PreC: A Prototypical Tool to Deal with Partial Preference Orders and Qualitative Uncertainty

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Abstract

Starting from an uniform declarative axiomatic treatment of qualitative uncertainty, we introduce a new tool that permits an *user-friendly* handling of uncertainty preference orders. The tool combines different programming paradigms by completing a declarative programming engine (based on Answer Set Programming) with a graphical interface.
Key words: Qualitative uncertainty, axiomatic approach, Answer Set Programming.

1 Introduction

It is a widespread opinion that qualitative approaches to uncertainty, in particular when relying on comparative preference elicitation, permit a more intuitive and, in some way, "more realistic" formalization of individual's believes [2, 7, 10, 11, 12, 14, 15]. Since qualitative approaches implicitly capture "imprecise" (i.e., not unique) models, they turn out to be extremely useful in sensitivity and sensibility analysis, also offering the advantage of a less complex modeling. The central idea of such methodologies is to grade uncertainty about the truth of propositions, through comparisons expressing the judgment "less (or more) believed to be true". This operationally translates into the use of order relations in place of numerical grades.

While they are differentiated by the specific way of combining distinct pieces of information, a common feature of all different qualitative frameworks is their axiomatic nature [6, 5]. Considering the human mental process yielding a judgment being guided by a number of heuristics which unconsciously "act behind the scenes", it seems reasonable that the different human mental behaviors can be captured by different formal ways (axioms) of combining believes.

In spite of their valuable features, any qualitative approach to uncertainty has to be "supported" by a corresponding numerical one, since quantitative approaches constitute anyhow valid and useful “reference stones”. For this reason the classification of qualitative uncertainty notions recalls to numerical uncertainty measures. Often, the terms *comparative* probabilities, *comparative* plausibility, etc., are used for the qualitative counterparts of numerical probabilities, numerical plausibility, and so on.

Because of their generality, in this article we focus on the so called *partial models*, i.e. qualitative assessments defined on *some* of the situations at hand and intended to be restrictions of some complete models. (Then, we deal with partial probabilities, partial plausibility, and so on.) On the one hand, this approach allows the analyst of the problem to focus his/her evaluation on the situations really judged relevant. On the other hand, it leaves open the possibility to enlarge the model to other elements that could enter the scene later.

We proceed by first introducing an uniform axiomatic treatment of qualitative uncertainty notions. This uniformity greatly helps in providing the representability results for each uncertainty framework. Then we introduce a new tool, named *PreC* (standing for *Pre*ference *C*runcher), that permits an user-friendly management of qualitative uncertainty assessments. Such a tool constitutes a concretization of both the theoretical ideas expressed in [3, 6, 7] and the practical intentions proposed in [4, 5]. As we will see, it combines different programming paradigms by completing a declarative programming engine (based on Answer Set Programming) with an user-friendly graphical interface (written in Java). The former component directly supports plain implementation of basic reasoning tasks (in our case, qualitative framework detection and qualitative inference). The latter component instead permits to define events, impose logical constraints and specify preference relations among events in a direct and intuitive manner.

The first reasoning task we deal with is *qualitative framework detection*. It consists in ascertaining which are the reasonable rules to work with, given an assessment over a finite set of events. This is realized by detecting which of the axioms characterizing representable preferences assessments can be applied. Hence we actually invert the usual attitude towards qualitative management of uncertainty, where usually a precise framework is chosen in advance and the models are adapted to it. The *qualitative inference* task consists in deducing new knowledge on the basis of the properties of a partial model, when a new event enters into play. More specifically, once an uncertainty framework for the domain has been chosen (or detected), we are interested in determining which is the “minimal” extension of the model, induced by the new event and still belonging to the chosen class of orderings.

2 Weak preference structures

In dealing with (partial) preference assessments, we consider the domain of discernment as represented by a finite set of events $\mathcal{E} = \{E_1, \ldots, E_n\}$ (among them, ϕ and Ω denote the impossible and the sure event, respectively). Such events are those relevant propositions on which the subject of the analysis expresses his/her opinion. Hence, $\mathcal{E}$ does not necessarily represent a full model, i.e. it does not comprehend all elementary situations and all of their combinations. For this reason, a crucial component of partial assessments are the logical relationships (incompatibilities, implications, combinations, equivalences, etc.) holding among events. Such relationships are expressed by means of a collection $\mathcal{C}$ of constraints on (conjunctions and disjunctions of) events. By taking into account $\mathcal{C}$, the family $\mathcal{E}$ spans a minimal Boolean algebra $\mathcal{A}_{\mathcal{E}}$ containing $\mathcal{E}$ itself. Note that $\mathcal{A}_{\mathcal{E}}$ is only implicitly defined via $\mathcal{E}$ and $\mathcal{C}$ and it is not a part of the assessment. Anyway, $\mathcal{A}_{\mathcal{E}}$ can be seen as a supporting structure inducing a lattice structure on $\mathcal{A}$. The *atoms* of $\mathcal{A}_{\mathcal{E}}$ are the minimal elements of the (sub-)lattice $\mathcal{A}_{\mathcal{E}} \setminus \{\phi\}$. Then, each event corresponds to a set of atoms and $\mathcal{A}_{\mathcal{E}}$ is (partially) ordered by set inclusion. We have the following notion:

Definition 1 *Let $\mathcal{A}_{\mathcal{E}}$ be an algebra of events. A binary relation $\preccurlyeq^*$ over $\mathcal{A}$ is a* total preference order *if it satisfies the following conditions:*[1]

(A1) $\preccurlyeq^*$ *is a pre-order, i.e. it is reflexive, transitive, and total;*

(A2) $\phi \preceq^* \Omega$ *and* $\neg(\Omega \preceq^* \phi)$ *(*non-triviality*);*

(A3) *for all events* X, Y, $X \subseteq Y \rightarrow (X \preccurlyeq^* Y)$ *(*monotonicity*).*

If $\preccurlyeq^*$ is a total preference order, $\sim^*$ and $\prec^*$ are its symmetric *and* asymmetric factors*, respectively.*

To deal with partial assessments, we consider two distinct but correlated relations, modeling weak and strict user's preferences, respectively. More formally:

Definition 2 *Let $\preccurlyeq$ and $\prec$ be binary relations over a set of events $\mathcal{E}$, such that $E_1 \prec E_2 \rightarrow E_1 \preccurlyeq E_2$. The pair $\langle \preccurlyeq, \prec \rangle$ is a* weak preference structure *for $\mathcal{E}$ (*w.p.s.*, for short) if it exists a total preference order $\preccurlyeq^*$ over $\mathcal{A}_{\mathcal{E}}$ such that:* $\forall E_1, E_2 \in \mathcal{E} \big((E_1 \preccurlyeq E_2 \rightarrow E_1 \preccurlyeq^* E_2) \wedge (E_1 \prec E_2 \rightarrow E_1 \prec^* E_2)\big)$.

Notice that Definition 2 does not require neither $\preccurlyeq$ or $\prec$ to be total orders, nor $\prec$ to be the asymmetric factor of $\preccurlyeq$. On the other hand, it is required that $\preccurlyeq^*$ extends $\preccurlyeq$, and that $\prec^*$ (the asymmetric factor of $\preccurlyeq^*$) extends $\prec$.

Example 1 Consider the following decision making problem in gastroenterology.[2] Patients might suffer from three possible diseases: Peptic ulcer, gastric cancer, and biliar disease. The symptoms that might be associated to such diseases are jaundice, weight loss, and dark stools. Presence of jaundice indicates biliar disease, weight loss can be associated to gastric cancer, dark stools might indicate peptic ulcer or gastric cancer. From data provided by the hospitals we know that the incidence of peptic ulcer is greater than the incidence of gastric cancer, while biliar disease affect the majority of the patients. Information useful to make a diagnosis includes age and sex of the patient: Peptic ulcer and

[1] in what follows, given any binary relation R, the writing $\neg(X\mathsf{R}Y)$ means that the pair $\langle X, Y \rangle$ does not belong to R.

[2] This example is hypothetical and for illustrative purpose only. It is not intended to express any clinical competence.

gastric cancer are more frequent in men; biliar diseases are more often complained by women. Moreover, in male population, incidence of ulcer is greater then the incidence of biliar disease. As regards age, we can reasonably affirm that older people are more subject to peptic ulcer or gastric cancer than young people. This scenario can be so represented:

$\mathsf{GC} \equiv$ *The real state of suffering from gastric cancer*	$\mathsf{JA} \equiv$ *Jaundice symptoms*
$\mathsf{PU} \equiv$ *The real state of suffering from peptic ulcer*	$\mathsf{WL} \equiv$ *Weight loss symptoms*
$\mathsf{BD} \equiv$ *The real state of suffering from biliar disease*	$\mathsf{DS} \equiv$ *Dark stools*

Let M (resp., W) denote the event *The patient is male* (resp., *female*), and OA (resp., YA) denote the event *The patient is old* (resp., *young*). Finally, let us assume that any patient suffers from at most one disease. The knowledge about diseases and symptoms can be so described in terms of logical constraint: $\mathsf{JA} \cap \mathsf{GC} = \mathsf{JA} \cap \mathsf{PU} = \mathsf{WL} \cap \mathsf{PU} = \phi$, $\mathsf{WL} \cap \mathsf{BD} = \mathsf{DS} \cap \mathsf{BD} = \phi$, $\mathsf{GC} \cap \mathsf{PU} = \mathsf{GC} \cap \mathsf{BD} = \mathsf{PU} \cap \mathsf{BD} = \phi$, $\mathsf{GC} \cup \mathsf{PU} \cup \mathsf{BD} = \Omega$, $\mathsf{OA} \cap \mathsf{YA} = \mathsf{M} \cap \mathsf{W} = \phi$, $\mathsf{OA} \cup \mathsf{YA} = \mathsf{M} \cup \mathsf{W} = \Omega$. Due to events' meaning, it seems reasonable to describe a w.p.s. as follows: $\phi \prec \mathsf{GC} \prec \mathsf{PU} \prec \mathsf{BD} \prec \Omega$, $\mathsf{GC} \cap \mathsf{W} \prec \mathsf{GC} \cap \mathsf{M}$, $\mathsf{PU} \cap \mathsf{W} \prec \mathsf{PU} \cap \mathsf{M}$, $\mathsf{BD} \cap \mathsf{M} \prec \mathsf{BD} \cap \mathsf{W}$, $\mathsf{BD} \cap \mathsf{M} \prec \mathsf{PU} \cap \mathsf{M}$, $\mathsf{YA} \cap (\mathsf{GC} \cup \mathsf{PU}) \preccurlyeq \mathsf{OA} \cap (\mathsf{GC} \cup \mathsf{PU})$. □

As regards w.p.s., we have the these counterparts of conditions (A1)–(A3):

Proposition 1 *For any w.p.s.* $\langle \preccurlyeq, \prec \rangle$ *for* $\mathcal{E}$*, the following properties hold:*

(A1') *if there exist* $E_1, \ldots, E_n \in \mathcal{E}$ *such that* $E_1 \preccurlyeq E_2 \preccurlyeq \ldots \preccurlyeq E_n \preccurlyeq E_1$*, then* $\neg(E_i \prec E_j)$ *for all* $i, j \in \{1, \ldots, n\}$*;*

(A2') $\neg(\Omega \preccurlyeq \phi)$*;*

(A3') *for all* $E_1, E_2 \in \mathcal{E}$*,* $E_1 \prec E_2 \to E_2 \not\subseteq E_1$*.*

Conditions (A1')–(A3') ensure the existence of a total preference order $\preccurlyeq^*$ which enlarges $\langle \preccurlyeq, \prec \rangle$. Considering numerical approaches to uncertainty, Capacities measures constitute the most general framework, as they express "common sense" behaviors. Any reasonable relation $\preccurlyeq$ must be representable by a partial Capacity (i.e., a restriction of a Capacity measure to the set of events at hand). This corresponds to the satisfaction of the conditions (A1')–(A3'). Notice that, in the light of (A3'), in what follows we assume all orders being closed under monotonicity (i.e. for all $E_1, E_2 \in \mathcal{E}$ $E_1 \subseteq E_2 \to E_1 \preccurlyeq E_2$).

Differentiations among uncertainty notions are done by considering the specific way of combining distinct pieces of information (e.g. for Probabilities additivity is adopted). Within the numerical context, this yields a taxonomy of numerical measures that reflects on a diversification of preference relations.

The correspondence between a qualitative uncertainty notion and a numerical measure is given in terms of a representability result.

Definition 3 *Let* $\mathcal{E}$ *be a set of events. A total preference order* $\preccurlyeq^*$ *over* $\mathcal{A}_{\mathcal{E}}$ *is said to be* representable *by a numerical measure* $f : \mathcal{A}_{\mathcal{E}} \to [0, 1]$ *if for all* $E_1, E_2 \in \mathcal{A}_{\mathcal{E}}$ *it holds that* $E_1 \preccurlyeq^* E_2 \leftrightarrow f(E_1) \leqslant f(E_2)$*.*

A w.p.s. $\langle \preccurlyeq, \prec \rangle$ *for* $\mathcal{E}$ *is said to be* representable *by a partial uncertainty measure* $g : \mathcal{E} \to [0,1]$ *if it admits an enlargement* $\preccurlyeq^*$ *over* $\mathcal{A}_\mathcal{E}$ *which is representable by an uncertainty measure* $g^* : \mathcal{A}_\mathcal{E} \to [0,1]$ *extension of* g *to* $\mathcal{A}_\mathcal{E}$.

By following [11, 14, 15, 3, 6], we refer to any specific class of preference orders according to their agreement with the numerical models. The starting assumption is a w.p.s. $\langle \preccurlyeq, \prec \rangle$ for $\mathcal{E}$ satisfying (A1')–(A3'). Then, any of the following specific axiom expresses a necessary and sufficient condition for the existence of an enlargement $\preccurlyeq^*$ over $\mathcal{A}_\mathcal{E}$ which is representable by an uncertainty measure:

(B') (Comparative belief)
for all $X, Y, Z, W \in \mathcal{E}$ s.t. $X \subset Y$, $Z \subset W \subset Y$, $W \setminus Z \subseteq Y \setminus X$ it holds that
$$X \sim Y \to \neg(Z \prec W).$$

(0M') (Comparative 0-monotonicity)
for all $X, Y, Z \in \mathcal{E}$ s.t. $X \subset Y, Z \subseteq Y \setminus X$ it holds that
$$X \sim Y \to \neg(\phi \prec Z).$$

(PL') (Comparative plausibility)
for all $X, Y, Z, W \in \mathcal{E}$ s.t. $X \subset Y$, $Z \subset W \subset Y$, $W \setminus Z = Y \setminus X$ it holds that
$$X \prec Y \to \neg(Z \sim W).$$

(0A') (Comparative 0-alternation)
for all $X, Y \in \mathcal{E}$ s.t. $X \subset Y$, it holds that
$$X \sim Y \to \neg(X \cup (\Omega \setminus Y) \prec \Omega).$$

Unfortunately, to the best of our knowledge, there exists no purely qualitative characterization of comparative probabilities. This notion seems to have an intrinsically numerical character. Among the possible characterizations proposed in literature, the following one is drawn from [7]:

(CP) (Comparative probability)
for any $X_1, \ldots, X_n, Y_1, \ldots, Y_n \in \mathcal{E}$, with $Y_i \preccurlyeq X_i, \forall i = 1, \ldots, n$, such that for some $r_1, \ldots, r_n > 0$ $\sup\left(\sum_{i=1}^n r_i(a_i - b_i)\right) \leq 0$, implies that $X_i \sim Y_i$, for all $i = 1, \ldots, n$ (where a_i, b_i are the indicator functions of X_i, Y_i, resp.).

Axiom (CP) involves quantitative notions (e.g., indicator functions and summations) and its verification requires numerical elaborations. Nevertheless, it is possible to (qualitatively) state a *necessary*, but not sufficient, condition for representability of an order through a probability function:

(WC) (*Weak* comparative probability)
for all $X, Y, Z \in \mathcal{E}$ s.t. $X \cap Z = Y \cap Z = \phi$, it holds that
$$X \preccurlyeq Y \to \neg(Y \cup Z \prec X \cup Z).$$

Example 2 Let A, B, and C be three companies, each of them potential buyer of a firm that some other company wants to sell. Even being distinct, both A and C belong to the same holding. Hence, the following uncertainty order, about which company will be the buyer could reflect specific information about the companies' strategies (by abuse of notation, let A denote the event "the company A buys the firm", and similarly for B and C): $\phi \prec A \prec B \prec B \cup C \prec A \cup C \prec \Omega$. Since A, B and C are incompatible, the order relation is not representable by a probability because it violates axiom (WC), while it can be managed in line with belief-functions behaviors because it agrees with axiom (B'). □

3 Reasoning tasks for preference orders

In this section we describe two reasoning tasks to be seen as the basic constituents of any expert system or decision-support tool that needs to handle qualitative knowledge in form of comparative assessments.

Qualitative framework detection. This is a classification task: Given a (partial) assessment (which means a description of domain of discernment, constraints, and preferences), the goal consists in detecting which is the most stringent among all compatible uncertainty frameworks. As mentioned, by proceeding in this way, we invert the usual attitude towards qualitative management of uncertainty. In fact, specific axioms are usually set in advance, so that only relations satisfying them are admitted. Here, on the contrary, given a fixed preference relation, the goal consists in ascertain which are the reasonable rules to work with. Considering an assessment as the outcome of a reasoning process performed by an agent (human or not), detecting the correct uncertainty framework provides useful information about the cognitive schema of the agent. This guides one in determining agent's conceptualization of uncertainty (i.e., its way of expressing lack of information and variability of phenomena) and its (implicit) model of the problem at hand. Such a detection process can be, for instance in a multi-agent system, of great help in constructing more informed representation of (other) agents' models of reality. This translates in better strategies in agent modeling, decision making, and plan recognition (i.e., the attempt of inferring the plans of other agents by communicating with them or by observing their behaviors). The following is an example of framework detection.

Example 3 A physician wants to perform a preliminary evaluation of the reliability of a test for SARS (Severe Acute Respiratory Syndrome). Up to his/her knowledge, the diagnosis is based on moderate or severe respiratory symptoms and on the positivity or indeterminacy of a clinical test about the presence of the SARS-associated antibody coronavirus SARS-CoV. The elements appearing in such analysis are: $\mathsf{A} \equiv$ *Normal respiratory symptoms*, $\mathsf{B} \equiv$ *Moderate respiratory symptoms*, $\mathsf{C} \equiv$ *Severe respiratory symptoms*, $\mathsf{D} \equiv$ *Moderate or severe respiratory symptoms*, $\mathsf{E} \equiv$ *Death from pulmonary diseases*, $\mathsf{F} \equiv$ *Positive or indeterminate clinical test*, subject to these (logical) restrictions: $\mathsf{A}\cap\mathsf{B}=\emptyset$, $\mathsf{B}\cap\mathsf{C}=\emptyset$, $\mathsf{A}\cap\mathsf{C}=\emptyset$, $\mathsf{A}\cup\mathsf{B}\cup\mathsf{C}=\Omega$, $\mathsf{D}=\mathsf{B}\cup\mathsf{C}$, $\mathsf{E}\subset\mathsf{C}$, $\mathsf{F}\cap\mathsf{A}=\emptyset$. Due to events' meaning, the w.p.s. $\langle\preccurlyeq,\prec\rangle$ so described: $\emptyset\prec\mathsf{C}$, $\mathsf{C}\prec\mathsf{B}$, $\mathsf{B}\preccurlyeq\mathsf{A}$, $\mathsf{C}\prec\mathsf{D}$, $\mathsf{E}\prec\mathsf{C}$, $\mathsf{E}\prec\mathsf{D}$, $\mathsf{F}\prec\mathsf{A}$, $\mathsf{A}\cup\mathsf{E}\sim\mathsf{A}\cup\mathsf{C}$, seems reasonable. Such w.p.s. agrees with the basic axioms of Sec. 2; but it cannot be managed by using neither a Probability nor a Belief, since it does not satisfies the corresponding axioms. However, one can use comparative plausibility, 0-monotone or 0-alternating functions. (See Example 5 below.) □

Qualitative inference. Strictly related to the previous one, this task consists in exploiting the properties of a partial model to infer new knowledge. This ultimately amounts to finding an extension of a preference relation so as to take into account one or more events extraneous to the initial assessment. Clearly, this should be achieved in a way that the extension retains the same character of the initial order (e.g., both should satisfy the same axioms). More precisely, let be given an initial (partial) assessment expressed as a w.p.s. $\langle \preceq, \prec \rangle$ over set of known events $\mathcal{E}$. Assume that $\langle \preceq, \prec \rangle$ satisfies the axioms characterizing a specific class, say $\mathcal{C}$, of orders. Consider a new event S (not in $\mathcal{E}$), implicitly described by a collection $\mathcal{C}'$ of set-theoretical constraints involving the events of $\mathcal{E}$. In the spirit of [7, Thm. 3], the problem can be formulated as: *Determine which is the "minimal" extension* $\langle \preceq^+, \prec^+ \rangle$ *(over* $\mathcal{E} \cup \{\mathsf{S}\}$*) of* $\langle \preceq, \prec \rangle$*, induced by the new event, which still belongs to the class* $\mathcal{C}$. In other words, we are interested in ascertaining how the new event S must relate to the members of $\mathcal{E}$ in order that $\langle \preceq^+, \prec^+ \rangle$ still is in $\mathcal{C}$. To this aim we want to determine the sub-collections $\mathcal{L}_\mathsf{S}$, $\mathcal{WL}_\mathsf{S}$, $\mathcal{U}_\mathsf{S}$, and $\mathcal{WU}_\mathsf{S}$, of $\mathcal{E}$ so defined:

$$\begin{array}{rll} E \in \mathcal{L}_\mathsf{S} & \text{iff} & \text{no extension } \preceq^* \text{ of } \preceq \text{ can infer that } \mathsf{S} \preceq^* E \\ E \in \mathcal{WL}_\mathsf{S} & \text{iff} & \text{no extension } \preceq^* \text{ of } \preceq \text{ can infer that } \mathsf{S} \prec^* E \\ E \in \mathcal{U}_\mathsf{S} & \text{iff} & \text{no extension } \preceq^* \text{ of } \preceq \text{ can infer that } E \preceq^* \mathsf{S} \\ E \in \mathcal{WU}_\mathsf{S} & \text{iff} & \text{no extension } \preceq^* \text{ of } \preceq \text{ can infer that } E \prec^* \mathsf{S} \end{array}$$

Consequently, in order to satisfy the axioms characterizing $\mathcal{C}$, any w.p.s. $\langle \preceq^+, \prec^+ \rangle$ extending $\langle \preceq, \prec \rangle$ must, at least, impose that:

$$\begin{array}{llll} E \prec^+ \mathsf{S} & \forall E \in \mathcal{L}_\mathsf{S}, & E \preceq^+ \mathsf{S} & \forall E \in \mathcal{WL}_\mathsf{S}, \\ \mathsf{S} \prec^+ E & \forall E \in \mathcal{U}_\mathsf{S}, & \mathsf{S} \preceq^+ E & \forall E \in \mathcal{WU}_\mathsf{S}. \end{array}$$

Example 4 Consider the w.p.s. of Example 3 and the new event $\mathsf{S} \equiv$ *The real state of suffering from SARS*, subject to $\mathsf{S} \subset \mathsf{F}$ and $\mathsf{F} \cap \mathsf{E} \subset \mathsf{S}$. Since in Example 3 we discovered that the initial preference relation satisfies axiom (PL'), we want to impose such axiom and compute the extension of the initial order. We will see in Example 6, that to satisfy (PL'), the following relationships among events have to be verified: $\mathsf{S} \prec \mathsf{A} \cup \mathsf{C}$, $\mathsf{S} \prec \mathsf{A} \cup \mathsf{E}$, $\mathsf{S} \prec \mathsf{D}$, $\mathsf{S} \prec \mathsf{A}$, $\mathsf{S} \prec \Omega$, $\emptyset \preceq \mathsf{S}$, $\mathsf{S} \preceq \mathsf{F}$. In other words, we have $\emptyset \in \mathcal{WL}_\mathsf{S}$, $\{\mathsf{A}\cup\mathsf{C}, \mathsf{A}\cup\mathsf{E}, \mathsf{D}, \mathsf{A}, \Omega\} \subseteq \mathcal{U}_\mathsf{S}$, and $\mathsf{F} \in \mathcal{WU}_\mathsf{S}$. Then, apart from obvious relations induced by monotonicity, no significant constraint involving S can be inferred. Since S and E can be freely compared, this result suggests that, either further investigation about relevance of the clinical test or a revision of the initial preference relation, should be done. □

The availability of automated tools able to extend preference orders, whenever new knowledge is acquired, directly suggests applications in expert systems and decision-support tools. In automated diagnosis, planning, or problem solving, to mention some examples, one could easily imagine scenarios where knowledge is not entirely available from the beginning. We could outline how a rudimental inference process could develop, by identifying the basic steps an automated agent should perform:

0) Acquisition of an initial collection of observations about the object of the analysis, together with a (qualitative) partial preference assessment;
1) Detection of which is the most adequate (i.e., the most discriminant) uncertainty framework;
2) Whenever new knowledge becomes available, refine agent's description of the real world by performing order extension.

The results of step 2) can be used to guide further investigations on the real world and to obtain new knowledge; the process iterates until further pieces of information can be obtained or an enough accurate degree of believe is achieved.

4 *PreC*: Working with preference orders

In this section we briefly describe the tool *PreC*. We start by providing an executable declarative specification of the uncertainty notions introduced in Sec. 2. In doing this we exploit the framework of Answer Set Programming (ASP). A detailed treatment of such a form of logic programming can be found, for instance, in [1]. To fit the purposes of this article, we limit ourselves to present a simplified form of ASP (namely, we do not deal with *classical* negation). In this logical framework, a problem can be encoded—by using a function-free logic language—as a set of properties and constraints which describe the (candidate) solutions. More specifically, an *ASP-program* is a collection of *rules* of the form

$$L_1; \cdots; L_k; not\, L_{k+1}; \cdots; not\, L_\ell \;\; \text{:-} \;\; L_{\ell+1}, \ldots, L_m, not\, L_{m+1}, \ldots, not\, L_n$$

where $n \geqslant m \geqslant \ell \geqslant k \geqslant 0$ and each L_i is an atom A (note that in generic ASP each L_i could be an atom A or the classical negation of an atom $\neg A$). Here "*not*" stands for negation-as-failure.[3] The left-hand side and the right-hand side of the rule are said *head* and *body*, respectively. A rule with empty head is a *constraint*. (Intuitively speaking, the literals in the body of a constraint cannot be all true, otherwise they would imply falsity.)

Semantics of ASP is expressed in terms of *answer sets*. Consider first the case of an ASP-program P which does not involve negation-as-failure (i.e., $\ell = k$ and $n = m$). In this case, a set X of atoms is said to be closed under P if for each rule in P, whenever $\{L_{\ell+1}, \ldots, L_m\} \subseteq X$, it holds that $\{L_1, \ldots, L_k\} \cap X \neq \emptyset$. If X is inclusion-minimal among the sets closed under P, then it is said to be an answer set for P. Such a definition is extended to any program P containing negation-as-failure by considering the *reduct* P^X (of P). P^X is defined as the set of rules of the form $L_1; \cdots; L_k$:- $L_{\ell+1}, \ldots, L_m$, for all rules of P such that X contains all the atoms $L_{k+1}, \ldots, L_\ell$, but does not contain any of the atoms $L_{m+1}, \ldots, L_n$. Clearly, P^X does not involve negation-as-failure. The set X is an answer set for P if it is an answer set for P^X.

Once a problem is described as an ASP-program P, its solutions (if any) are represented by the answer sets of P. Notice that an ASP-program may have none, one, or several answer sets. As a simple example, let us consider the program P consisting of the two rules $p; q$:-. and r :- p. Such a program has two

[3] In ASP syntax, $','$ and $';'$ stand for logical conjunction and disjunction, respectively.

answer sets: $\{p, r\}$ and $\{q\}$. If we add the rule (actually, a constraint) :- q. to P, then we rule-out the second of them, because it violates the new constraint. This simple example reveals the core of the usual approach followed in formalizing/solving a problem with ASP. Intuitively speaking, the programmer adopts a "generate-and-test" strategy: First (s)he provides a set of rules describing the collection of (all) potential solutions. Then, the addition of constraints rules-out all those answer sets that are not desired real solutions. Expressive power of ASP, as well as, its computational complexity have been deeply investigated (see [9], among others). Several ASP-solvers have became available and can be used to find the solutions of an ASP-program [16].

Apart from qualitative probabilities, all axioms in Sec. 2 involve only logical and preference relations and are of direct declarative reading. It is then rather immediate to provide an ASP specification of them. We start by defining in ASP the predicates prec(·,·), precneq(·,·), and equiv(·,·), to render the relators $\preceq$, $\prec$, and $\sim$, respectively. Potential legal answer sets are characterized by asserting properties of prec(·,·), precneq(·,·), and equiv(·,·). (Auxiliary predicates/functions set-theoretical constructs, such as event(·), subset(·,·), diffset(·,·), are introduced and are of immediate reading.) For instance (A3') is rendered by weeding out all answer sets where $X \subseteq Y \wedge Y \prec X$ holds for some X and Y:

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:- event(X), event(Y), subset(X,Y), precneq(Y,X).
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As regards preference classification, let us consider one of the axioms of Sec. 2, say (B'). The following rule is also of immediate reading:

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failsB :- event(X), event(Y), event(Z), event(W),
          subset(X,Y), X!=Y, subset(Z,W), Z!=W, subset(W,Y), W!=Y,
          subset(diffset(W,Z),diffset(Y,X)), equiv(X,Y), precneq(Z,W).
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Namely, the fact failsB is true (i.e., belongs to the answer set) whenever there exist events falsifying (B'). All other axioms can be treated similarly.

When an ASP-solver is fed with such program and a description of a preference relation (i.e., a set of facts of the forms prec(·,·), precneq(·,·), equiv(·,·)), different outcomes may be obtained. Namely, if no answer set is produced, then the input w.p.s. violates some basic requirement, such as axioms (A1')–(A3'). Otherwise, if an answer set is generated, there exists a numerical (partial) model representing the input w.p.s. The presence in the answer set of a fact of the form fails$\mathcal{C}$ (say failsB), witnesses that the corresponding axiom ((B') in the case) is violated. Consequently, the given order (as well as any of its extensions) is not compatible with the uncertainty framework ruled by $\mathcal{C}$.

Example 5 Consider the Example 3. The w.p.s. $\langle \preceq, \prec \rangle$ can be so described in the syntax of ASP:[4]

```
precneq(N,C) :- empty(N).              precneq(C,B). precneq(C,D). precneq(E,C).
equiv(unionset(A,E),unionset(A,C)). precneq(E,D). precneq(F,A). prec(B,A).
```

As expected, if such ASP specification is given as input to Smodels, the an-

[4]For the sake of readability, in Examples 5 and 6 we use the symbols A, B, C, D, E, F, and S to denote specific events. In the real ASP-program such symbols have to be replaced by the proper constants chosen to represent the events at hand.

swer set found includes the facts failsB1 and failsWC, testifying that the given assessment does not satisfy both axioms (B') and (WC). □

In the case of order extension, the input knowledge consists in a set of events together with a collection of logical constraints and preferences, a description of a the new event, and one or more axioms to be imposed. The handling of the imposed axioms is done by ASP-rules of the form:

```
:- holdsB, event(X), event(Y), event(Z), event(W),
   subset(X,Y), X!=Y, subset(Z,W), Z!=W, subset(W,Y), W!=Y,
   subset(diffset(W,Z),diffset(Y,X)), equiv(X,Y), precneq(Z,W).
```

Compare this constraint with the similar rule introduced to implement the classification task. Since a constraint is satisfied when at least one of the atoms in its body is false, by such rule we declare "undesirable" any extension for which an axiom ((B'), in this case) is violated. Intuitively speaking, whenever the fact holdsB is true, in order to satisfy the above rule, at least one of the other facts must be false. (Notice that, these facts are all true exactly when (B') is violated.) In order to activate this constraint (i.e. to impose axiom (B')) it suffices to add the fact holdsB to the input of the solver. In general, more than one extension is possible, hence the collections $\mathcal{L}_\mathsf{S}$, $\mathcal{WL}_\mathsf{S}$, $\mathcal{U}_\mathsf{S}$, and $\mathcal{WU}_\mathsf{S}$ can be obtained by computing the intersection *Cn* of all the answer sets (Or, equivalently, by computing the set of logical consequences of the ASP-program, if this feature is offered by the specific ASP-solver under consideration.) This allows one to detect the minimal extension of the preference relation which is mandatory for each total order.

Example 6 Consider the Example 4. The following facts belong to each answer set and are obtained by filtering Smodels' output: precneq(S,AC), precneq(S,AE), precneq(S,D), precneq(S,A), precneq(S,O), prec(E,S), prec(S,F), where AC, AE, O, and E are instantiated to the events A∪C, A∪E, Ω, and ∅, respectively. □

The executable specifications we outlined in this section (together with the ASP-solver and a C-library of functions designed to efficiently handling sets and operations on sets) constitute the core inference-engine of the prototypical tool *PreC*. This tool is aimed at assisting the user in interactively dealing with (partial) preference orders and qualitative uncertainty. *PreC* offers an user-friendly and mouse-oriented interface to input, modify, and manage assessments; to activate the reasoning tasks; and to handle order extensions as described in the previous sections. In this manner the user does not interact directly with the ASP-solver and does not handle any ASP-specification. Actually, due to the declarative nature of the ASP rules, it is rather immediate to plug a (new) specification of an axiom within the application. Moreover, both the solver and the ASP code could be changed or improved in a transparent manner to improve and extend the tool. Figure 1(l) depicts the main window of *PreC*, through which the user can describe, manage, and modify his/her assessments. The visualization of preference orders in form of graphs (cf., Figure 1(r)) allows the user to modify or extend the set of events, for instance in preparation of the execution of one of the inference tasks of Sec. 3.

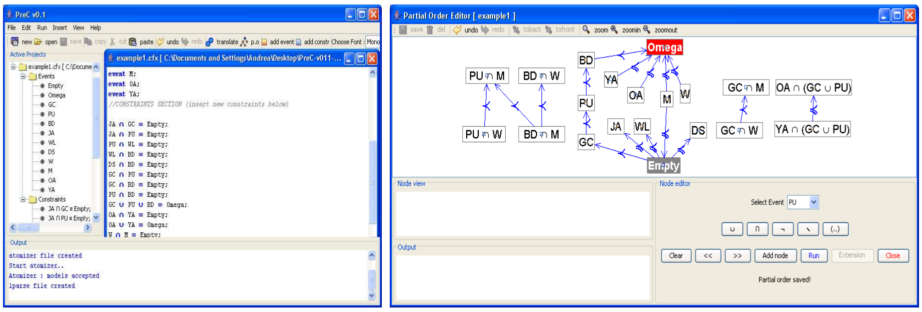

Figure 1: Browsing (l) and visual management (r) of preferences in *Prec*.

Conclusions

In this article we proposed a straightforwardly translation of partial preference axiomatizations into an executable specification in a declarative programming framework. This allowed us to explore the potentialities offered by Answer Set Programming for building decision support systems based on qualitative judgments. Hence, an implementation of what could be thought as a kernel of an inference engine sprouted almost naturally. Moreover, the highly declarative character of the encoding of the axioms into executable ASP-specifications makes it possible to easily modify and extend the treatment to deal with new notions of uncertainty. Similarly, the discovery of alternative axiomatizations of an uncertainty notion can be immediately "plugged-in" the framework just modifying or adding suitable ASP rules (consider, for instance, the case of comparative Probabilities, for which no qualitative characterization has been found to date). Alternatively, a challenging goal for future research consists in completing our approach so as to handle comparative Probabilities through integration with efficient numerical approaches such as linear optimization tools (e.g., column generation techniques, cf. [13], among others). More in general, we envisage the design of a full-blown automated system which integrates different (in some way complementary) techniques and methods for uncertainty management; comprehending mixed numerical/qualitative assessments and extending the range of applicability to conditional probability frameworks [8].

References

[1] C. Baral. (2003) *Knowledge representation, reasoning and declarative problem solving.* Cambridge University Press.

[2] R. I. Brafman and M. Tennenholtz. (1997) Modeling agents as qualitative decision makers. *Artif. Intel.*, **94**, 217–268.

[3] A. Capotorti, G. Coletti, and B. Vantaggi. (1998) Non additive ordinal relations representable by lower or upper probabilities. *Kybernetika*, **34** (1), 79–90.

[4] A. Capotorti and A. Formisano. (2006) Management of Uncertainty Orderings through ASP. In B. Bouchon-Meunier, G. Coletti, R. R. Yager, eds., *Modern Information Processing: From Theory to Applications*, Elsevier.

[5] A. Capotorti and A. Formisano. (2006) Qualitative uncertainty orderings revised. To appear in *Electron. Notes Theoret. Comput. Sci.*

[6] A. Capotorti and B. Vantaggi. (2000) Axiomatic characterization of partial ordinal relations. *Internat. J. Approx. Reason.*, **24**, 207–219.

[7] G. Coletti. (1990) Coherent qualitative probability. *J. Math. Psych.*, **34**, 297–310.

[8] G. Coletti, B. Vantaggi. (2006) Representability of Ordinal Relation on a Set of Conditional Events. *Theory and Decision*, **60**, 137–174.

[9] E. Dantsin, T. Eiter, G. Gottlob, and A. Voronkov. (2001) Complexity and expressive power of logic programming. *ACM Comput. Surveys*, **33** (3), 374–425.

[10] G. de Cooman. (1997) Confidence relations and ordinal information. *Inform. Sci.*, **104**, 241–278.

[11] D. Dubois, H. Fargier, and P. Perny. (2003) Qualitative decision theory with preference relations and comparative uncertainty: An axiomatic approach. *Artif. Intel.*, **148**, 219–260.

[12] C. R. Fox. (1999) Strength of evidence, judged probability, and choice under uncertainty. *Cognitive Psych.*, **38**, 167–189.

[13] B. Jaumard, P. Hansen, and M. Poggi de Aragão. (1991) Column Generation Methods for Probabilistic Logic. *J. on Computing*, **3** (2), 135–148.

[14] P. Walley and T. Fine. (1979) Varieties of modal (classificatory) and comparative probability. *Synthese*, **41**, 321–374.

[15] S. K. M. Wong, Y. Y. Yao, P. Bollmann, and H. C. Bürger. (1991) Axiomatization of qualitative belief structure. *IEEE Trans. Sys. Man. Cyber.*, **21**, 726–734.

[16] Web sites for ASP. DLV: `www.dbai.tuwien.ac.at/proj/dlv`, Cmodels: `www.cs.utexas.edu/users/tag/cmodels`, Smodels: `www.tcs.hut.fi/Software/smodels`.