ON PROBABILISTIC MODELING IN FAULT DETECTION

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Abstract

This paper provides a unified background of probabilistic modeling methods which should serve to model-based fault detection (FD). This background should help in orientation within within the set of the problems to be addressed when developing a novel method as well as within a wide set of available methods and in recognition of their suitability to a particular problem in hands.

1 Introduction

Early recognition/isolation of a faulty behavior of a dynamic system is the main task of fault detection (FD). Its importance, which cannot be exaggerated, stimulated a lot of engineering/research activities which led to a chain of general as well as specific results. For reviews see [1, 2]. Availability of plenty of clever methods and their variants makes the choice of an appropriate method and its tuning difficult. A lack of a unified formalized framework is partially responsible for it. This paper makes a step in this respect. The presented, seemingly abstract, formulation help us to understand the individual elements involved and put a proper perspective on them.

2 Role of modeling in FD

The final goal considered is model-based fault detection. Definitely, the *aim of modeling* has to be considered in the model building since it determines the desired accuracy and complexity of the model. The model inaccuracy can be and will be represented as the uncertainty of the model. Traditionally, modeling (in a broad sense) includes a description of deterministic relationships (modeling in a narrow sense) as well as disturbance representations, which are adequate with respect to the modeling goal and reflect knowledge incompleteness. The key elements of the model-based fault detection are:

Modeling of relationships of observable data, system state and fault labels varies according to the degree and way of knowledge exploitation.

White box models describe the relationships using available physical knowledge (first engineering principles) *only.*

Note that robust models may belong to this class if the model structure and uncertainty boundaries are set using a priori physical knowledge only.

Grey box models tend to describe these relationships in the same manner as white box models do. However, for various reasons, the description is incomplete and some parts of the system state remain unknown. The consequences of the incomplete knowledge have to be suppressed by a learning process. Learning can be viewed as an experience accumulation with the help of the observed data and other information sources. Temporal changes of the learned unknowns have to be much slower than the learning process itself. Otherwise, no success can be expected. This implies that the unknown quantities may undergo only known changes. Typically, they are supposed not to change at all. Then they are called parameters.

Black box models face an incomplete knowledge similarly as grey box models do. They are constructed without relying on physical insight. Instead of it, a sufficiently rich parameterized class of models is used. It should have "universal" approximation property, i.e. the ability to describe a sufficiently wide class of studied relationships.

Description of uncertainties should respect **objective & subjective** ingredients. Incomplete knowledge met in modeling can be labeled as subjective uncertainty as it can be (potentially) removed by subject's learning. Objective uncertainties are also present in the majority of the real cases. These uncertainties cannot be removed completely. Their "expected" influence can only be considered when designing a fault detection rule. In fact, they do not differ from the subjective uncertainties in an operational sense. At least during the learning period, they can be treated identically with subjective uncertainties.

The concept of grey box modeling has emerged in many sub-fields, often independently. Its handling, however, differs according to the background of the model builder. Here, we restrict ourselves to *probabilistic paradigm*: All uncertainties are treated as random variables and described in probabilistic terms [3]. Prior information on the physical phenomena behind the data can be taken into account both by selecting the appropriate parameterized systems model and by setting a suitable prior distribution to the parameters. For computational reasons, the parameterized model is often of black box type and the prior distribution is used only for enforcing physical constraints on the parameters involved [4].

3 Fault detection problem

This section formulates fault detection in a general formal way which allows us to understand the problem better. The presentation essentially translates the general decision-problem under uncertainty [5] to FD.

3.1 Formalization of fault detection

The following general notation is adopted:

Sets are marked by *. It means X^* is a set of all Xs.
Time is denoted by $t, \tau \in t^*$ where $t^* = \{0, 1, \ldots, T\}$ in the discrete time case and $t^* = [0, T]$ in the continuous time case. The horizon T may be infinite.
Trajectory of a variable x_τ on the closed time interval $0 \le \tau \le t$ is denoted $X[t]$. The symbol $X[t)$ is used if the half-closed time interval $0 \le \tau < t$ is considered.
Symbol $p(\bullet)$ denotes a probability (density) function (p(d)f); $p(\bullet|\bullet)$ is a conditional version.
Indicator symbol $\chi_{Y^*}(\bullet)$ denotes a characteristic function of the set $Y^* \subseteq \Re^Y$.

The problem formulation uses the following elements:

System $\mathcal{S}$ is a part of the real world we are interested in, here, in order to detect faults. Some data trajectories $D[T] \in D[T]^*$ are observed on it. They are used for deciding whether the system behavior differs substantially from a normal one. Formally, $\mathcal{S} : (X[T], D[T])^* \to (X[T], D[T])^*$ where $X[T]$ is a trajectory of the (internal) state x_t of the system.

The system is specified by the user who asks for solving the fault detection/isolation problem. The specifications of system "boundaries", of the measured data and their sampling, influence definitely the final detection problem. They are assumed to be given for the task addressed.
State classificator $\mathcal{C}$ labels errors $f_t \in f_t^* \equiv \{\text{non-faulty}, \text{fault}_1, \ldots, \text{fault}_{m_F}\}$ to the system state. Formally, $\mathcal{C} : X[T]^* \to F[T]^*$. The trajectory $F[T]$ consists of faulty-state labels f_t.

The error classification is a part of the technical problem specification provided by the user.
Model $\mathcal{M}$ describes formally the system behavior by a mapping $\mathcal{M} : (X[T], D[T])^* \to (X[T], D[T])^*$ which relates unknown system state and observed data. It serves as an auxiliary tool for deciding whether the system is in a good or faulty state. Its construction is a key discussed part of the detection design.
Detection rule or detection strategy $\mathcal{R}[T] \equiv \mathcal{R}(\hat{F}[T], D[T])$ is a collection of mappings $\mathcal{R}_t : (\hat{F}[T], D[T])^* \to \hat{f}_t^*,\ t \in t^*$. It generates estimates $\hat{f}_t$ of the fault labels f_t without knowing the internal state of the system and the "true" labels of faults. The set of possible label estimates equals to the set of faults.
Causal detection rule is a rule fulfilling

$$\hat{f}_t \equiv \mathcal{R}_t(\hat{F}[T], D[T]) = \mathcal{R}_t(\hat{F}[t), D[t)),\ t \in t^* \text{ where}$$

$\hat{F}[t)$ is the restriction of the trajectory $\hat{F}[T]$ to time instants before t, $\{0 \le \tau < t\}$. $D[t)$ is defined similarly.

The (usually causal) detection rule is the main outcome of the fault detection design. Modeling activities discussed here serve to its construction.
Ranking of detection rules $\mathcal{O}$ provides preferences among different detection rules. Usually, it is specified by a loss function measuring a distance of the involved trajectories of the true labels and their estimates:

$$\mathcal{O} : \mathcal{C}(X[T])^* \otimes \mathcal{R}(\hat{F}[T], D[T])^* \to [0, \infty].$$

The interpretation of the loss-function-specified rating is as follows. For a given system state and observed data realization $(X[T], D[T])$, a detection rule $\mathcal{R}$ acts better than the detection rule $\tilde{\mathcal{R}}$ if

$$\mathcal{O}(\mathcal{C}(X[T]), \mathcal{R}(\hat{F}[T], D[T])) \le \mathcal{O}(\mathcal{C}(X[T]), \tilde{\mathcal{R}}(\tilde{\hat{F}}[T], D[T]))$$

The rating reflects user's wishes and as such it is provided by him. The designer of the FD strategy has to take care of the feasibility of the tasks influenced by this option.

3.2 The problem formalization

A systematic design of the best or at least good fault detection strategy is the final technical problem addressed. The design is to be based on the elements specified above. They do not allow us to give a consistent formulation yet. Any understanding of the "best" or a "good" strategy is inherently based on the distance of a considered strategy behavior to the optimum one. It cannot, however, be measured as the system state (and thus correct classification) is unknown *a priori* and often even a posteriori.

Two interrelated activities are used to overcome this:

Modeling (in a narrow sense) quantifies the relationships of the observed data to the (unknown) system state or at least to images of sets corresponding to various fault types. Ideally, it should replace the missing information on the trajectory of the system state.
"Expectation"- based rating is used for completion of the partial rating induced by $\mathcal{O}$. For each trajectory of data and labels generated by a detection rule,

$\hat{F}[T] \equiv \mathcal{R}(\hat{F}[T], D[T])$, $\mathcal{O}(\mathcal{C}(X[T]), \hat{F}[T])$ is a function of $X[T]$ belonging to the set $\mathcal{O}^* = \{X[T]^* \to [0, \infty]\}$. It is completely ordered by an "expectation"

$$\mathcal{E} : \mathcal{O}^* \to [0, \infty].$$

It means that the values of the rating $\mathcal{O}$, which are unknown because of the lack of knowledge about labels $\mathcal{C}(X[T])$, are "replaced" by their "expected" values. This *completion of the partial rating* is a specialized sub-part of modeling, the *modeling of uncertainties*.

These steps cover **modeling in a broad sense**.

With the given elements and the selected "expectation" operator, *the design of the best detection rule* $\mathcal{R}^{[o]}$ *becomes the optimization task*

$$\mathcal{R}^{[o]} \in Arg \min_{\mathcal{R} \in \mathcal{R}^*} \mathcal{E}\mathcal{O}(C(\bullet), \mathcal{R}(\bullet, \bullet)).$$

3.3 Free options

The presented general formulation of fault detection offers some free options. They can be categorized as:

Model of deterministic relationships describes connections of involved quantities, i.e. oobserved data and unknown state. It may fall in the white, grey as well as black box category. The adequate option is dictated by the available knowledge and by the restrictions on the complexity of the final model and, consequently, of the optimal fault detection rule.

Model of uncertainties introduces a specific description of incomplete knowledge. There is no unique way of "substituting" expectations instead of unknown quantities, no unique way of describing uncertainties. A consistent way should, however, fulfill:

Monotonicity requirement If $\forall\ X[T] \in X[T]^*$ and given $\mathcal{R}$, $\tilde{\mathcal{R}}$ holds

$$\mathcal{O}(\mathcal{C}(X[T]), \mathcal{R}(\hat{F}[T], D[T])) \leq \mathcal{O}(\mathcal{C}(X[T]), \tilde{\mathcal{R}}(\hat{\tilde{F}}[T], D[T]))$$
$$\text{then} \qquad \mathcal{E}[\mathcal{O}(\mathcal{C}(\bullet), \mathcal{R}(\bullet, \bullet))] \leq \mathcal{E}[\mathcal{O}(\mathcal{C}(\bullet), \tilde{\mathcal{R}}(\bullet, \bullet))].$$

Without meeting this condition, we might take as the optimal a detection rule which is uniformly worse than another detection rule. This requirement leaves still a lot of freedom in choosing the expectation operator. The arbitrariness can be decreased by:

Objectivity requirement

> The selected description of uncertainties should be (almost) independent of the rating $\mathcal{O}$, i.e. it should be applicable to a rich set of ratings.

Unlike the previous one, this requirement is not an absolute necessity. It reflects either a pragmatic viewpoint (a model should serve for various criteria) or an objectivist's one (there are uncertainties inherent to the considered system and state of the knowledge about it).

4 Probabilistic modeling

Under general technical conditions, the monotonicity and objectivity requirement single out so called *stochastic rating* [6, 7] induced by the *mathematical expectation*, i.e. by a positive linear functional on possible ratings.

$$\mathcal{E}[\mathcal{O}(\bullet)] \equiv \int \mathcal{O}(\mathcal{C}(X), \mathcal{R}(\hat{F}, D))\, p(X, \hat{F}, D)\, dX\, d\hat{F}\, dD.$$

The time argument is suppressed in order to see the structure of operations. The symbol $\int$ denotes multivariate integration over the set $(X, \hat{F}, D)^*$. The weighting factor $p(\cdot)$ is *probability density function* (pdf). For simplicity, the same notation and terms are used even for discrete valued arguments for which the integration reduces to summation and $p(\cdot)$ is the *probability function* (pf). Note that such pdf is justified by the solved decision problem, not by frequency considerations.

For sake of simplicity, we shall mostly restrict ourselves to the discrete-time case with at most countable time set $t^* = \{0, 1, \ldots, T\}$. Recall that the values of particular quantities at particular time-instants $\tau, t \in t^*$ are denoted x_τ for $X[T]$ and similarly for other variables.

Proposition 1 [Algebra of pdfs] *It holds*

Chain rule: $p(X[T], \hat{F}[T], D[T]) = \prod_{t=0}^{T}$

$$p(d_t|X[t], \hat{F}[t], D(t))p(f_t|X[t], \hat{F}[t), D(t))p(x_t|X[t), \hat{F}[t), D(t))$$

Marginalization:
$p(\hat{F}[T], D[T]) = \int p(X[T], \hat{F}[T], D[T])\, dX[T]$.

Bayes rule: $p(x_0|F[0), D[0)) \equiv p(x_0)$, $p(x_t|\hat{F}[t], D[t]) \propto$

$$p(d_t|x_t, \hat{F}[t], D(t))p(f_t|x_t, \hat{F}[t), D(t))p(x_t|\hat{F}[t), D(t))$$

4.1 Demands implied by design

Let us consider the design of *causal* detection rules. The optimization of the expected rating is described by the following proposition whose proof is based on monotonicity of the expectation and chain rule only:

Proposition 2 [Dynamic programming]
Let the indicated operations be well defined. Then, the strategy $\mathcal{R}^{[o]}[T]$ *generating the minimizing arguments in*

$$\mathcal{H}(t-1) = \min_{\hat{f}_t \in \hat{f}_t^*} \mathcal{E}[\mathcal{H}(t)|\hat{F}[t], D[t]] \text{ is } \mathcal{E} - \text{optimal}.$$

$\mathcal{H}(T) \equiv \mathcal{O}(\bullet)$ *starts the backward recursion.*

At the time T, the evaluation of the expectation in the dynamic programming requires to know the pdf

$$p(X[T], d_T|\hat{F}[T], D(T)) = \tag{1}$$
$$= \quad p(X[T]|\hat{F}[T], D[T])p(d_T|\hat{F}[T], D(T)).$$

The right-hand side of (1) is implied by the chain rule.

For any $t < T$, the evaluation of the expectation in the dynamic programming requires to know the pdf

$$p(d_t|\hat{F}[t], D[t)).$$

This pdf is *one-step ahead predictor of data* trajectory. It is object is denoted by $\mathsf{p}(d_t; \hat{f}_t) \equiv p(d_t|\hat{F}[t], D[t))$ ($\propto$ denotes proportionality). This main input of the design is gained from the modeling (in a broad sense). It is discussed in next section dealing with $p(X[T]|\hat{F}[T], D[T])$.

4.2 Filtering and prediction

The factor $p(X[T]|\hat{F}[T], D[T])$ entering the design of the optimal fault-detection strategy can be interpreted as an estimate of the unknown-state trajectory $X[T]$. This section outlines how it can be constructed from simpler elements. Essentially, state filtering and prediction tasks are solved. The inspection of general (non-causal) fault detection rules would lead to the task of smoothing, too.

The chain rule implies the following factorization of the pdf of interest $p(X[T]|\hat{F}[T], D[T]) \propto \prod_{t=1}^{T}$

$$\underbrace{p(d_t|X[t], \hat{F}[t], D[t))}_{\textbf{c)}}\underbrace{p(f_t|X[t], \hat{F}[t), D[t))}_{\textbf{b)}}\underbrace{p(x_t|X[t), \hat{F}[t), D[t))}_{\textbf{a)}}$$

The factor a) The quantity x_t can be called system state if its value determines fully the corresponding future trajectory. Formally, it means that

$$p(x_t|X[t), \hat{F}[t), D[t)) = p(x_t|x_{t-1}) \equiv \mathsf{m}(x_t; x_{t-1}).$$

The function m which describes time-evolution of the system state can be called *state model* and has to result from the system modeling.

The factor b) The pdf $p(\hat{f}_t|X[t], \hat{F}[t), D[t))$ describes the probability with which a fault label is selected. It extends detection rules to *randomized detection rules*.

As the system states are assumed to be inaccessible to the considered rules the rules have to fulfill so called *natural conditions of control* [3]

$$p(\hat{f}_t|X[t], \hat{F}[t), D[t)) = p(\hat{f}_t|\hat{F}[t), D[t)) \equiv \mathsf{r}(\hat{f}_t).$$

The factor c) It relates the observable data to the unknown system states. The definition of the state implies that d_t is fully determined by $\hat{f}_t$ and x_t only.

$$p(d_t|\hat{F}[t], X[t], D[t)) = p(d_t|\hat{f}_t, x_t) \equiv \mathsf{o}(d_t; \hat{f}_t, x_t).$$

This *observation model* is the main input to be supplied from modeling. The given *state model* $\mathsf{m}(\bullet)$, the *initial pdf of the state* $p(x_0) \equiv \mathsf{i}(x_0)$ and the *observation model* $\mathsf{o}(\bullet)$ determine the predictor $\mathsf{p}(\bullet)$ (see next Proposition).

> The pdfs m, o, i, forming the complete probabilistic model of the system, are the key ingredients needed for the determination of the optimal (randomized) fault-detection rule r.

Proposition 3 [Filtering and prediction] *Under the natural conditions of control, the* output predictor *is*

$$\mathsf{p}(d_t; \hat{f}_t) = \int \mathsf{o}(d_t; \hat{f}_t, x_t)\mathsf{s}(x_t)\, dx_t$$

where the state predictor *is described by* $\mathsf{s}(x_0) = \mathsf{i}(x_0)$ *and*

$$\begin{aligned}\mathsf{s}(x_{t+1}) &\equiv p(x_{t+1}|\hat{F}[t], D[t]) = \\ &= \frac{\int \mathsf{m}(x_{t+1}; x_t)\mathsf{o}(d_t; \hat{f}_t, x_t)\mathsf{s}(x_t)\, dx_t}{\mathsf{p}(d_t; \hat{f}_t)}.\end{aligned}$$

Note the non-standard dependence of models on the decision made: formally, it is often omitted, however, practically our decision of fault state leads to related action (e.g. stopping of the production process).

5 Problems of modeling

This section discusses practical problems of the probabilistic modeling and their possible remedies. The crucial problem of mismodeling when the system $\mathcal{S}$ is not in the set of models $\mathcal{M}^*$ is discussed in the next section.

5.1 Completeness and complexity

The *complete probabilistic model* consists of the state model $\mathsf{m}(x_t; x_{t-1})$, of the pdf of the initial state $\mathsf{i}(x_0)$ and of the observation model $\mathsf{o}(d_t; \hat{f}_t, x_t)$. Its construction does not belong to the standard engineer's tool set.

At the same time, the deterministic modeling is a well developed art which provides equations relating the involved quantities. Typically, the relationships are given in the form of parameterized differential equations

$$dx_t = \mathcal{F}(x_t, \theta, u_t)\, dt, \quad y_t = \mathcal{G}(\theta, x_t)$$

where $\mathcal{F}$ and $\mathcal{G}$ are nonlinear vector functions. Data items $d_t = (u_t, y_t)$ $t \in t^*$ are split into the system inputs u_t (manipulable part of data) and system outputs y_t. The introduced unknown parameters θ can be interpreted as a time-invariant part of the state.

The random influences (objective uncertainties) in the state equation can be relatively universally introduced by interpreting the state equations as stochastic differential equations. External influences are taken as a part of the system state. The random sources have to be Wiener-type process [8] if the requirement of a stationary influence of the system environment is satisfied and x_t is the system state. The distribution of $X[T]$ results from the transformation of this process by the system dynamics.

The choice of the initial pdf $\mathsf{i}(x_0)$ and of the observation model $\mathsf{o}(d_t; f_t, x_t)$ is less definite. Mostly, a range of the involved variables can be a priori judged and expectation of some simple functions of the involved quantities guessed. For instance, the output equation is extended to

the stochastic case by relating the expected output to the parameters and state

$$\mathcal{E}[y_t|\theta, x_t] = \mathcal{G}(\theta, x_t).$$

The options available for the completion to full pdfs are:

- Black box type approach selects a rich class of pdfs parameterized by this expectation and some additional parameters, say σ. Consequently, the overall model contains the unknown parameters $\Theta = (\theta, \sigma)$.
- The observation o (initial i) pdf is found by optimizing some additional criterion (like entropy) within the class restricted by the given form of the expectation.
- Whole set of pdfs is considered in one shot, e.g. if they are specified by supports only, the problem can formally be described by fuzzy type evaluations [9].

All these *completion procedures can be understood as a sort of "smooth" interpolation between firm knowledge items.*

It is fair to say that the problem of *computational complexity* drives the modeling art in majority of cases. It stems from the fact that probability density functions are infinite-dimensional objects which either have to be treated analytically or handled approximately in digital computer. To be more specific let the considered system be described by the uncontrolled Itô differential equation

$$dx_t = \mathcal{F}(x_t, t)\, dt + \mathcal{H}(x_t, t)\, d\beta_t$$

where $\mathcal{F}$, $\mathcal{H}$ are sufficiently smooth vector functions and β_t is the vector Wiener process with the unit intensity.

Under mild technical conditions, it can be shown that this equation generates Markov process (x_t is state) with the transition pdf (state model) $\mathsf{x}(x_t; x_\tau)$, $\tau < t$, solving the forward Kolmogorov equation

$$\frac{\partial \mathsf{x}(x_t; x_\tau)}{\partial t} = \mathcal{L}(\mathsf{x}(x_t; x_\tau)).$$

The introduced forward diffusion operator $\mathcal{L}$ has the form

$$\mathcal{L}(\bullet) = -\nabla_{x_t}(\bullet\mathcal{F}) + 0.5\nabla^2_{x_t}(\bullet\mathcal{H}'\mathcal{H})$$

where ∇ denotes gradient, ∇^2 the Laplace operator and $'$ transposition. The same equation is fulfilled by the marginal pdf $p(x_t)$ of the state x_t.

This form demonstrates explicitly that even for systems with a finite dimensional state (lumped parameter case), the probabilistic description requires to solve partial differential equation. The *modeling and consequently estimation and fault-detection problems are generally computationally hard.* Possible ways out are discussed below.

5.2 Analytically tractable cases

For the fault detection design, the (state) estimation problem has to be solved. For specificity, let us think of the most realistic case of continuous time system and discrete time measurements (digital processing).

Having a given state estimate $p(x_t|\hat{F}[t], D[t])$, the measurement updates it at time t according to the Bayes rule

$$p(x_t|\hat{F}[t], D[t]) \propto \mathsf{o}(d_t; \hat{f}_t, x_t) p(x_t|\hat{F}[t), D[t)).$$

The state estimate evolves according to the Kolmogorov equation between measurement moments. Thus, its solution is a part of the estimation.

It is known that the separation of variables is the only sufficiently general technique for analytic solution of such equations. This observation forms the basis of the analysis made in [10] where the analytically tractable cases were characterized. Essentially, the analytic solution – if it exists – belongs to the so called exponential family

$$p(x_t|\hat{F}[t), D[t)) = \exp[s'(x_t) h(\hat{F}[t), D[t))]$$

where s, h are finite-dimensional vector functions acting on x_t^* and $(\hat{F}[t), D[t))^*$.

It restricts very substantially the tractable system models. The linear-Gaussian model (LG) is a dominant member of this set. LG model is described by the linear time-variant stochastic differential equation

$$dx_t = (A_t x_t + B_t u_t)\, dt + H_t d\beta_t, \quad y_t = C_t x_t + G_t \beta_t$$

where A, B, H, C, G are *known functions of time.* The state estimation is described by Kalman filter [8].

Note, that even a partial lack of knowledge of some entries in A_t, B_t, H_t, C_t, G_t makes the analytical treatment impossible. These entries become the part of the state. Consequently, products of state entries occur in the model.

6 Local and multiple modeling

Previous section dealt with the standard but unrealistic assumption that the system $\mathcal{S}$ belongs to the assumed model set $\mathcal{M}^*$. But at least the model approximations, necessary for reaching the computation feasibility, cause that this conditions is violated as a rule. It can be shown [6] that the predictor constructed is in a sense the best possible within the model class considered. The best, however, need not be good enough: the error is given by the relationship between reality an model set. In this respect, only an improved modelling may help but up the boundary created by the "curse of dimensionality". The best we can hope for are *locally valid models.* They are, however, insufficient for fault detection as it inspect just the situations when the modelled quantities go out of their usual range. It means that a bunch of local models is needed for fault detection purposes: this adds an explanation to the fact why multi-model approaches are so popular for solving fault detection tasks.

Usually, single local (tractable) model is sufficient for describing single faulty mode. Then, we can use a novel quasi-Bayes estimator of mixtures [11] for detecting faults

at least for input-output models parameterized by unknown parameters $\theta^{[f_t]}$ and "selected by the unknown time-varying fault label f_t occurring with unknown probability α_{f_t}. Let us sketch this case. The considered observation model is $\prod_{f\in f^*}[\mathsf{o}(d_t;\hat{f}_t,\theta^{[f]})]^{\delta(f,f_t)}$ where Kronecker δ is used. Let the prior pdf on unknown $\Theta = [\theta^{[1]},\dots\theta^{[f[])},\alpha_1,\dots\alpha_{m_{f^*}}]$ has a product form

$$\mathsf{s}(\Theta,f_{t+1}) = \prod_{f\in f^*}\mathsf{s}(\theta^{[f]})\alpha_f^{\kappa_f+\delta(f_{t+1},f)-1}$$

with some pdfs $p(\theta^{[f]}|\hat{F}[t+1),D[t+1)) = \mathsf{s}(\theta^{[f]})$ and positive scalars κ_f. The Bayes rule gives the posterior pdf

$$p(\Theta,f_{t+1}|\hat{F}[t+1],D[t+1]) \propto \prod_{f\in f^*}\mathsf{s}(\theta^{[f]})^{\delta(f_{t+1},f)}\alpha_f^{\kappa_f+\delta(f_{t+1},f)-1}$$

By Marginalization we get probabilities of particular faults

$$p(f_{t+1}=f|\hat{F}[t+1],D[t+1]) \propto \mathsf{p}(d_{t+1};\hat{f}_{t+1})\kappa_f$$

where p is predictor computed for the fth parameterized model as given in Proposition 3.

It remains to update the probabilities on individual models. The exact evaluation destroys the nice product form. The used quasi-Bayes approach [11] replaces simply the unobservable $\delta(f_{t+1},f)$ by its conditional expectation, i.e. by $w_{t+1}(f)\equiv p(f_{t+1}=f|\hat{F}[t+1],D[t+1])$. Then the product form is preserved and updating is performed by the weighted Bayes rule

$$p(\theta^{[f]}|\hat{F}[t+1],D[t+1]) \propto [\mathsf{s}(\theta^{[f]})]^{w_{t+1}(f)}p(\theta^{[f]}|\hat{F}[t+1),D[t+1)).$$

The "counters" κ_f are increased by $w_{t+1}(f)$: i.e. all models are updated with a weight on data that is proportional to probability that the model should be applied at this moment. The point estimate of probability of occurrence of fth error can be shown to be proportional to κ_f.

The procedure is tractable when analytically tractable parameterized models are considered as factors.

7 Conclusions

This paper inspects modelling for fault detection purposes.

The problem is studied from the probabilistic perspective. It underlines less stressed fact that the trajectory of occurrences of faults has to be modelled.

The probabilistic description is undoubtedly the most general and universal one. It suffers, however, from computational burden related to it. For this reasons, available options for reaching acceptable degree of computational complexity are judged. Use of multiple local models taken from analytically tractable classes seems to be most promising. Essentially, single local model is attached to each among finite amount of faulty-states. Then, the complexity problem become the problem of on-line estimation of the overall model obtained by a random jumps between them. It is, however, solvable by a novel quasi-Bayes estimation of mixtures.

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