi_7 = c_{22},$$

$$\psi_8 = \mathrm{Re}(c_{31}c_{12}^2),$$

$$\psi_9 = \mathrm{Im}(c_{31}c_{12}^2),$$

$$\psi_{10} = \mathrm{Re}(c_{40}c_{12}^4),$$

$$\psi_{11} = \mathrm{Im}(c_{40}c_{12}^4).$$

In the case of third-order and fourth-order invariants, the bases are determined unambiguously. However, there are various possibilities as to apply Theorem 2 when constructing higher-order bases. The difference is in the choice of the indices p_0 and q_0. Although it is not strictly required, it is highly desirable always to choose p_0 and q_0 as small as possible, because low-order moments are less sensitive to noise than the higher-order ones.

5. Skew invariants

In this section, we investigate the behavior of the rotation invariants under reflection. The invariants, which do not change their values under reflection are traditionally called the *true invariants* while the others are called *skew invariants* [1] or *pseudoinvariants* [35]. Skew invariants distinguish between the mirrored images of the same object that is useful in some applications but may be undesirable in other cases. In the following text we show which invariants of those introduced in Theorem 1 are skew and which are the true ones.

Let us consider an invariant of the type (7) and let us investigate its behavior under the reflection across an arbitrary line. Due to the rotation and shift invariance, we can restrict ourselves to the reflection across the x-axis. Without loss of generality, we consider the invariants from the basis only.

Let $\bar{f}(x, y)$ be a reflected version of $f(x, y)$, i.e. $\bar{f}(x, y) = f(x, -y)$. It follows from Eq. (3) that

$$\overline{c_{pq}} = c_{pq}^*.$$

Thus, it holds for any basic invariant $\Phi(p, q)$

$$\overline{\Phi(p, q)} = \overline{c_{pq}c_{q_0p_0}^{p-q}} = c_{pq}^*(c_{q_0p_0}^*)^{p-q} = \Phi(p, q)^*.$$

This indicates that the real parts of the basic invariants are true invariants. On the other hand, the imaginary parts of them are skew invariants, because they change their signs under reflection.

6. Summary and conclusion

In this paper, the problem of the independence and completeness of the rotation moment invariants was discussed. Although the moment invariants have attracted a significant attention of pattern recognition community within the last thirty years, they have not been studied from this point of view as yet.

A general method how to derive the rotation invariants of any order was described first. Then the theorem showing what the basis of the invariants looks like was formulated and proven. This is the major theoretic result of the paper. Finally, the relationship to the previous works was demonstrated. As an interesting consequence of our results, it was shown that Hu's system of moment invariants is dependent and incomplete.

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Appendix A. Proof of Theorem 2

Completeness of $\mathscr{B}$: Let I be an arbitrary element of $\mathscr{I}$. Thus

$$I = \prod_{i=1}^{n} c_{p_i q_i}^{k_i},$$

where $c_{p_i q_i} \in \mathscr{M} \cup \mathscr{M}^*$. The product can be decomposed into two factors according to the relation between p_i and q_i:

$$I = \prod_{i=1}^{n_1} c_{p_i q_i}^{k_i} \prod_{i=n_1+1}^{n} c_{p_i q_i}^{k_i}$$

where $p_i \geqslant q_i$ if $i \leqslant n_1$ and $p_i < q_i$ if $i > n_1$.

Let us construct another invariant J from the elements of $\mathscr{B}$ only as follows:

$$J = \prod_{i=1}^{n_1} \Phi(p_i, q_i)^{k_i} \prod_{i=n_1+1}^{n} \Phi(q_i, p_i)^{*k_i}.$$

Grouping the factors $c_{q_0 p_0}$ and $c_{p_0 q_0}$ together we get

$$J = c_{q_0 p_0}^{\sum_{i=1}^{n_1} k_i(p_i - q_i)} c_{p_0 q_0}^{\sum_{i=n_1+1}^{n} k_i(q_i - p_i)} \prod_{i=1}^{n_1} c_{p_i q_i}^{k_i} \prod_{i=n_1+1}^{n} c_{p_i q_i}^{k_i}$$

$$= c_{q_0 p_0}^{\sum_{i=1}^{n_1} k_i(p_i - q_i)} c_{p_0 q_0}^{\sum_{i=n_1+1}^{n} k_i(q_i - p_i)} I.$$

Since I is assumed to be an invariant, it must hold

$$\sum_{i=1}^{n_1} k_i(p_i - q_i) + \sum_{i=n_1+1}^{n} k_i(p_i - q_i) = 0$$

and, consequently,

$$\sum_{i=1}^{n_1} k_i(p_i - q_i) = \sum_{i=n_1+1}^{n} k_i(q_i - p_i) = K.$$

Now I can be expressed as a function of the elements of $\mathscr{B}$:

$$I = \Phi(p_0, q_0)^{-K} J.$$

Thus, I has been proven to be dependent on $\mathscr{B}$.

Independence of $\mathscr{B}$: Let us assume $\mathscr{B}$ is dependent, i.e. there exists $\Phi(p, q) \in \mathscr{B}$, such that it depends on $\mathscr{B} - \{\Phi(p, q)\}$. As follows immediately from the mutual independence of the moments, it must hold $p = p_0$ and $q = q_0$. That means, according to the above assumption, there exist invariants $\Phi(p_1, q_1), \ldots, \Phi(p_n, q_n)$ and $\Phi(s_1, t_1), \ldots, \Phi(s_m, t_m)$ from $\mathscr{B} - \{\Phi(p_0, q_0)\}$ such that

$$\Phi(p_0, q_0) = \frac{\prod_{i=1}^{n_1} \Phi(p_i, q_i)^{k_i} \prod_{i=n_1+1}^{n} \Phi(p_i, q_i)^{*k_i}}{\prod_{i=1}^{m_1} \Phi(s_i, t_i)^{\ell_i} \prod_{i=m_1+1}^{m} \Phi(s_i, t_i)^{*\ell_i}}. \tag{A.1}$$

Substituting into Eq. (9) and grouping the factors $c_{p_0 q_0}$ and $c_{q_0 p_0}$ together, we get

$$\Phi(p_0, q_0) = \frac{c_{q_0 p_0}^{\sum_{i=1}^{n_1} k_i(p_i - q_i)} c_{p_0 q_0}^{\sum_{i=n_1+1}^{n} k_i(q_i - p_i)} \prod_{i=1}^{n_1} c_{p_i q_i}^{k_i} \prod_{i=n_1+1}^{n} c_{q_i p_i}^{k_i}}{c_{q_0 p_0}^{\sum_{i=1}^{m_1} \ell_i(s_i - t_i)} c_{p_0 q_0}^{\sum_{i=m_1+1}^{m} \ell_i(t_i - s_i)} \prod_{i=1}^{m_1} c_{s_i t_i}^{\ell_i} \prod_{i=m_1+1}^{m} c_{t_i s_i}^{\ell_i}}. \tag{A.2}$$

Comparing the exponents of $c_{p_0 q_0}$ and $c_{q_0 p_0}$ on the both sides we get the constraints

$$K_1 = \sum_{i=1}^{n_1} k_i(p_i - q_i) - \sum_{i=1}^{m_1} \ell_i(s_i - t_i) = 1 \tag{A.3}$$

and

$$K_2 = \sum_{i=n_1+1}^{n} k_i(q_i - p_i) - \sum_{i=m_1+1}^{m} \ell_i(t_i - s_i) = 1. \tag{A.4}$$

Since the rest of the right-hand side of Eq. (A.2) must be equal to 1 and since the moments themselves are mutually independent, the following constraints must be fulfilled for any index i:

$$n_1 = m_1, \quad n = m, \quad p_i = s_i, \quad q_i = t_i, \quad k_i = \ell_i.$$

Introducing these constraints into Eqs. (A.3) and (A.4), we get $K_1 = K_2 = 0$ that is a contradiction $\square$.

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